IJPAS | INTERNATIONAL JOURNAL PURE AND APPLIED SCIENCES
ISSN: 2149-0910

Vol. 10(1); 283-289 (2024)
DOI: 10.29132/ijpas.1480871

Research Article

Teleparallel Energy Density within the Framework of Rainbow Gravitation Theory for A Spatial Self-Similar, Local Rotational Symmetric Model

Sibel KORUNUR [1,*] and Murat KORUNUR [2]

Received: 08.05.2024

Accepted: 16.06.2024

[1] Tunceli Vocational School, Munzur University, Tunceli, Türkiye; skorunur@munzur.edu.tr

[2] Faculty of Engineering, Munzur University, Tunceli, Türkiye; mkorunur@munzur.edu.tr

*Corresponding author

Abstract: It is known that the general theory of relativity provides valuable answers about our universe. General relativity theory is used to describe space, time, and mass-energy interactions, while quantum theory is used to explain the behavior and interactions of microscopic particles. The gap between these two theories reveals the need to develop a unified theory of "quantum gravity". However, so far no universal theory has yet been found that fully resolves this conflict. This is a big puzzle that physicists have been working on for a long time, and unifying these two theories harmoniously is one of the biggest challenges in modern physics. One of the theories put forward for this purpose is the "Rainbow" theory of gravity. In this study, Einstein, Bergmann-Thomson and Landau-Lifshitz energy densities are calculated for a spatial self-similar, locally rotationally symmetric model using teleparallel geometry within the framework of the Rainbow theory of gravity. However, the results obtained are evaluated using rainbow functions that are well known in the literature. The obtained results are rewritten as explicit forms of energy densities for Einstein, Bergman-Thomson and Landau-Liftshitz representations using $f_1(\chi) = 1/(1-\chi)$ and $f_2(\chi) = 1$ rainbow functions. Accordingly, it has been shown that the test particle changes its energy density for the Einstein and Bergmann-Thomson energy-momentum prescriptions but does not change the energy density for the Landau-Liftshitz energy-momentum prescription.

Keywords: Teleparallel theory; rainbow theory of gravity; energy-momentum

Uzaysal Öz-Benzer, Yerel Dönel Simetrik Model için Gökkuşağı Kütleçekim Kuramı Çerçevesinde Teleparalel Enerji Yoğunluğu

Özet: Genel görelilik kuramının evrenimiz hakkında çok değerli cevapları verdiğini biliyoruz. Genel görelilik, uzayı, zamanı ve kütle-enerji etkileşimlerini açıklamak için kullanılırken, kuantum kuramı, mikroskobik parçacıkların davranışını ve etkileşimlerini açıklamak için kullanılır. Bu iki kuram arasındaki uçurum, birleşik bir teori olan "kuantum kütleçekimi" teorisinin geliştirilmesi gerekliliğini ortaya koyar. Ancak, şu ana kadar bu çatışmayı tam olarak çözen evrensel bir teori henüz bulunamamıştır. Bu, fizikçilerin uzun zamandır üzerinde çalıştığı büyük bir bulmacadır ve bu iki kuramın uyumlu bir şekilde birleştirilmesi, modern fizikteki en büyük zorluklardan biridir. Bu amaca yönelik olarak ortaya konan kuramlardan birisi de “Gökkuşağı” kütleçekim kuramıdır. Bu çalışmada Gökkuşağı kütleçekim kuramı çerçevesinde tele paralel geometri kullanılarak uzaysal öz-benzer, yerel dönel simetrik model için Einstein, Bergmann-Thomson ve Landau-Lifshitz enerji yoğunlukları hesaplanmaktadır. Bununla birlikte elde edilen sonuçlar için literatürde iyi bilinen gökkuşağı fonksiyonları kullanılarak bir değerlendirme yapılmaktadır. Elde edilen sonuçlar $f_1(\chi) = 1/(1-\chi)$ ve $f_2(\chi) = 1$ gökkuşağı fonksiyonları kullanılarak Einstein, Bergman-Thomson ve Landau-Liftshitz gösterimleri için enerji yoğunluklarının açık halleri yeniden yazılmıştır. Buna göre, Einstein ve Bergmann-Thomson enerji momentum gösterimleri için test parçacığının enerji yoğunluğunu değiştirdiği ancak Landau-Liftshitz enerji momentum gösterimi için enerji yoğunluğunu değiştirmediği gösterilmiştir.

Anahtar Kelimeler: Teleparalel kuram; gökkuşağı kütleçekim kuramı; enerji-momentum

1. Introduction

Our universe is expanding rapidly, as can be seen from Einstein field equations and the data obtained by the Hubble telescope. This expansion can also be defined as the expansion of space time. Within the framework of this expansion, what the energy-momentum density of our universe is or whether localization is possible has been a subject of research for theoretical physicists. As a result of Einstein's efforts to combine the theory of gravity and electromagnetism, the foundations of the teleparallel theory, an alternative theory to the general theory of relativity, were laid. In 1961, after Moller [1]'s study containing his perspective on the energy-momentum puzzle, the solution of the problem accelerated with the writing of the Lagrangian equation for the teleparallel theory. In the late 20th and early 21st centuries, Einstein, Bergmann-Thomson, Landau-Lifshitz, Moller, Weinberg, Tolman, Qadir-Sharif notations were used by physicists such as Virbhadra [2], Xulu [3], Sharif [4], Salti [5], Aydogdu [6] have addressed the energy momentum localization problem within the framework of general relativity theory. However, Vargas [7], Pereira [8], Sharif [9], Salti [10], Aydogdu [11] and Aygün [12] re-investigated the problem within the framework of teleparallel theory. Solutions approaches to the problem have been modified and expanded in both general relativity and teleparallel theory. For these two theories, it can be said that the concept of torsion in the theory of absolute parallelism corresponds to the curvature in general relativity. While the general theory of relativity is successful in explaining many unknown phenomena, it remains incomplete at some points. The most important of these shortcomings is the inability to combine gravity and quantum theory. At this point, the "Rainbow Theory of Gravity" (RTG) appears with a structure that includes quantum contributions. According to the theory, the deformation caused by a test particle in the structure of space-time is discussed. The energy momentum distribution relation for RTG under non-linear Lorentz transformations is given as follows [13]:

$$f_1^2(\chi)E^2 - f_2^2(\chi)p^2 = m^2. \tag{1.1}$$

The symbols m, E, p here indicate the mass, energy and momentum of the test particle, respectively. In addition, $f_1(\chi), f_2(\chi)$ are rainbow functions, E_P is the Planck energy represented by $\chi = E/E_P$. In case $\chi \to 0$ the normal distribution relation is obtained.

In this study, by mapping $dt \to \frac{dt}{f_1(\chi)}$ and $dx_j \to \frac{dx_j}{f_2(\chi)}$ within the line element depicting space time, Einstein, Bergmann-Thomson and Landau-Lifshitz energy densities are calculated in teleparallel geometry.

2. Teleparallel Energy Momentum Prescriptions

In general, the metric tensor $(g_{\mu\nu})$ plays a very important role in formulating theories of gravity. Tetrad $(h^a{}_\mu)$ provides the connection between curved and flat space time, used to describe the structure of space time in the theory of string-parallel gravity:

$$h^a{}_\mu h_a{}^\nu = \delta^\nu_\mu , \quad h^a{}_\mu h_b{}^\mu = \delta^a_b, \quad g_{\mu\nu} = \eta_{ab} h^a{}_\mu h^b{}_\nu . \tag{2.1}$$

Here δ^ν_μ is the well-known Kronocker-Delta function and $\eta_{ab} = diag(-1, +1, +1, +1)$ is the Minkowski flat space-time metric.

Using tetrads, the Weitzenböck coefficients, which are the basic coefficients of this geometry, can be calculated with the following relation [14]:

$$\Gamma^\lambda{}_{\mu\nu} = h_a{}^\lambda \partial_\nu h^a{}_\mu = -h^a{}_\mu \partial_\nu h_a{}^\lambda . \tag{2.2}$$

Using the Weitzenböck coefficients, the torsion tensor is written as follows, with anti-symmetric properties:

$$\mathrm{T}^\lambda{}_{\mu\nu} = \Gamma^\lambda{}_{\nu\mu} - \Gamma^\lambda{}_{\mu\nu} . \tag{2.3}$$

Freud’s superpotentials can be defined using torsion tensor components:

$$U_\beta{}^{\nu\lambda} = h g_{\beta\mu} \left[m_1 T^{\mu\nu\lambda} + \frac{m_2}{2} (T^{\nu\mu\lambda} - T^{\lambda\mu\nu}) + \frac{m_3}{2} (g^{\mu\lambda} T^{\beta\nu}_\beta - g^{\nu\mu} T^{\beta\lambda}_\beta) \right]. \tag{2.4}$$

Here m_1, m_2, m_3 are dimensionless coupling constants and ($m_1 = \frac{1}{4},\ m_2 = \frac{1}{2},\ m_3 = -1$) makes general relativity and teleparallel theory equivalent. The Einstein, Bergmann-Thomson and Landau-Lifshitz energy-momentum density in the teleparallel gravity frame are written as follows, respectively;

$$hE^\mu_\nu = \frac{1}{4\pi} \partial_\theta (U_\nu{}^{\mu\theta}) \tag{2.5}$$

$$hB^{\mu\nu} = \frac{1}{4\pi} \partial_\theta (g^{\mu\lambda} U_\lambda{}^{\nu\theta}) \tag{2.6}$$

$$hL^{\mu\nu} = \frac{1}{4\pi} \partial_\theta (h g^{\mu\lambda} U_\lambda{}^{\nu\theta}). \tag{2.7}$$

Here E^0_0, B^{00}, L^{00} are components of the energy density and E^i_0, B^{0i}, L^{0i} are components of the momentum density.

3. Teleparallel Energy for a Self-Similar, Local Rotational Symmetric Model in a RTG Framework

The line element for a self-similar, locally rotationally symmetric model in curved spacetime is given by [15]:

$$ds^2 = e^{-2\lambda x}[-dt^2 + A^2(t)dx^2 + B^2(t)e^{-2\alpha x}(dy^2 + \kappa^{-2}\sin(\kappa y)dz^2)]. \qquad (3.1)$$

The parameters expressed by λ, α, κ define the symmetry groups of various models. *A* and *B* are time dependent functions. Line element in RTG framework is expressed in the form

$$ds^2 = e^{-2\lambda x}\left[-\frac{1}{f_1^2}dt^2 + \frac{A^2(t)}{f_2^2}dx^2 + \frac{B^2(t)e^{-2\alpha x}}{f_2^2}(dy^2 + \kappa^{-2}\sin(\kappa y)dz^2)\right] \qquad (3.2)$$

Now, one can obtain the metric tensor $g_{\mu\nu}$, and its inverse $g^{\mu\nu}$ from the line element (3.2) as follows,

$$g_{\mu\nu} = \begin{bmatrix} -\frac{e^{-2\lambda x}}{f_1^2} & 0 & 0 & 0 \\ 0 & \frac{A^2(t)e^{-2\lambda x}}{f_2^2} & 0 & 0 \\ 0 & 0 & \frac{B^2(t)e^{-2(\lambda+\alpha)x}}{f_2^2} & 0 \\ 0 & 0 & 0 & \frac{B^2(t)e^{-2(\lambda+\alpha)x}\sin(\kappa y)}{f_2^2\kappa^2} \end{bmatrix} \qquad (3.3)$$

$$g^{\mu\nu} = \begin{bmatrix} -f_1^2 e^{2\lambda x} & 0 & 0 & 0 \\ 0 & \frac{f_2^2 e^{2\lambda x}}{A^2(t)} & 0 & 0 \\ 0 & 0 & \frac{f_2^2 e^{2(\lambda+\alpha)x}}{B^2(t)} & 0 \\ 0 & 0 & 0 & \frac{f_2^2\kappa^2 e^{2(\lambda+\alpha)x}}{B^2(t)\sin(\kappa y)} \end{bmatrix} \qquad (3.4)$$

equations are obtained. Using equation (2.1) $h^a{}_\mu$ and $h_a{}^\mu$ can be written as

$$h^a{}_\mu = \begin{bmatrix} \frac{e^{-\lambda x}}{f_1} & 0 & 0 & 0 \\ 0 & \frac{A(t)e^{-\lambda x}}{f_2} & 0 & 0 \\ 0 & 0 & \frac{B(t)e^{-(\lambda+\alpha)x}}{f_2} & 0 \\ 0 & 0 & 0 & \frac{B(t)e^{-(\lambda+\alpha)x}\sqrt{\sin(\kappa y)}}{f_2\kappa} \end{bmatrix} \qquad (3.5)$$

$$h_a{}^\mu = \begin{bmatrix} f_1 e^{\lambda x} & 0 & 0 & 0 \\ 0 & \frac{f_2 e^{\lambda x}}{A(t)} & 0 & 0 \\ 0 & 0 & \frac{f_2 e^{(\lambda+\alpha)x}}{B(t)} & 0 \\ 0 & 0 & 0 & \frac{f_2\kappa e^{(\lambda+\alpha)x}}{B(t)\sqrt{\sin(\kappa y)}} \end{bmatrix} \qquad (3.6)$$

Using metric tensor and tetrads, Weitzenböck coefficients are obtained as follows:

$$\Gamma^0{}_{01} = \Gamma^1{}_{11} = -\lambda,\ \Gamma^1{}_{10} = \frac{\dot{A}(t)}{A(t)}, \quad \Gamma^2{}_{20} = \Gamma^3{}_{30} = \frac{\dot{B}(t)}{B(t)},$$

$$\Gamma^2{}_{21} = \Gamma^3{}_{30} = -\alpha - \lambda,\ \Gamma^3{}_{32} = \frac{\kappa}{2}\cot\kappa y \tag{3.7}$$

Here $\dot{A}(t) \equiv \frac{dA}{dt}$, $\dot{B}(t) \equiv \frac{dB}{dt}$ and the torsion tensors are written by

$$T^0{}_{12} = -T^0{}_{21} = \lambda, \qquad T^1{}_{01} = -T^1{}_{10} = \frac{\dot{A}}{A}$$

$$T^2{}_{02} = -T^2{}_{20} = T^3{}_{03} = -T^3{}_{30} = \frac{\dot{B}}{B},$$

$$T^2{}_{12} = -T^2{}_{21} = T^3{}_{13} = -T^3{}_{31} = -\alpha - \lambda,$$

$$T^3{}_{23} = -T^3{}_{32} = \frac{\kappa}{2}\cot\kappa y \tag{3.8}$$

Using torsion tensors, Freud superpotentials are obtained from equation (2.4) as follows.

$$U_0{}^{01} = -U_0{}^{10} = -\frac{B^2(t)(\alpha+\lambda)e^{-2x(\alpha+\lambda)}\sqrt{\sin\kappa y}}{f_1 f_2 \kappa A(t)},$$

$$U_0{}^{02} = -U_0{}^{20} = U_1{}^{12} = -U_1{}^{21} = \frac{A(t)e^{-2x\lambda}\ \cos\kappa y}{4 f_1 f_2 \kappa\sqrt{\sin\kappa y}}$$

$$U_1{}^{01} = -U_1{}^{10} = \frac{f_1 B(t)A(t)e^{-2x(\alpha+\lambda)}\sqrt{\sin\kappa y}\dot{B}(t)}{f_2^3\kappa}$$

$$U_2{}^{02} = -U_2{}^{20} = U_3{}^{03} = -U_3{}^{30} = \frac{f_1 B(t)e^{-2x(\alpha+\lambda)}\sqrt{\sin\kappa y}[B(t)\dot{A}(t)+A(t)\dot{B}(t)]}{f_2^3\kappa}$$

$$U_2{}^{12} = -U_2{}^{21} = U_3{}^{13} = -U_3{}^{31} = \frac{B^2(t)(\alpha+2\lambda)e^{-2x(\alpha+\lambda)}\sqrt{\sin\kappa y}}{2 f_1 f_2 \kappa A(t)}. \tag{3.9}$$

After the finding of Freudian superpotentials and using them into the equations (2.5)-(2.6)-(2.7), Einstein, Bergmann-Thomson and Landau-Lifshitz energy-momentum densities were determined respectively.

$$hE_0^0 = \frac{e^{-2x(\alpha+\lambda)}\left[e^{2x\alpha}\kappa^2 A^2(t)(\cos 2\kappa y - 3) + 32(\alpha+\lambda)^2 B^2(t)\sin^2\kappa y\right]}{64\pi f_1 f_2 \kappa A(t)\sin^{3/2}\kappa y} \tag{3.10}$$

$$hB^{00} = \frac{f_1\kappa^2 A^2(t)(3-\cos 2\kappa y) - 32 e^{-2x\alpha} f_1 \alpha(\alpha+\lambda)B^2(t)\sin^2\kappa y}{64\pi f_2 \kappa A(t)\sin^{3/2}\kappa y} \tag{3.11}$$

$$hL^{00} = \frac{e^{-4x(\alpha+\lambda)}B^2(t)[e^{2x\alpha}\kappa^2 A^2(t) - 16(\alpha+\lambda)^2 B^2(t)]\sin\kappa y}{16\pi f_2^4\kappa^2} \tag{3.12}$$

4. Conclusion and Suggestions

In this study, energy densities of Einstein, Bergmann-Thomson and Landau-Lifshitz are calculated for a self-similar, local rotationally symmetric model within the framework of RTG. Teleparallel geometry is used for the results obtained. As can be seen in Equations (3.10)-(3.11)-(3.12), all densities depend on rainbow functions. For the Einstein representation, the energy density varies inversely with both the f_1 and f_2 rainbow functions. Bergmann-Thomson energy density varies directly with f_1 and inversely with f_2 rainbow functions. It is noticeable that the Landau-Lifshitz energy density changes only inversely depending on the f_2 rainbow function. If the energy densities are rewritten for one of the well-known rainbow functions in the literature [16], $f_1 = \frac{1}{1-\frac{E}{E_P}}$, $f_2 = 1$, then,

Einstein energy densities are obtained as:

$$hE_0^0 = \frac{(1-\frac{E}{E_P})e^{-2x(\alpha+\lambda)}\left[e^{2x\alpha}\kappa^2 A^2(t)(\cos 2\kappa y-3)+32(\alpha+\lambda)^2 B^2(t)\sin^2\kappa y\right]}{64\pi\kappa A(t)\sin^{3/2}\kappa y} \tag{4.1}$$

As seen in Equation (4.1), increasing the energy of the test particle will cause a decrease in the Einstein energy density.

Bergmann-Thomson energy density is obtained as:

$$hB^{00} = \frac{\kappa^2 A^2(t)(3-\cos 2\kappa y)-32e^{-2x\alpha}\alpha(\alpha+\lambda)B^2(t)\sin^2\kappa y}{64\pi(1-\frac{E}{E_P})\kappa A(t)\sin^{3/2}\kappa y} \tag{4.2}$$

Increasing the energy (E) of the test particle will cause an increase in the Bergmann-Thomson energy density.

Landau-Lifshitz energy density is written as:

$$hL^{00} = \frac{e^{-4x(\alpha+\lambda)}B^2(t)[e^{2x\alpha}\kappa^2 A^2(t)-16(\alpha+\lambda)^2 B^2(t)]\sin\kappa y}{16\pi\kappa^2} \tag{4.3}$$

Here f_2, one of the rainbow functions, being 1 causes that the test particle will not affect the Landau-Lifshitz energy density. However, if the f_2 function is different from 1, the change in energy density is an undeniable fact.

An application of Rainbow gravity theory has been made to unify the theory of gravity and quantum theory, which is one of the problems that general relativity suffers from. The study is especially important in terms of shedding light on both the energy-momentum localization problem and the problems of combining Rainbow gravity theory and quantum theory.

Acknowledgment

This study was presented as an oral presentation at the “Avrasya 10th International Conference on Applied Sciences” on May 2 - 5, 2024, Tbilisi, Georgia.

Conflict of Interest

The Author report no conflict of interest relevant to this article.

Research and Publication Ethics Statement

The author declares that this study complies with research and publication ethics.

References

[1] Moller, C. (1958). On the localization of the energy of a physical system in the general theory of relativity. Ann. Phys. 4, 347-371.

[2] Virbhadra, K. S. (1990). Energy associated with a Kerr-Newman black hole. Physical Review D, 41(4), 1086-1090.

[3] Xulu, S. S. (2003). The Energy-Momentum Problem in General Relativity, arXiv:hep-th/0308070.

[4] Sharif M. and Fatima T. (2005). Energy-momentum distribution: a crucial problem in general relativity, Int. J. Mod. Phys. A, 20, 4309-4330.

[5] Salti M. and Havare A. (2005). Energy-momentum in viscous Kasner-type universe in Bergmann-Thomson formulations, Int. J. Mod. Phys. A, 20, 2169-2177.

[6] Aydogdu O. and Salti M. (2006). Energy density associated with the Bianchi type-II space-time, Prog. Theor. Phys., 115, 63-71.

[7] Vargas, T. (2004). The energy of the universe in teleparallel gravity. General Relativity and Gravitation, 36, 1255-1263.

[8] Pereira, J. G., Vargas, T. and Zhang, C. M. (2001). Axial-vector torsion and the teleparallel Kerr spacetime. Classical and Quantum Gravity, 18(5), 833-842.

[9] Sharif, M. and Jamil Amir, M. (2007). Teleparallel energy–momentum distribution of lewis–papapetrou spacetimes. Modern Physics Letters A, 22(06), 425-434.

[10] Salti, M. and Aydogdu, O. (2006). Energy in the Schwarzschild-de Sitter spacetime. Foundations of Physics Letters, 19, 269-276.

[11] Aydogdu, O., Saltı, M. and Korunur, M. (2005). Energy in Reboucas-Tiomno-Korotkii-Obukhov and Godel-type space-times in Bergmann-Thomson's formulations. Acta Phys. Slov., 55, 537-548.

[12] Aygün, S. and Tarhan, İ. (2012). Energy–momentum localization for Bianchi type-IV Universe in general relativity and teleparallel gravity. Pramana - J Phys., 78, 531–548.

[13] Magueijo, J. and Smolin, L. (2004). Gravity's rainbow. Classical and Quantum Gravity, 21(7), 1725-1736.

[14] Hayashi K. and Shirafuji, T. (1979). New general relativity. Phys. Rev. D 19(12), 3524-3553.

[15] Chao, W. Z. (1981). Self-similar cosmological models. General Relativity and Gravitation, 13, 625-647.

[16] Feng, Z. W. and Yang, S. Z. (2017). Thermodynamic phase transition of a black hole in rainbow gravity. Physics Letters B, 772, 737-742.