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Sekizinci Sınıf Öğrencilerinin Matematiksel Muhakemeleri İle Uzamsal Yetenekleri Arasındaki İlişki

The Relationship Between Mathematical Reasoning and Spatial Ability of Eighth Grade Students

Ramazan GÜRBÜZ[a], Emrullah ERDEM[a], Mehmet GÜLBURNU[b]
[a]*Adıyama Üniversitesi, Eğitim Fakültesi, Adıyaman, Türkiye.*
[b]*Milli Eğitim Bakanlığı, Türkiye.*

Özet

Bu çalışmanın amacı matematiksel muhakeme ile uzamsal yetenek arasındaki ilişkiyi belirlemektir. Araştırma, 324 sekizinci sınıf öğrencisinin katılımıyla gerçekleştirilmiştir. "Matematiksel Muhakeme Testi" ve "Uzamsal Yetenek Testi" veri toplama araçları olarak kullanılmıştır. Verilerin analizinde her bir katılımcının test puanları arasındaki Pearson korelasyon katsayısı hesaplanmıştır. Araştırmanın sonuçları, matematiksel muhakemeyle uzamsal yetenek arasında pozitif yönde anlamlı bir ilişki olduğunu göstermiştir. Bu iki matematiksel süreç becerisi arasında ortaya çıkan ilişki, ortaokul öğrencilerinin matematiksel muhakemeleriyle uzamsal yeteneklerinin eş güdümlü geliştirilmesi açısından değerli görülebilir.

Anahtar Kelimeler: *Matematiksel muhakeme, uzamsal yetenek, sekizinci sınıf öğrencileri*

Abstract

The aim of this study is to determine the relationship between mathematical reasoning and spatial ability. The present study was carried out with 324 eighth-grade students. "Mathematical Reasoning Test" and "Spatial Ability Test" were used as data collection tools. In analyzing the data, Pearson's correlation coefficient between participants' scores of each test was computed. Results show that there is a significant positive correlation between mathematical reasoning and spatial ability. The relationship that emerges between these two mathematical process abilities can be seen as valuable in terms of co-ordinated development of mathematical reasoning with the spatial skills of middle school students.

Keywords: *Mathematical reasoning, spatial ability, eighth graders*

1. Introduction

It is possible to see reasoning, association, estimation, intuition, shapes in space, formula and symbols in math building. Short thinking can be insufficient in understanding mathematics that is complex, abstract, and multidimensional because mathematics requires reasoning ability which is an high-level thinking act. Polya (1981) report that reasoning is considerably important in understanding and associating mathematics topics. Spatial ability can be shortly described as a multidimensional thinking activity, especially in geometry. It enables to comprehend and make concrete spaces by animating, motioning them in mind, and see and draw the all parts of a geometric model. Not only spatial components are integral parts of the structure of mathematics, but spatial representations are being increasingly included in the mathematics. Most concrete and pictorial representations of arithmetical, geometrical and algebraic ideas appear to be heavily reliant on spatial attributes (Fennema, 1974).

Mathematical Resoning

In mathematics, scientific truths are arrived at through reasoning–not by experiment or observation (Umay and Kaf, 2005), and mathematical concepts and operations are associated via reasoning (Ball and Bass, 2003). Mathematical reasoning is defined as the process of reaching a decision by using critical, creative and logical thinking (Erdem and Gürbüz, 2015). Mathematical reasoning is highlighted in a great deal in both national curriculum (MNE, 2013) and international reforms about mathematics education (NCTM, 2000) as well as mathematics education researchers (English, 1998; Lithner, 2008). The reasoning includes abilities like following and assessing chains of arguments, knowing what a proof is and how it differs from other kinds of reasoning, uncovering the basic ideas in a given line of argument, and devising formal and informal arguments (Niss, 2003). People who reason and think analytically tend to note patterns, structure, or regularities in both real-world situations and symbolic objects; they ask if those patterns are accidental or if they occur for a reason; and they conjecture and prove (NCTM, 2000).

Reasoning starts with comparison of beings or objects at early ages; changes and develops according to interests as growing (Gürbüz and Erdem, 2014). Lithner (2008) states that reasoning can possibly be considered as a process of thinking, output of this process or both and visualizes reasoning process in Figure 1: a vertex v_n represents both a momentary state of knowledge and of the (sub)task. The reasoner makes a strategy choice among the edges leading from v_n. The strategy implementation is represented by a transition edge $e_{n,m}$. Here knowledge not already accessed in v_n is recalled or constructed and added up to form the new knowledge state in v_m, where the task is partially resolved and therefore a new task state is formulated. A reason is the motivation supporting transitions between vertices (p. 257).

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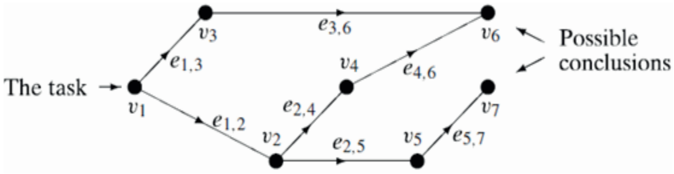


Figure 1. Reasoning process

To solve such a mathematical task or problem, it is necessary to visually process the given problem and to perform a number of calculation steps before coming to a decision and selecting the appropriate response (Vansteensel et al., 2014). Since the thinking strategy can vary in efficiency and elegance depending on the sophistication of the individual's understanding (McIntosh, Reys and Reys, 1997), making right decision occurs through higher-order thinking, namely, person-specific mathematical reasoning. That is to say, mathematical reasoning is high-level of thinking aims to reach a reasonable result by considering all aspects of a problem or case. Even if an upper level thinking is not based on information basis, it is not justified and it does not include reasonable approach, it cannot be accepted as reasoning. Thus, people who can reason about a case are well-informed about that case sufficiently and they examine the situation newly confronted with the all dimensions, discover, associate this with the previous information, make reasonable estimation and assumptions, justify his/her thinkings, reach results, can explain these results and defend them (Umay, 2003). Basic sign of reasoning is to show the ability of seeing the relationships between mathematics concepts (Mandacı Şahin, 2007). One of these signs is the spatial ability that is processed to notice spatial relationships in geometry that is a sub-branch of mathematics.

Spatial Ability

For at least 50 years, researchers have showed that spatial ability contributes in an important way to the learning mathematics and enhancing mathematics achievement (Ekstrom, French, Harman, and Dermen, 1976). Lohman (1993) defined spatial ability as the ability to generate, retain, retrieve, and transform well-structured visual images. According to Olkun (2003), spatial visualization, that is a term related closely to spatial ability, is the mental manipulation and integration of stimuli consisting of more than one part or movable parts.

Spatial ability requires commenting, drawing, creating mental images, visualizing and generalizing (Hatfield, Edwards and Bitter, 1997). While low-level spatial abilities were defined as requiring the visualization of two-dimensional configurations, but no mental transformations of these visual images, high-level ones were characterized as requiring the visualization of three dimensional configurations, and the mental manipulation of these visual images (Guay and McDaniel, 1977). If running a mental model of a physical device involves spatial visualization, then individuals who are high in spatial visualization ability should be more successful at such mental-model processing than individuals who are low-level in spatial visualization ability (Hegarty and Sims, 1994). One broad area of mathematics which is closely related to spatial visualization is Geometry. Children have opportunities to engage with geometric shapes from very early ages. Also, mathematics textbooks in early elementary grades include pictures of geometric shapes so that children could extract spatial relations within and across various shapes (Hallowell, Okamoto, Romo, and La Joy, 2015). However, children may have difficulties when they visually process geometric shapes. The main reason of these difficulties is that students have difficulties in comprehending three dimensional and static appearence of geometric objects (Accascina and Rogora, 2006). To overcome these difficulties greatly depends on being developed spatial ability of students. Relatively, NCTM (2000) suggests using images for students to create models and solve problems from early ages. Fennema (1974) suggested that without knowledge of mathematical ideas, it is impossible to learn mathematics, explaining that since the only way to add simple mathematical ideas to one's cognitive structure at early developmental levels is by interaction with concrete or pictorial materials which represent those ideas, and since those representations depend heavily on spatial attributes, if for same reason one is hampered in perception of those spatial attributes then one is hampered in learning those early the ideas. Fennema and Tartre (1985) suggested that one should question the idea that spatial visualization skills are highly important in the learning of mathematics and that the development of such skills should become a major goal of mathematics education. There are two reasons for paying attention to spatial ability that begins to develop from early ages and continues in middle school and further. The first is that a student can solve real life problems by using visual images and comprehension, reconstruction and movement of objects, namely, through spatial ability; the second is that there may be a positive correlation between spatial ability and mathematical reasoning.

As understood from the above-mentioned literature, mathematical reasoning and spatial ability have important places in understanding mathematics and both of them is associated with high level of thinking. This generates the idea that both of them can have a relation. Although most studies (Cheng and Mix, 2014; Ganley and Vasilyeva, 2011) were conducted on the relationship between mathematics achievement and spatial ability, the literature is not clear regarding the relationship between mathematical reasoning and spatial ability of students. The question whether being able to think spatially is dependent on mathematical reasoning has not been definitely answered. With this study, it is revealed that these two intertwined abilites are important from early ages, that is, mathematical reasoning provides to understand mathematics in detail and make reasonable decisions, and spatial ability contributes to make the multi-dimensional world understandable. In this context, the present study aims at determining the relationship between mathematical reasoning and spatial ability of eighth graders.

2. Method

Research Design

The current study was carried out using the correlational model among relational survey models because the relation between student's mathematical reasoning and spatial ability was studied. In this model, relationships between the variables are searched for, and the levels of these relationships are determined (Fraenkel et al., 2012).

Subjects

The participants were 324 eighth-grade students (13-14 year olds) and 173 of them were girl, 151 of them were boy. They were selected from different middle schools that served low and middle socioeconomic areas (by receiving opinions of school principals in that city) in five different cities of Turkey. The cities are from three different regions of Turkey, three of them are from Southeastern Anatolia Region that has mainly schools in low level. These students were given code names such as "S1", "S2", "S3", …

Instruments

Mathematical reasoning was measured with Mathematical Reasoning Test (MRT) composed of 33 questions and were developed with the help of the researchers (Erdem, 2016; Fast,1997; Gürbüz, 2010). MRT generally consists of non-routine problems that are probabilistic or may have more than one result (e.g. the shortest one? At most? At least?). Spatial ability was measured with Spatial Ability Test (SAT) developed by Turğut (2007) and composed of 29 questions. SAT includes processes such as bird's-eye view of the 3-dimensional objects, images from a corner, the number of cubes in an a building that cubes were used, imagining a building given with bird's eye view. While in MRT, minimum score is 0 and maximum score is 33, in SAT, minimum score is 0 and maximum score is 29. Two experienced mathematics teachers and two mathematics educators confirmed the validity of the instruments. Their opinions were taken in order for determining whether the questions in MRT neccessitate mathematical reasoning and in SAT spatial visualization or not. Also, both tests were applied to five 8th grade students not taking part in real applying. And then, it was corrected some expressions. As a result of pre-application doing with these five students, it was decided that students were given one hour (60 minutes) for each test to answer. Kuder–Richardson–20 coefficient of the MRT was found to be .84, and that of the SAT was found to be .89.

Data Analysis

Student responses were analyzed by using Statistical Package for Social Sciences (SPSS) program. The answers of questions in MRT and SAT were analyzed by accepting "1" point for each true answer and "0" point for each false answer or empty ones. Total points of all students taken from each test were calculated. If taken points are between 0-10 from SAT, spatial ability is thought to be low level; if between 10-20, it is mid-level; if between 20-29, it is high level. If taken points from MRT are between 0-11, mathematical reasoning is thought to be low level; if between 11-22, it is mid-level; if between 22-33, it is high level. The correlation between the total scores of the students taken from the tests was analyzed by calculating the Pearson Correlation Coefficient (r). To explain this relationship in detail, some of the students' answers in each test were discussed in the results section.

3. Results

The result of this study shows that there is a significant relationship between mathematical reasoning and spatial ability of the students.

Table 1. Avarage points from SAT and MRT

	Mean	Std. Deviation	N
SAT	13.31	5.35	324
MRT	11.39	3.75	324

As seen in table 1, all students' avarage point from SAT is 13.31, the ones of MRT is 11.39. According to these results, participants are in mid-level both in terms of mathematics reasoning and spatial ability.

Table 2. Relationship coefficient between mathematical reasoning and spatial ability

		Mathematical Reasoning	Spatial Ability
Mathematical Reasoning	Pearson Correlation	1	.804 (**)
	Sig. (2-tailed)		.000
	N	324	324
Spatial Ability	Pearson Correlation	.804 (**)	1
	Sig. (2-tailed)	.000	
	N	324	324

*** Correlation is significant at the 0.01 level (2-tailed).*

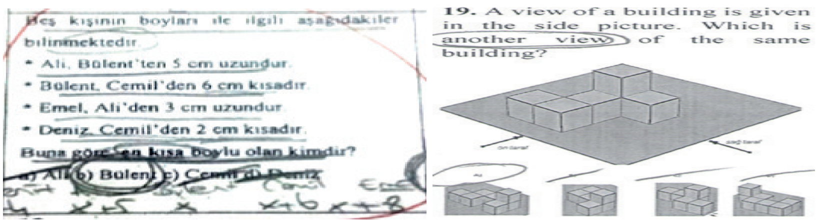
When Table 2 is examined, it can be seen that there is a significant relationship between the students'mathematical reasoning and spatial ability (r=.804, p=.000). The fact that correlation value is .65 or over .65 in education studies shows that it represents the prefered relation correctly (Fraenkel et al., 2012). From this finding, it can be said that if a student's mathematical reasoning is in high level, his/her spatial ability can be seen in high level, too. This relationship between mathematical reasoning and spatial ability was described by variance analysis and results were showed in Table 3.

Table 3. Results of variance analysis

	Sum of Squares	Df	Mean Square	F	Sig.
Regression	2941,372	1	2941,372	587,523	,000[a]
Residual	1612,060	322	5,006		
Total	4553,432	323			

a Predictors: (Constant), spatial ability,b. Dependent Variable: mathematical reasoning

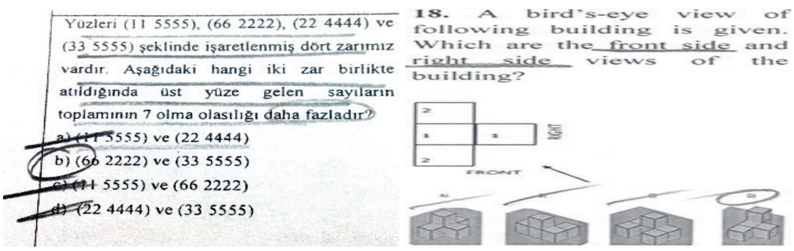
As seen in Table 3, in the results of variance analysis, spatial ability is predicted by mathematical reasoning and it is seen that percentage of this prediction is meaningful (F=587,523, p<.05). With this finding, it is confirmed that there is close and linear relation between mathematical reasoning and spatial ability. In addition, when some students' answers to questions in both tests are examined in detail and correlated, it could be seen that there is a highly positive relationship between mathematical reasoning and spatial ability. Below are some of the students' answers to some of the questions in each test; the students' answers in each test are interpreted in relation to each other.



a) b)

Figure 2. Responses by S83 to Q10 in MRT and Q19 in SAT

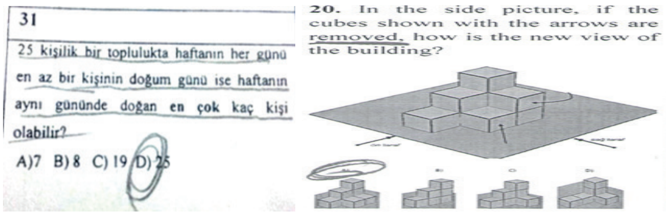
In Figure 2a, when student's answer to Q10 in MRT is examined, it is seen that he understood the question truly and found the correct answer by using algebra. It can be said that his mathematical reasoning is good because he determined statures of all five with algebra as following: Deniz's is x+4, Ali's is x+5, Bülent's is x, Cemil's x+6 and Emel's is x+8 and he compares them correctly. This student took 25 points from MRT. This point comes up to "high" level interval (22-33). Therefore, it can be said that the student's, S83, mathematical reasoning is in high level. In Figure 2b, when his answer in SAT is examined, it is seen that he determined the image of the building from different aspects correctly. He considered other answer choices one by one and eliminated false ones. This picture tells that he has a good performance in terms of spatial visualization. His point is 21 in SAT. This point comes up to "high" level interval (20-29). Therefore, it can be said that his spatial ability also is in high level. In Figure 2, when his answers and his points from both test are evaluated, it can be concluded that there is a linear relationship between mathematical reasoning and spatial ability.



a) b)

Figure 3. Responses by S101 to Q25 in MRT and Q18 in SAT

In Figure 3a, when the student's answer to Q25 in MRT is examined, it could be seen that she didn't give expected answer. That is to say, when the dice designed as (66 2222) and (11 5555) in choice "C", the results of total '7' are (1,6), (1,6), (1,6), (1,6) [4 times], (5,2), (5,2), (5,2), … (5,2) [16 times] . There are 20 results in total. But, the student's answer was as (2,5), (2,5), (2,5), … (2,5) [16 times]. There were 16 results according to her reasoning. Even if the results were not sufficient, it can not be said that she did not make mathematical reasoning. She took 15 points from MRT. This point comes up to "mid" level interval (11-22). Therefore, it can said that her mathematical reasoning is in midlevel. In Figure 3b, when the student's answer to Q18 in SAT is examined, it is seen that she determined the image of the building from front and right correctly. This correct determination gives information about her spatial ability. But, because spatial ability can not be determined from only this answer, her point from all test must be taken into account. She took 14 points in SAT. This point comes up to "mid" level interval (10-20). Therefore, it can be said that her spatial ability is in midlevel. In Figure 3, when her answers and her points from both test are examined, it can be said that the linear relationship between mathematical reasoning and spatial ability arised again.



a) b)

Figure 4. Responses by S86 to Q31 in MRT and Q20 in SAT

In Figure 4a, when the student's answer to Q31 in MRT is examined, it is seen that he made a reasoning that all people are born on the same day of the week. But, in question, it is stated that at least one person is born on each day of a week. It is understood from his answer that he couldn't make reasoning within expected way. This shows that his mathematical reasoning is not considerably sufficient. The point of him from the test confirms this judgement. He took 8 points in MRT. This point comes up to "low" level interval (0-11). Therefore, it can be said that his mathematical reasoning is in low level. In Figure 4b, when the student's answer to Q20 in SAT, there is not a cube showed with K in his answer but there is a cube showed with L. Although the answer including the cube with L showed with arrow mark, he didn't tick up correct answer. This tellss that his spatial ability is not efficient. His point from the test confirms this judgement. He took 7 points from SAT. This point comes up to "low" level interval. Therefore, it can be said that his spatial ability is in low level. In Figure 4, when his answers and his points from both tests were examined, it can be said that there is a linear relation betweeen mathematics reasoning and spatial ability.

4. Discussion and Conclusions

The present study provides a detailed picture of the relationship between mathematical reasoning and spatial ability of 8th grade students. As a result of analysis, it was found out that there is a significant relationship between 8th graders' mathematical reasoning and spatial ability (r=.804, p=.000). Cheng and Mix (2014), Ganley and Vasilyeva (2011) and Holmes et al. (2008) are substantially in agreement with this result, that is, individuals who show good performance in commenting spatial relations correctly make mathematics better. When the role of mathematical reasoning on doing mathematics is considered, it is possible to agree with that inference.

In the result of variance analysis, it was determined that mathematical reasoning can be predicted with spatial ability and the percentage of prediction is meaningful (F=536,5, p< .05). This result indicates that students who are good at spatial ability from the view of visualization and mental spinning are good at mathematical reasoning, too. Researchers (Hallowell et al., 2015; Hatfield et al., 1997; Olkun, 2003) support this finding, explaning that spatial ability necessitates skills such as commenting, drawing, creating mental images, mental transformation, visualizing and generalizing. Undoubtedly, these skills are essentail in order for doing mathematical reasoning efficiently. Indeed, mathematical reasoning and spatial ability are two interacting higher-order thinking activities because mathematical reasoning is essential in seeing and drawing the all parts from visible parts of a shape or object. From this point of view, it can be said that spatial ability triggers the structuring of mathematical reasoning and contributes to developing.

When compared many students' performances in MRT and SAT qualitatively, the relationship between two abilites was confirmed. It was observed that students whose spatial abilities are good use thinking activities such as justification, inference and interpreting while they are solving a problem. Likewise; it was seen that students who are bad in terms of mathematical reasoning usually dealt with appearing surface of visuals and they can't make spatial visualizing. In some studies made about mental activities, it was drew attention that similar areas of brain are active while solving problems with mathematical reasoning and spatial ability (Hubbard, Piazza, Pinel, and Dehaene, 2005; Umilta, Priftis, and Zorzi, 2009). Some researches (Lubinski, 2010; Newcombe, 2010) see this relation between spatial perception and mathematics as a resource of performance in Science, Technology, Engineering, and Mathematics (STEM). In other words, the success in STEM includes mental processes depending on the quality of relation between mathematics and spatial ability. On the other hand, another result in this study is that grade 8 studens' spatial ability and mathematical reasoning are in midlevel. It is possible to encounter with studies reporting that middle school students' spatial ability is considerably low (Turğut and Yılmaz, 2012), low and middle (Ganley and Vasilyeva, 2011) mathematical reasoning ability is middle.

It is thought that the linear relationship between mathematical reasoning and spatial ability is a dynamic structure developing each other. When spatial ability is considered to be structured in early ages, it can be said that spatial visualization training enables students to solve non-standard problems, and thus, develop their mathematical reasoning. In this context, Fennema and Tartre (1985) reported that spatial visualization skills are highly important in the learning of mathematics and that the development of such skills should become a major goal of mathematics education. At the same parallel NTCM (2000) advices that the applications developing both spatial ability and mathematical reasoning must be integrate to mathematics curriculum from primary schools. Primary schools even pre-schools should do activities developing both mathematical reasoning and spatial ability. This can be a crucial role in increasing their performance in mathematics. Through this way, there will be a culture in which students can make sense of the world in detail and differently and make reasable decisions.

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