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Mechanical Behavior of Functionally Graded Pressure Vessels Under the Effect of Poisson's Ratio

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Abstract

Functionally graded materials (FGM) belong to advanced composite materials class whose mechanical properties vary continuously from one surface to another at macro level. Thick walled annular structures are widely used in industry as nuclear reactors, long pipes used for carrying gases/oil, pressure vessels etc. Elastic analysis of FG thick walled cylindrical and spherical pressure vessels subjected only to internal pressure are studied. The material properties (Young' s modulus-Poisson' s ratio) are assumed to obey the power-law function through the wall thickness. Under these assumptions the governing equations of the FG vessels become a two-point boundary value problem. The analytical solution of such an equation can not be obtained except for simple grading functions. Complementary Functions Method is performed to solve governing equation in order to obtain displacement and stress distributions depending on inhomogeneity parameters.

Keyword: Pressure Vessels, Functionally Graded Material, Complementary Functions Method, Poisson Ratio, Stress Analysis.

1. INTRODUCTION

The notion of functionally graded materials was introduced in 1984 by a group of material scientists in an attempt to prepare thermal barrier materials in Japan. Functionally graded materials (FGM) are multifunctional composite materials whose mechanical properties vary continuously and smoothly from one side to the other. In contrast to layered structures the mechanical properties of these materials consisting of two or more components vary continuously depending on the specific function in a specific direction or plane [1]. Mechanical properties such as shear modulus, elastic modulus, Poisson's ratio coefficient of thermal expansion and material density change continuously and uniformly in the preferred direction in the FGM's. These materials are generally consist of the ceramic-metal compositions and well suited to applications with tough working conditions. For instance, one side of the material may have high mechanical strength while the other side may have high thermal resistance. Functional graded materials utilize various applications such as, rocket heat protectors, heat exchanger tubes, abrasion-resistant primers for transporting heavy abrasive parts, thermoelectric generators, heat motor parts, protective surfaces of fusion reactors and pressure vessels [2].

Homogeneous isotropic hollow pressure vessel subjected to internal pressure problem has been solved for various cases [3]. Most of the studies concerned with functional graded thick-walled pressure vessels focus on the elastic behavior of the vessels. Güven and Baykara specified the distributions of elasticity modulus of functionally graded isotropic spheres with internal pressure according to the constant circumferential stress, and presented corresponding stress distributions in the spheres [4]. Closed form solutions for the stresses through the thickness in FGM hollow cylinders and spheres was obtained by Tütüncü and Öztürk [5]. They used standard type Euler-Cauchy equations in order to solve the problem. Elastic analysis of Internally pressurized thick walled spherical pressure vessels of FGMs was analyzed elastically by You et al. [6]. Two types of pressure vessel are adopted in their study: one consists of two homogeneous layers near the inner and outer surfaces of the vessel and functionally graded layer in the middle; the other consists of the functionally graded material only. In the paper, the modulus of elasticity is assumed in the form of exponential function. Tütüncü [7] derived power series solution for displacement and stresses in the thick-walled cylinders with exponentially varying properties and subjected to internal pressure only by using infinitesimal theory of elasticity. The same solution procedure was applied for spherical vessels by Yontar et. al. [8]. Based on the constant Poisson's ratio assumption and the modulus of elasticity is an exponential function of radius, stresses and displacements in FG cylindrical and spherical

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pressure vessels were analyzed numerically by Chen and Lin [9] and analytically by Eraslan and Akis [10]. A hollow FG sphere subjected to radial pressure was analyzed for an arbitrary varying continuous function of the elasticity modulus by Li et. al. [11]. In this paper the problem reduces to a Fredholm integral equation. By using plane elasticity theory and complementary functions method, Tütüncü and Temel [12] obtained axisymmetric displacements and stresses in functionally graded hollow cylinders, disks and spheres subjected to uniform internal pressure. As an alternative numerical solution, Chen and Lin [13] suggested transmission matrix method to obtain displacement and stresses in thick-walled FG cylinders and spheres. In the governing equation, displacement function was assumed as unknown and elasticity modulus was taken arbitrary. A technique to tailor materials for FG linear elastic hollow cylinders and spheres was presented by Nie et al. [14] to attain both the constant circumferential stress and the constant in-plane shear stress through the thickness. It was assumed that the volume fractions of two phases of a FGM vary only with the radius. A few more studies can be found in the literature [15,18] where different varying functions and solution procedures have been used.

The objective of this work is to emphasize the effect of Poisson's ratio on the distribution of stresses and displacement for pressurized thick walled cylindrical and spherical pressure vessel made of FGM with power law varying properties. Actually, the variation of Poisson's ratio effects some mechanical behaviours of the FG vessels, which will be clearly shown in the present paper.

2. MATERIALS AND METHODS

The stress and displacement distribution in thick walled heterogeneous pressure vessels will be considered. *a* and *b* represents inner and outer radius of the vessels. The Poisson ratio and stiffness are presumed to vary as , through the wall thickness. Here, and care the Poisson ratio and stiffness at the outer surface , and are the inhomogeneity parameters determined experientially. Vessels are under only internal pressure and there is no heat source effect.

2.1 Basic Formulation of Cylindirical Pressure Vessel

Employing the plain strain assumption and axisymmetry, the strain-displacement and constituve equations are [5],

$$\varepsilon_r = \frac{du}{dr}, \qquad \varepsilon_\theta = \frac{u}{r} \tag{1}$$

$$\gamma_{r\theta} = 0 \tag{2}$$

$$\sigma_r = C_{11}(r)\varepsilon_r + C_{12}(r)\varepsilon_\theta \tag{3}$$

$$\sigma_\theta = C_{12}(r)\varepsilon_r + C_{11}(r)\varepsilon_\theta \tag{4}$$

where u is the radial displacement and the stiffness terms $C_{11}(r)$ and $C_{12}(r)$ depend on radial direction as,

$$C_{11}(r) = \left(\frac{E_0(1-\nu_0 r^\gamma)}{(1+\nu_0 r^\gamma)(1-2\nu_0 r^\gamma)}\right) r^\beta, \qquad C_{12}(r) = \left(\frac{E_0 \nu_0 r^\gamma)}{(1+\nu_0 r^\gamma)(1-2\nu_0 r^\gamma)}\right) r^\beta$$

The only nontrivial equilibrium equation under assumptions can be written in the following form [5],

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \tag{5}$$

with boundary conditions in radial directions

$$\sigma_r(a) = -P, \qquad \sigma_r(b) = 0 \tag{6}$$

According to stress equilibrium equations, the governing equation of displacement becomes,

$$K_c u'' + L_c u' + M_c u = 0 \tag{7}$$

with boundary conditions that come from radial stress,

$$\left(C_{11}(r)\frac{du}{dr} + C_{12}(r)\frac{u}{r}\right)\Bigg|_{r=a} = -P, \qquad \left(C_{11}(r)\frac{du}{dr} + C_{12}(r)\frac{u}{r}\right)\Bigg|_{r=b} = 0 \tag{8}$$

where $K_c(r)$, $L_c(r)$, $M_c(r)$ obtained as below depends on the *r*. Subscript *c* represents cylindrical vessel.

$$K_c(r) = \frac{r^{\beta}\, E_0(1 - \nu_0 r^{\gamma})}{(1 - 2\nu_0 r^{\gamma})(1 + \nu_0 r^{\gamma})}$$

$$L_c(r) = \frac{r^{\beta-1}\, E_0(1 + \beta + \nu_0 r^{\gamma}(-3 - 2\beta + 2\gamma + \nu_0 r^{\gamma}(-\beta - 2\gamma + 2r^{\gamma}(2 + \beta + \gamma)\nu_0)))}{(1 - 2\nu_0 r^{\gamma})^2(1 + \nu_0 r^{\gamma})^2}$$

$$M_c(r) = \frac{r^{\beta-2}\, E_0(-1 + \nu_0 r^{\gamma}(2 + \beta + \gamma + \nu_0 r^{\gamma}(3 + \beta + 4\gamma + 2r^{\gamma}\gamma\nu_0)))}{(1 - 2\nu_0 r^{\gamma})(1 + \nu_0 r^{\gamma})^2}$$

2.2. Basic Formulation of Spherical Pressure Vessel

The strain-displacement and constitutive equations for spherical case are [5],

$$\varepsilon_r = \frac{du}{dr}, \qquad \varepsilon_\theta = \varepsilon_\phi = \frac{u}{r} \tag{9}$$

$$\gamma_{r\theta} = \gamma_{r\phi} = \gamma_{\theta\phi} = 0 \tag{10}$$

$$\sigma_r = C_{11}(r)\varepsilon_r + C_{12}(r)\varepsilon_\theta + C_{12}(r)\varepsilon_\phi \tag{11}$$

$$\sigma_\theta = \sigma_\phi = C_{12}(r)\varepsilon_r + C_{11}(r)\varepsilon_\theta + C_{12}(r)\varepsilon_\phi \tag{12}$$

The only nontrivial equilibrium equation is [5],

$$\frac{d\sigma_r}{dr} + \frac{2(\sigma_r - \sigma_\theta)}{r} = 0 \tag{13}$$

Similarly, according to stress equilibrium equations, the governing equation of displacement becomes

$$K_s u'' + L_s u' + M_s u = 0 \tag{14}$$

with boundary conditions that come from radial stress,

$$\left(C_{11}(r)\frac{du}{dr} + 2C_{12}(r)\frac{u}{r}\right)\Big|_{r=a} = -P\,, \qquad \left(C_{11}(r)\frac{du}{dr} + 2C_{12}(r)\frac{u}{r}\right)\Big|_{r=b} = 0 \tag{15}$$

where $K_s(r), L_s(r), M_s(r)$ obtained as below depends on the *r*. Subscript *s* represents spherical vessel.

$$K_s(r) = \frac{r^{\beta}\, E_0(1 - \nu_0 r^{\gamma})}{(1 - 2\nu_0 r^{\gamma})(1 + \nu_0 r^{\gamma})}$$

$$L_s(r) = \frac{r^{\beta-1}\, E_0\left(-2 + \beta + \nu_0 r^{\gamma}(-6 - 2\beta + 2\gamma + \nu_0 r^{\gamma}(-\beta - 2\gamma + 2r^{\gamma}(-4 + \beta + \gamma)\nu_0))\right)}{(1 - 2\nu_0 r^{\gamma})^2(1 + \nu_0 r^{\gamma})^2}$$

$$M_s(r) = \frac{r^{\beta-2}\, E_0(-2 + 4r^{\gamma} - \nu_0 r^{\gamma}(3 - 4r^{\gamma} + \beta + \gamma + \nu_0 r^{\gamma}(1 + \beta + 4\gamma + 2r^{\gamma}\gamma\nu_0)))}{(-1 + 2\nu_0 r^{\gamma})(1 + \nu_0 r^{\gamma})^2}$$

3. SOLUTION PROCEDURE

The Complementary functions method (CFM) transform the solution of two-point boundary value problems (BVP) to initial-value problems (IVP) [19]. Considering the equation (7 and 14) with their the boundary conditions (8-15);

$$Ku'' + Lu' + Mu = 0$$

On the point of CFM proposed complete solution is [12],

$$u = b_i\, U_i, \quad i = 1{,}2 \tag{16}$$

where is linearly independent homogeneous solution and are constants to be determined via the boundary conditions. For homogeneous solution transform the (BVP) to (IVP) by letting

$$U_i = Z_1^{(i)} \tag{17}$$

$$U_i' = Z_2^{(i)} \tag{18}$$

yields first order differential equation system

$$(Z_1^{(i)})' = Z_2^{(i)} \tag{19}$$

$$\left(Z_2^{(i)}\right)' = -\frac{L}{K}Z_2^{(i)} + \frac{M}{K}Z_1^{(i)} \tag{20}$$

Systems of equations (19) and (20) can be solved numerically for each homogeneous solution. The solution of the homogenous and partial equations system are solved with dummy initial conditions. This dummy initial conditions C_{ij};

$$Z_j^{(i)} = C_{ij}, \quad i, j = 1, 0 \tag{21}$$

should be chosen linearly independent to ensure the linear independence of the solutions. The numerical frame selected is the five-order Runge-Kutta method (RK5). The solutions are computed at 11 collocation points (10 division) through the thickness in interval of $0.6 < r < 1$. For inhomogeneity parameter; β assumed as and γ assumed as 0 and 0.6 for power-form variations to ensure physically meaningful results.

RESULTS

The competency and verity of the present numerical solution method is first checked with the analytical results obtained for thick walled homogeneous vessels with $\nu_0 = 0.3, E_0 = 1$. Comparisons are given in Table (1-2). It is clearly seen from comparison that the results are in good agreement with each other, which demonstrate the validity of the numerical solutions used in this work. By picking only 11 collacation points in RK5, exact numerical results have been obtained to six-digit accuracy.

Table 1. Comparison of displacement through the thickness for homogeneous cylindirical pressure vessel.

	Circumferential Stress		Radial Stress		Radial Displacement	
r	Analytic	Numeric	Analytic	Numeric	Analytic	Numeric
0.6	2.12500000	2.12499999	-1.00000000	-0.99999999	1.39425000	1.39424999
0.64	1.93579102	1.93579101	-0.81079102	-0.81079101	1.32977813	1.32977812
0.68	1.77897924	1.77897923	-0.65397924	-0.65397923	1.27426765	1.27426764
0.72	1.64756944	1.64756943	-0.52256945	-0.52256944	1.22622500	1.22622499
0.76	1.53635734	1.53635733	-0.41135734	-0.41135733	1.18447105	1.18447104
0.80	1.44140625	1.44140624	-0.31640625	-0.31640624	1.14806250	1.14806249
0.84	1.35969387	1.35969386	-0.23469388	-0.23469387	1.11623571	1.11623570
0.88	1.28886880	1.28886879	-0.16386880	-0.16386880	1.08836591	1.08836590
0.92	1.22707939	1.22707938	-0.10207940	-0.10207939	1.06393695	1.06393694
0.96	1.17285156	1.17285155	-0.04785156	-0.04785156	1.04251875	1.04251874
1	1.12500000	1.12499999	0	0	1.02375000	1.02374999

In order to underline the effect of inhomogeneity, stresses in FG vessels are normalised by corresponding stress in the homogeneous case. Figures (1, 4) show comparison of the methods and evolution of radial stress σ_r, circumferential stress σ_θ and radial displacement u. A positive β means increasing stiffness in the radial direction. It is obvious on the figures that dimensionless radial and circumferential stress and u increase with the increase of positive β. If only β is taken as a negative value suchlike , results start meeting technical expectations. Thus, we used β as -1.5 for further calculations.

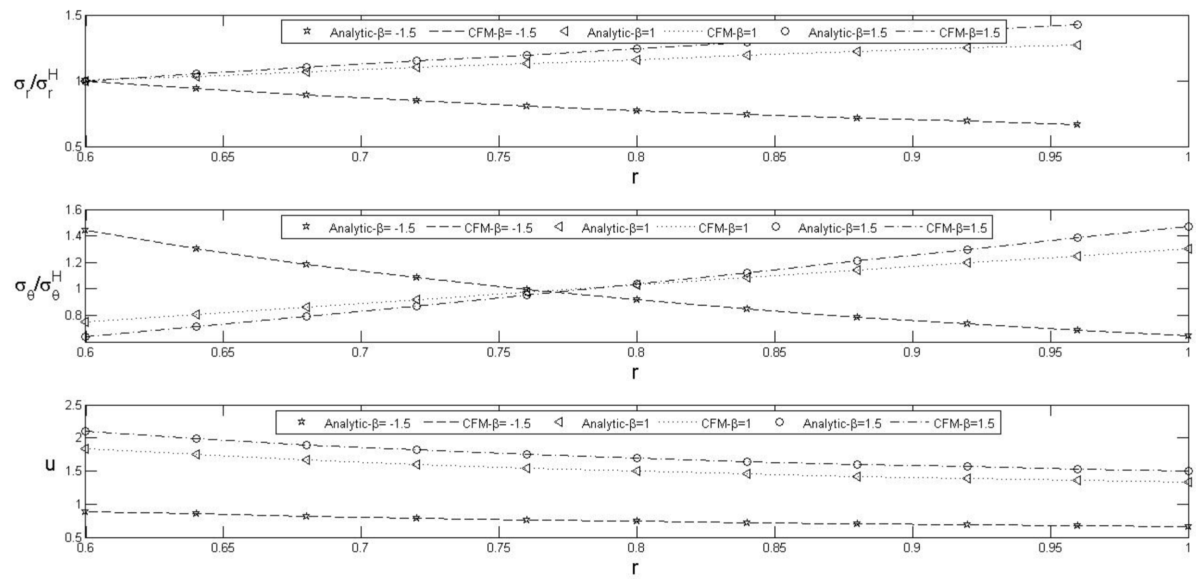

Fig. 1. Comparison of analytical and numerical methods for cylindrical vessel with $\nu = \nu_0 r^\gamma, E = E_0 r^\beta$, for $\beta = -1.5, 1, 1.5$

Fig. (2-3) show effect of changing Poisson ratio in the radial direction for cylindirical vessel and Fig. (5-6) for spherical vessel. The effect of changing Poisson ratio can be seen more specifically near the outer radius for radial stress but in circumferential stress there is no significiant difference. However, assuming Poisson ratio as constant makes it impossible to see its effect on displacement properly. The Fig. (3, 6) present that the radial displacement u highly depens on the variation of Poisson ratio, especially at the outer radius $(b = 1)$.

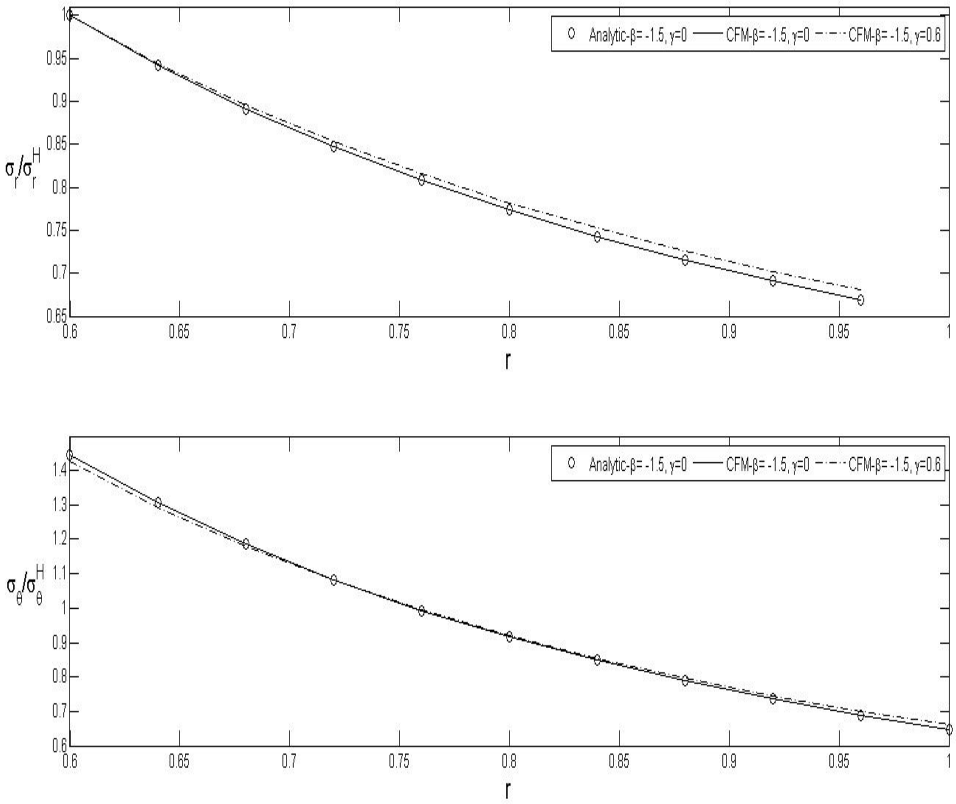

Fig. 2. Radial and circumferential stresses of cylindirical pressure vessel with $\nu = \nu_0 r^{\gamma}$, $E = E_0 r^{\beta}$ for $\beta = -1.5$ and $\gamma = 0, 0.6$

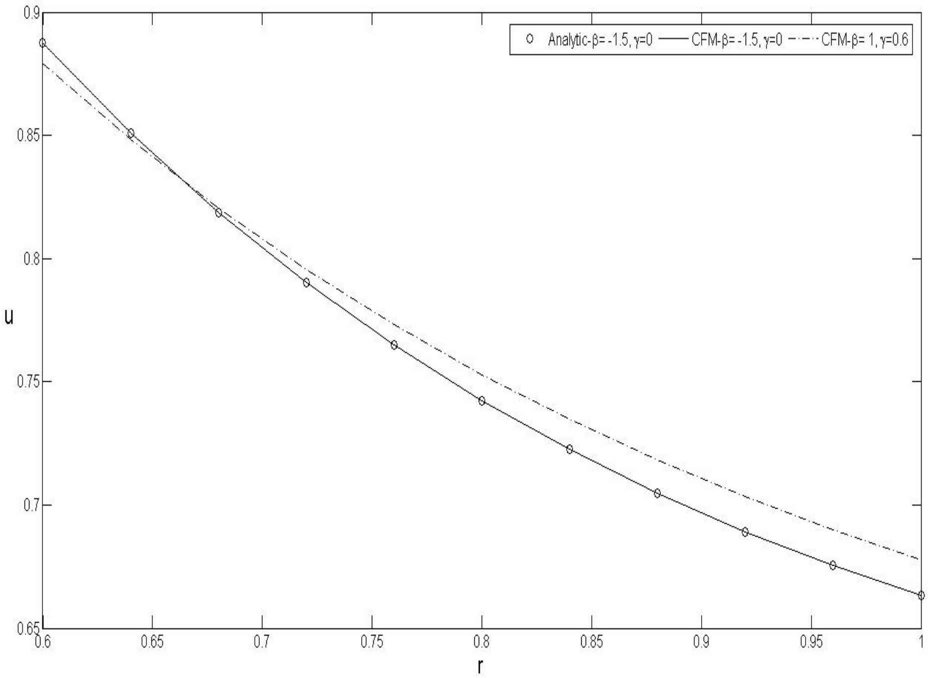

Fig. 3. Displacement of cylindirical pressure vessel with $\nu = \nu_0 r^{\gamma}$, $E = E_0 r^{\beta}$ for $\beta = -1.5$ and $\gamma = 0, 0.6$

Table 2. Comparison of displacement through the thickness for homogeneous spherical pressure vessel

	Circumferential Stress		Radial Stress		Radial Displacement	
r	**Analytic**	**Numeric**	**Analytic**	**Numeric**	**Analytic**	**Numeric**
0.6	0.91326530	0.91326527	-1.00000000	-0.99999999	0.56357142	0.56357141
0.64	0.80100421	0.80100416	-0.77547781	-0. 7754779	0.50774162	0.50774160
0.68	0.71361797	0.71361791	-0.60070533	-0.60070530	0.46222604	0.46222600
0.72	0.64458144	0.64458138	-0.46263227	-0.46263224	0.42479761	0.42479758
0.76	0.58932030	0.58932024	-0.35210999	-0.35210996	0.39379947	0.39379944
0.80	0.54456313	0.54456307	-0.26259566	-0.26259563	0.36797831	0.36797827
0.84	0.50792824	0.50792818	-0.18932587	-0.18932585	0.34637192	0.34637188
0.88	0.47765365	0.47765359	-0.12877669	-0.12877668	0.32823170	0.32823166
0.92	0.45241687	0.45241682	-0.07830313	-0.07830313	0.31296813	0.31296809
0.96	0.43121213	0.43121208	-0.03589365	-0.03589365	0.30011192	0.30011188
1	0.41326530	0.41326525	0	0	0.28928571	0.28928567

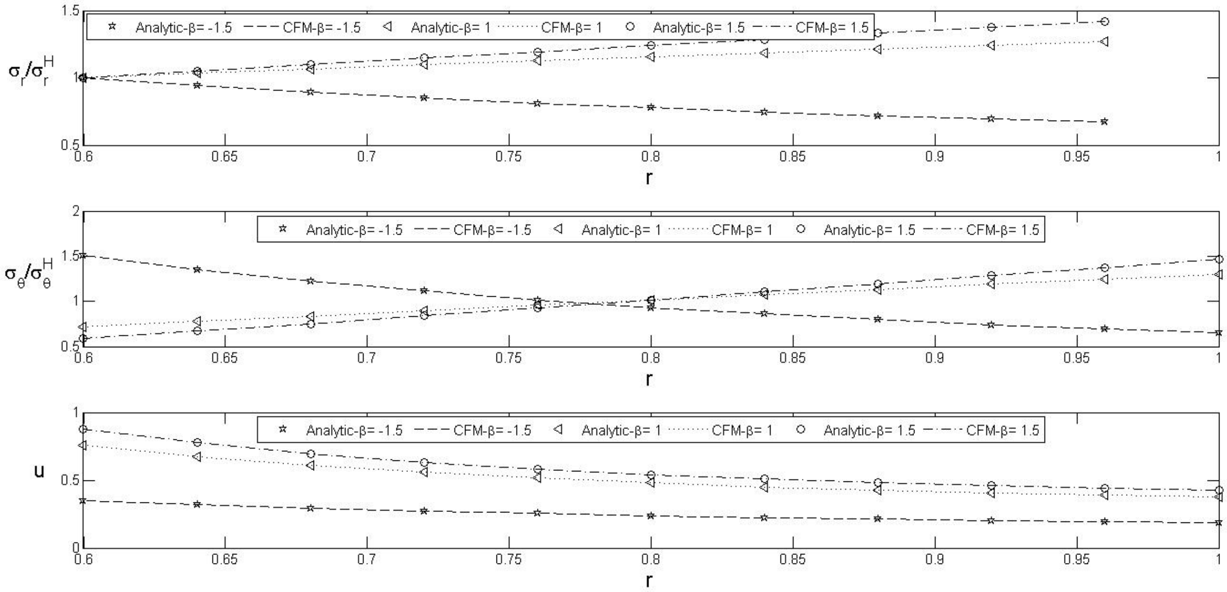

Fig. 4. Comparison of analytical and numerical methods for spherical vessel with , $\nu = \nu_0 r^{\gamma}$, $E = E_0 r^{\beta}$ for $\beta = -1.5, 1, 1.5$

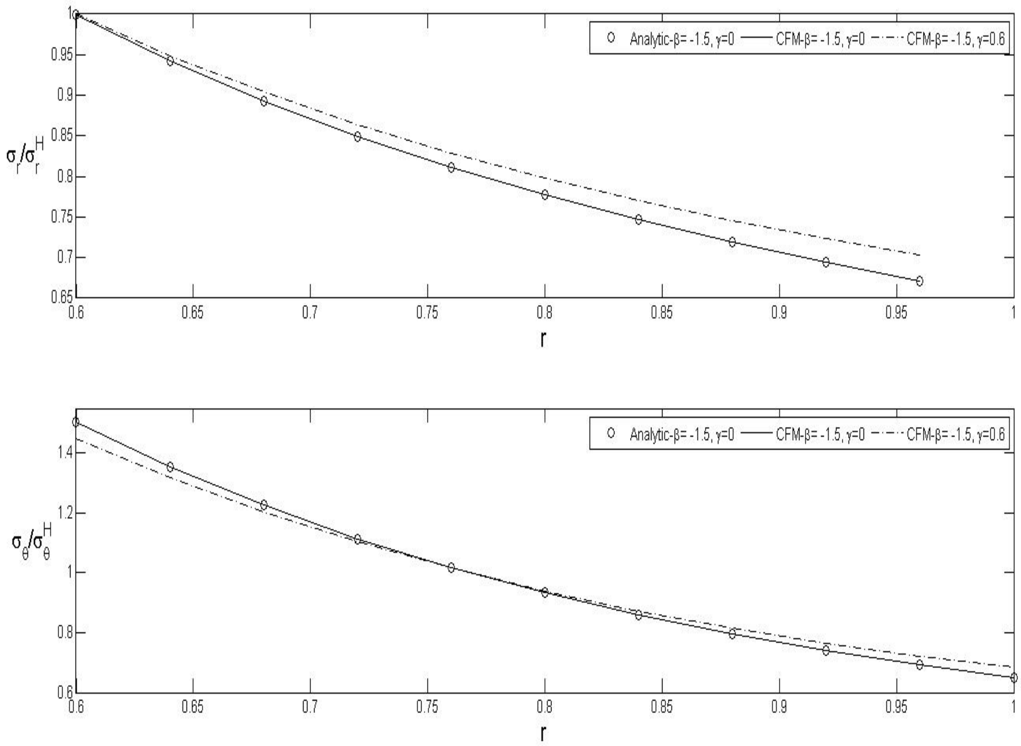

Fig. 5. Radial and circumferential stresses of spherical pressure vessel with $\nu = \nu_0 r^{\gamma}$, $E = E_0 r^{\beta}$ for $\beta = -1.5$ and $\gamma = 0, 0.6$

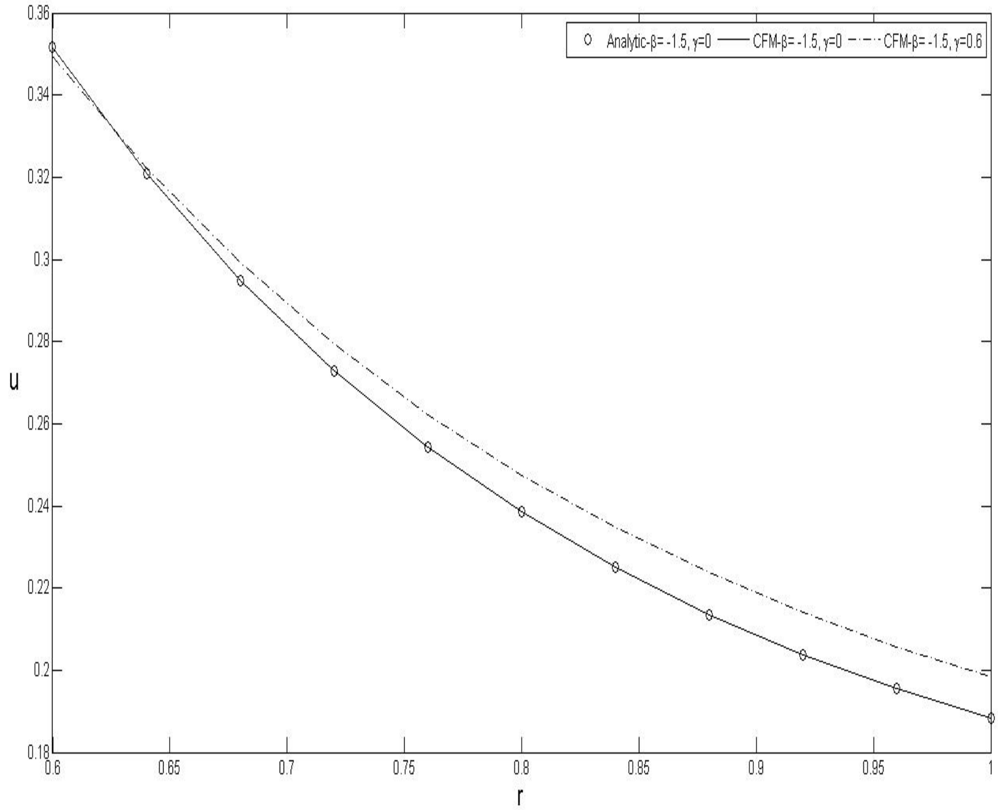

Fig. 6. Displacement of spherical pressure vessel with $\nu = \nu_0 r^{\gamma}$, $E = E_0 r^{\beta}$ for $\beta = -1.5$ and $\gamma = 0, 0.6$

CONCLUSION

Numerical model of FG pressure vessel for stresses and displacement are obtained and solved by Complementary Functions Method. The efficacy and adequacy of the present method is first compared to analytical results presented for constant Elastic Modulus and Poisson Ratio. The solution procedure can be applied to any choice of continuous grading functions. The solution procedure, besides satisfying accuracy with small computational costs, is also well structured, simple and efficient. The inhomogeneity parameter is a useful parameter for design perspective and can be tailored for specific applications so that the stress distributions and displacement amplitude can be controlled. Lastly, when sensitive solutions are made, the Poisson Ratio must be taken variable as other property functions.

NOMENCLATURE

a, b = Inner and outer radius
ν = Poisson ratio
ν_0 = Poisson ratio for outer wall
E = Modulus of elasticity
E_o = Modulus of elasticity for outer wall
$\varepsilon_r, \varepsilon_\theta, \varepsilon_\phi$ = Radial, circumferential and axial strain
$\sigma_r, \sigma_\theta, \sigma_\phi$ = Radial, circumferential and axial stress
γ, β = Inhomogeneity parameters
C_{11}, C_{12} = Stiffness terms
P = Pressure
u = Radial displacement

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