

# A New Application to Coding Theory via generalized Jacobsthal and Jacobsthal-Lucas Numbers

E. Akinci, S. Uygun\*

Department of Mathematics, Science Faculty,  
Gaziantep University, Campus, Gaziantep, Turkey

June 26, 2024

## Abstract

Coding/decoding algorithms carry out vital importance to providing information security since information security is very important for all people in recent years. In this study, we consider two new coding/decoding algorithms by means of  $J$ -matrices with elements generalized Jacobsthal and Jacobsthal-Lucas numbers. We used blocked message matrices for our models and we have different keys for the encryption of each message matrix. These new algorithms will give us opportunity for increasing the security of information and high correct ability.

*Keywords:* Encoding-Decoding algorithms, Jacobsthal sequence, Jacobsthal-Lucas sequence

*AMS Classifications:* 68P30, 11B83, 11B50, 14G50, 11T71

## 1 Introduction and Preliminaries

Integer sequences, such as Fibonacci, Lucas, Jacobsthal, Jacobsthal Lucas, Pell charm us with their abundant applications in science and art. Many properties of these sequences were deduced directly from elementary matrix algebra. Fibonacci coding and cryptography were studied in detail by the authors in [2]. The generalized relations among the code elements for Fibonacci coding theory were investigated in [4]. An application of mobile phone encryption based on Fibonacci structure of chaos was given in [5]. The generalization of golden cryptography based on  $k$ -Fibonacci numbers was denoted in [6]. A novel approach for information security with automatic variable key using Fibonacci  $Q$ -matrix was given in [7]. A Fibonacci-polynomial based coding method with error detection and correction was studied in [8]. In [10], Prasad expressed coding theory on Lucas  $p$ -numbers and the authors investigated a new coding/decoding algorithm

---

\*e-mail: suygun@gantep.edu.tr

using Fibonacci numbers in [11]. Right circulant matrices with generalized Fibonacci and Lucas polynomials for coding theory were demonstrated in [12]. A new application to coding theory via Fibonacci and Lucas numbers was denoted in [13].

In this study, we introduce two new coding/decoding algorithms using Jacobsthal  $J$ -matrices and  $C$ -matrices. The basic idea of our method depends on dividing the message matrix into the block matrices of size  $2 \times 2$ . Because of using mixed type algorithm and different numbered alphabet for each message, we have a more safely coding/decoding method. The alphabet is determined by the number of block matrices of the message matrix. Our method will not only increase the security of information but also has high correct ability for data transfer over communication channel.

The Jacobsthal and Jacobsthal Lucas sequences are defined recurrently by

$$\begin{aligned} j_n &= j_{n-1} + 2j_{n-2}, & (j_0 = 0, j_1 = 1) \\ c_n &= c_{n-1} + 2c_{n-2}, & (c_0 = 2, c_1 = 1) \end{aligned}$$

where  $n \geq 1$  any integer. These sequences can be generalized by preserving the relation of sequence, altering the initial conditions or by altering the relation of sequence preserving the initial conditions.

Let be  $n \in \mathbb{N}$ ,  $k > 0$  any real number. Then  $k$ -Jacobsthal sequence  $\{\hat{j}_{k,n}\}_{n \in \mathbb{N}}$  and  $k$ -Jacobsthal Lucas sequence  $\{\hat{c}_{k,n}\}_{n \in \mathbb{N}}$  are defined by the recurrence relation  $\hat{j}_{k,n} = k\hat{j}_{k,n-1} + 2\hat{j}_{k,n-2}$ , with initial conditions  $\hat{j}_{k,0} = 0$ ,  $\hat{j}_{k,1} = 1$  and  $\hat{c}_{k,n} = k\hat{c}_{k,n-1} + 2\hat{c}_{k,n-2}$ , with initial conditions  $\hat{c}_{k,0} = 2$ ,  $\hat{c}_{k,1} = k$  respectively. The relation  $\hat{c}_{k,n} = k\hat{j}_{k,n} + 4\hat{j}_{k,n-1}$  is established.

**Definition 1** For  $n \in \mathbb{N}$ ,  $k > 0$  any real number, then  $k$ -Jacobsthal matrix sequence  $(\hat{J}_{k,n})_{n \in \mathbb{N}}$  is defined by the following recurrence relation

$$\hat{J}_{k,n+2} = k\hat{J}_{k,n+1} + 2\hat{J}_{k,n}$$

with the initial conditions

$$\hat{J}_{k,0} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ and } \hat{J}_{k,1} = \begin{pmatrix} k & 2 \\ 1 & 0 \end{pmatrix}.$$

For  $n \in \mathbb{N}$ ,  $k > 0$  any real number, then  $k$ -Jacobsthal Lucas matrix sequence  $(\hat{C}_{k,n})_{n \in \mathbb{N}}$  is defined by the recurrence relation

$$\hat{C}_{k,n+2} = k\hat{C}_{k,n+1} + 2\hat{C}_{k,n}$$

with initial conditions

$$\hat{C}_{k,0} = \begin{pmatrix} k & 4 \\ 2 & -k \end{pmatrix} \text{ and } \hat{C}_{k,1} = \begin{pmatrix} k^2 + 4 & 2k \\ k & 4 \end{pmatrix}.$$

**Theorem 2** Let  $n$  be a positive integer and  $k > 0$  any real number, we have

$$\hat{J}_{k,n} = \begin{pmatrix} \hat{j}_{k,n+1} & 2\hat{j}_{k,n} \\ \hat{j}_{k,n} & 2\hat{j}_{k,n-1} \end{pmatrix}, \quad \hat{C}_{k,n} = \begin{pmatrix} \hat{c}_{k,n+1} & 2\hat{c}_{k,n} \\ \hat{c}_{k,n} & 2\hat{c}_{k,n-1} \end{pmatrix},$$

$$\begin{aligned} \hat{J}_{k,m+n} &= \hat{J}_{k,m} \cdot \hat{J}_{k,n} \\ \hat{J}_{k,n} &= \hat{J}_{k,1}^n \\ \hat{C}_{k,0} \hat{J}_{k,n} &= \hat{C}_{k,n} \\ \hat{C}_{k,n+1} &= \hat{C}_{k,1} \hat{J}_{k,n} \\ \hat{C}_{k,n} &= k\hat{J}_{k,n} + 4\hat{J}_{k,n-1}. \end{aligned}$$

### 1.1 A New Coding/Decoding Method using J-Matrix with Jacobsthal Numbers

In this part, we give a new coding/decoding algorithms using Jacobsthal and generalized  $P$ -numbers. We put our message in a matrix of even size adding a zero between two words and at the end of the message until the size of the message matrix is even. Dividing the  $2m \times 2m$  message matrix  $M$  into  $2 \times 2$  block matrices from left to right,  $B_i$  for  $i = 1, \dots, m^2$ , we get a new coding method.

If we demonstrate the symbols of the coding method, that matrices  $B_i, E_i$  and  $\hat{J}_{k,n}$  are of the following forms:

$$B_i = \begin{pmatrix} b_1^i & b_2^i \\ b_3^i & b_4^i \end{pmatrix}, \quad E_i = \begin{pmatrix} e_1^i & e_2^i \\ e_3^i & e_4^i \end{pmatrix}, \quad \hat{J}_{k,n} = \begin{pmatrix} \hat{j}_1 & \hat{j}_2 \\ \hat{j}_3 & \hat{j}_4 \end{pmatrix}.$$

The number of the block matrices  $B_i$  is denoted by  $b$ . According to  $b$ , we choose the number  $n$  as follows:

$$n = \begin{cases} 2, & b \leq 2 \\ b, & b > 2 \end{cases}$$

Using the choosen  $n$ , we write the following letter table according to  $mod 27$  (this table can be extended according to the used characters in the message matrix). We begin the “ $n$ ” for the first character.

A	B	C	D	E	F	G	H	I	J
n	n+1	n+2	n+3	n+4	n+5	n+6	n+7	n+8	n+9
K	L	M	N	O	P	R	Q	S	T
n+10	n+11	n+12	n+13	n+14	n+15	n+16	n+17	n+18	n+19
U	V	W	X	Y	Z	0			
n+20	n+21	n+22	n+23	n+24	n+25	n+26			

Now we explain the following new coding and decoding algorithms.

**Coding Algorithm (Jacobsthal Blocking Algorithm)**

**Step 1.** Divide the matrix  $M$  into blocks  $B_i$  ( $1 \leq m \leq 2$ ).

**Step 2.** Choose  $n$ .

**Step 3.** Determine  $b_j^i, (1 \leq j \leq 4)$

**Step 4.** Compute  $\det(B_i) \rightarrow d_i$ .

**Step 5.** Construct  $F = [d_i, b_j^i, ]_{j \in \{1,3,4\}}$ .

**Step 6.** End of algorithm.

**Decoding Algorithm**

**Step 1.** Compute  $\hat{J}_{k,n}$ .

**Step 2.** Determine  $\hat{j}_i$  ( $1 \leq i \leq 4$ ).

**Step 3.** Evaluate  $\hat{j}_1 b_3^i + \hat{j}_3 b_4^i \rightarrow e_3^i (1 \leq i \leq m^2)$

**Step 4.** Evaluate  $\hat{j}_2 b_3^i + \hat{j}_4 b_4^i \rightarrow e_4^i$

**Step 5.** Find  $(-2)^n d_i = e_4^i (\hat{j}_3 x_i + \hat{j}_1 b_1^i) - e_3^i (\hat{j}_4 x_i + \hat{j}_2 b_1^i)$

**Step 6.** Substitute for  $x_i = b_2^i$

**Step 7.** Construct  $B_i$ .

**Step 8.** Construct  $M$ .

**Step 9.** End of algorithm.

In the following examples we give applications of the above algorithm for  $b > 2$  and  $b \leq 2$  respectively.

**Example 3** Let's consider the message matrix for the following message text

"MAGIC NUMBERS".

We get the following message matrix  $M$ :

$$M = \begin{bmatrix} M & A & G & I \\ C & 0 & N & U \\ M & B & E & R \\ S & 0 & 0 & 0 \end{bmatrix}$$

**Step1.** We divide the message matrix  $M$  of size  $2 \times 2$  into the matrices, named  $B_i$  ( $1 \leq m \leq 2$ ), from left to right:

$$B_1 = \begin{bmatrix} M & A \\ C & 0 \end{bmatrix} \quad B_2 = \begin{bmatrix} G & I \\ N & U \end{bmatrix} \quad B_3 = \begin{bmatrix} M & B \\ S & 0 \end{bmatrix} \quad B_4 = \begin{bmatrix} E & R \\ 0 & 0 \end{bmatrix}$$

**Step2.**  $b = 4 > 2$ . For  $n = 4$ , we use the "character table" for the message matrix  $M$ :

M	A	G	I	M	B	E	R
16	4	10	12	16	5	8	21
C	0	N	U	S	0	0	0
6	3	17	24	22	3	3	3

**Step3.** The elements of the blocks  $B_i$  ( $1 \leq i \leq 4$ ) as follows:

$$\begin{array}{llll} b_1^1 = 16 & b_2^1 = 4 & b_3^1 = 6 & b_4^1 = 3 \\ b_1^2 = 10 & b_2^2 = 12 & b_3^2 = 17 & b_4^2 = 24 \\ b_1^3 = 16 & b_2^3 = 5 & b_3^3 = 22 & b_4^3 = 3 \\ b_1^4 = 8 & b_2^4 = 21 & b_3^4 = 3 & b_4^4 = 3 \end{array}$$

**Step4.** We calculate the determinants  $d_i$  of the blocks  $B_i$ :

$$\begin{aligned} d_1 &= \det(B_1) = 24 \\ d_2 &= \det(B_2) = 36 \\ d_3 &= \det(B_3) = -62 \\ d_4 &= \det(B_4) = -39 \end{aligned}$$

**Step5.** Using Step3 and Step4, we construct the  $F$  matrix:

$$F = \begin{bmatrix} 24 & 16 & 6 & 3 \\ 36 & 10 & 17 & 24 \\ -474 & 16 & 22 & 3 \\ -39 & 8 & 3 & 3 \end{bmatrix}$$

**Step6.** End of algorithm.

**Decoding Algorithm**

**Step1.** We choose  $k = 1$ ,  $n = 4$  and compute

$$\hat{J}_{k,n} = \begin{bmatrix} j_{1,5} & 2j_{1,4} \\ j_{1,4} & 2j_{1,3} \end{bmatrix} = \begin{bmatrix} j_1 & j_2 \\ j_3 & j_4 \end{bmatrix} = \begin{bmatrix} 11 & 10 \\ 5 & 6 \end{bmatrix}$$

**Step2.** We determine

$$j_1 = 11 \quad j_2 = 10 \quad j_3 = 5 \quad j_4 = 6$$

**Step3.** We compute the elements  $e_3^i$  to construct the matrix  $E_i$ :

$$e_3^1 = 81 \quad e_3^2 = 307 \quad e_3^3 = 257 \quad e_3^4 = 48$$

**Step4.** We compute the elements  $e_4^i$  to construct the matrix  $E_i$ :

$$e_4^1 = 78 \quad e_4^2 = 314 \quad e_4^3 = 238 \quad e_4^4 = 48$$

**Step5.** We calculate the elements  $x_i$ :

$$\begin{aligned} (-2)^4 \cdot 24 &= 78(16.11 + x_1.5) - 81(16.10 + x_1.6) \\ &\quad x_1 = 4 \\ (-2)^4 \cdot 36 &= 314(10.11 + x_2.5) - 307(10.10 + x_2.6) \\ &\quad x_2 = 12 \\ (-2)^4 \cdot (-62) &= 238(16.11 + x_3.5) - 257(16.10 + x_3.6) \\ &\quad x_3 = 5 \\ (-2)^4 \cdot (-39) &= 48(8.11 + x_4.5) - 48(8.10 + x_4.6) \\ &\quad x_4 = 21 \end{aligned}$$

**Step6.** We determine  $x_i$  as  $b_i$ :

$$x_1 = b_2^1 = 4 \quad x_2 = b_2^2 = 12 \quad x_3 = b_2^3 = 5 \quad x_4 = b_2^4 = 21$$

**Step7.** We construct the block matrices  $B_i$ :

$$B_1 = \begin{bmatrix} 16 & 4 \\ 6 & 3 \end{bmatrix} \quad B_2 = \begin{bmatrix} 10 & 12 \\ 17 & 24 \end{bmatrix} \quad B_3 = \begin{bmatrix} 16 & 5 \\ 22 & 3 \end{bmatrix} \quad B_4 = \begin{bmatrix} 8 & 21 \\ 3 & 3 \end{bmatrix}$$

**Step8.** We obtain the message matrix  $M$ :

$$M = \begin{bmatrix} 16 & 4 & 10 & 12 \\ 6 & 3 & 17 & 24 \\ 16 & 5 & 8 & 21 \\ 22 & 3 & 3 & 3 \end{bmatrix} = \begin{bmatrix} M & A & G & I \\ C & 0 & N & U \\ M & B & E & R \\ S & 0 & 0 & 0 \end{bmatrix}$$

**Step9.** End of algorithm.

**Jacobsthal Lucas Blocking Algorithm**

**Coding Algorithm**

**Step 1.** Divide the matrix  $M$  into blocks  $B_i$  ( $1 \leq m \leq 2$ ).

**Step 2.** Choose  $n$ .

**Step 3.** Determine  $b_j^i, (1 \leq j \leq 4)$ .

**Step 4.** Compute  $\det(B_i) \rightarrow d_i$ .

**Step 5.** Construct  $F = [d_i, b_j^i]_{j \in \{1,3,4\}}$ .

**Step 6.** End of algorithm.

**Decoding Algorithm**

**Step 1.** Compute  $\hat{C}_{n+1}$ .

**Step 2.** Determine  $\hat{C}_i$  ( $1 \leq i \leq 4$ ).

**Step 3.** Evaluate  $c_1 b_1^i + c_2 b_2^i \rightarrow e_1^i$  ( $1 \leq i \leq m^2$ ).

**Step 4.** Evaluate  $c_3 b_1^i + c_4 b_2^i \rightarrow e_3^i$ .

**Step 5.** Find  $(2k^2 + 16)(-2)^n d_i = e_1^i (c_3 x_i + c_4 b_4^i) - e_3^i (c_1 x_i + c_2 b_4^i)$ .

**Step 6.** Substitute for  $x_i = b_2^i$ .

**Step 7.** Construct  $B_i$ .

**Step 8.** Construct  $M$ .

**Step 9.** End of algorithm.

In the following examples we give applications of the above algorithm for  $b > 2$  and  $b \leq 2$  respectively.

**Example 4** Let's consider the message matrix for the following message text

"JACOBSTHAL LUCAS BLOCKING ALGORITHM"

We get the following message matrix  $M$ :

$$M = \begin{bmatrix} J & A & C & O & B & S \\ T & H & A & L & 0 & L \\ U & C & A & S & 0 & B \\ L & O & C & K & I & N \\ G & 0 & A & L & G & O \\ R & I & T & H & M & 0 \end{bmatrix}$$

**Step1.** We divide the message matrix  $M$  of size  $6 \times 6$  into the matrices, named  $B_i$  ( $1 \leq m \leq 2$ ), from left to right, each of size is  $2 \times 2$

$$\begin{aligned}
 B_1 &= \begin{bmatrix} J & A \\ T & H \end{bmatrix} & B_2 &= \begin{bmatrix} C & O \\ A & L \end{bmatrix} & B_3 &= \begin{bmatrix} B & S \\ 0 & L \end{bmatrix} & B_4 &= \begin{bmatrix} U & C \\ L & O \end{bmatrix} \\
 B_5 &= \begin{bmatrix} A & S \\ C & K \end{bmatrix} & B_6 &= \begin{bmatrix} 0 & B \\ I & N \end{bmatrix} & B_7 &= \begin{bmatrix} G & 0 \\ R & I \end{bmatrix} & B_8 &= \begin{bmatrix} A & L \\ T & H \end{bmatrix} \\
 B_9 &= \begin{bmatrix} G & O \\ M & 0 \end{bmatrix}
 \end{aligned}$$

**Step2.**  $b = 9 > 2$ . For  $n = 9$ , we use the "character table" for the message matrix  $M$ :

$J$	$A$	$C$	$O$	$B$	$S$	$U$	$C$	$A$	$S$	$0$	$B$	$G$	$0$	$A$	$L$	$G$	$O$
18	9	11	13	10	0	2	11	9	0	8	10	15	8	9	20	15	13
$T$	$H$	$A$	$L$	$0$	$L$	$L$	$O$	$C$	$K$	$I$	$N$	$R$	$I$	$T$	$H$	$M$	$0$
1	16	9	20	8	20	20	13	11	19	17	22	26	17	1	16	21	8

**Step3.** The elements of the blocks  $B_i$  ( $1 \leq i \leq 9$ ) as follows:

$$\begin{aligned}
 b_1^1 &= 18 & b_2^1 &= 9 & b_3^1 &= 1 & b_4^1 &= 16 \\
 b_1^2 &= 11 & b_2^2 &= 13 & b_3^2 &= 9 & b_4^2 &= 20 \\
 b_1^3 &= 10 & b_2^3 &= 0 & b_3^3 &= 8 & b_4^3 &= 20 \\
 b_1^4 &= 2 & b_2^4 &= 11 & b_3^4 &= 20 & b_4^4 &= 13 \\
 b_1^5 &= 9 & b_2^5 &= 0 & b_3^5 &= 11 & b_4^5 &= 19 \\
 b_1^6 &= 8 & b_2^6 &= 10 & b_3^6 &= 17 & b_4^6 &= 22 \\
 b_1^7 &= 15 & b_2^7 &= 8 & b_3^7 &= 26 & b_4^7 &= 17 \\
 b_1^8 &= 9 & b_2^8 &= 20 & b_3^8 &= 21 & b_4^8 &= 8 \\
 b_1^9 &= 15 & b_2^9 &= 13 & b_3^9 &= 21 & b_4^9 &= 8
 \end{aligned}$$

**Step4.** We calculate the determinants  $d_i$  of the blocks  $B_i$ :

$$\begin{aligned}
 d_1 &= \det(B_1) = 279 \\
 d_2 &= \det(B_2) = 103 \\
 d_3 &= \det(B_3) = -200 \\
 d_4 &= \det(B_4) = -194 \\
 d_5 &= \det(B_5) = -171 \\
 d_6 &= \det(B_6) = -6 \\
 d_7 &= \det(B_7) = -47 \\
 d_8 &= \det(B_8) = -124 \\
 d_9 &= \det(B_9) = -153
 \end{aligned}$$

**Step5.** Using Step3 and Step4 we construct the  $F$  matrix:

$$F = \begin{bmatrix} 279 & 18 & 1 & 16 \\ 103 & 11 & 9 & 20 \\ 200 & 10 & 8 & 20 \\ -194 & 2 & 20 & 13 \\ 171 & 9 & 11 & 19 \\ 6 & 8 & 17 & 22 \\ 47 & 15 & 26 & 17 \\ 124 & 9 & 1 & 16 \\ -153 & 15 & 21 & 8 \end{bmatrix}$$

**Step6.** End of algorithm.

**Decoding Algorithm**

**Step1.** We choose  $k = 1$  ,  $n = 2$  and compute

$$C_{n+1} = \begin{bmatrix} c_{1,4} & 2c_{1,3} \\ c_{1,3} & 2c_{1,2} \end{bmatrix} = \begin{bmatrix} C_1 & C_2 \\ C_3 & C_4 \end{bmatrix}$$

**Step2.** We determine

$$C_1 = 17 \quad C_2 = 14 \quad C_3 = 7 \quad C_4 = 10$$

**Step3.** We compute the elements  $e_1^i$  to construct the matrix  $E_1$ :

$$\begin{array}{cccccc} e_1^1 = 320 & e_1^2 = 313 & e_1^3 = 282 & e_1^4 = 314 & e_1^5 = 307 \\ e_1^6 = 374 & e_1^7 = 619 & e_1^8 = 167 & e_1^9 = 549 & \end{array}$$

**Step4.** We compute the elements  $e_3^i$  to construct the matrix  $E_3$ :

$$\begin{array}{cccccc} e_3^1 = 136 & e_3^2 = 167 & e_3^3 = 150 & e_3^4 = 214 & e_3^5 = 173 \\ e_3^6 = 226 & e_3^7 = 365 & e_3^8 = 73 & e_3^9 = 315 & \end{array}$$



**Step5.** We calculate the elements  $x_i$ :

$$\begin{aligned}
 (-2)^2.18.279 &= 320(16.10 + x_1.7) - 136(16.14 + x_1.17) \\
 &\quad x_1 = 9 \\
 (-2)^2.18.103 &= 313(20.10 + x_2.7) - 167(20.14 + x_2.17) \\
 &\quad x_2 = 13 \\
 (-2)^2.18.200 &= 282(20.10 + x_3.7) - 150(20.14 + x_3.17) \\
 &\quad x_3 = 0 \\
 (-2)^2.18.(-194) &= 314(13.10 + x_4.7) - 214(13.14 + x_4.17) \\
 &\quad x_4 = 11 \\
 (-2)^2.18.171 &= 307(19.10 + x_5.7) - 173(19.14 + x_5.17) \\
 &\quad x_5 = 0 \\
 (-2)^2.18.6 &= 374(22.10 + x_6.7) - 226(22.14 + x_6.17) \\
 &\quad x_6 = 10 \\
 (-2)^2.18.47 &= 619(17.10 + x_7.7) - 365(17.14 + x_7.17) \\
 &\quad x_7 = 8 \\
 (-2)^2.18.124 &= 167(16.10 + x_8.7) - 73(16.14 + x_8.17) \\
 &\quad x_8 = 20 \\
 (-2)^2.18.(-153) &= 549(8.10 + x_9.7) - 315(8.14 + x_9.17) \\
 &\quad x_9 = 13
 \end{aligned}$$

**Step6.** We determine  $x_i$  as  $b_i$ :

$$\begin{aligned}
 x_1 = b_2^1 = 9 & \quad x_2 = b_2^2 = 13 & \quad x_3 = b_2^3 = 0 & \quad x_4 = b_2^4 = 11 & \quad x_5 = b_2^5 = 0 \\
 x_6 = b_2^6 = 10 & \quad x_7 = b_2^7 = 8 & \quad x_8 = b_2^8 = 20 & \quad x_9 = b_2^9 = 13
 \end{aligned}$$

**Step7.** We construct the block matrices  $B_i$ :

$$\begin{aligned}
 B_1 &= \begin{bmatrix} 18 & 9 \\ 1 & 16 \end{bmatrix} & B_2 &= \begin{bmatrix} 11 & 13 \\ 9 & 20 \end{bmatrix} & B_3 &= \begin{bmatrix} 10 & 0 \\ 8 & 20 \end{bmatrix} & B_4 &= \begin{bmatrix} 2 & 11 \\ 20 & 13 \end{bmatrix} \\
 B_5 &= \begin{bmatrix} 9 & 0 \\ 11 & 19 \end{bmatrix} & B_6 &= \begin{bmatrix} 8 & 10 \\ 17 & 22 \end{bmatrix} & B_7 &= \begin{bmatrix} 15 & 8 \\ 26 & 17 \end{bmatrix} & B_8 &= \begin{bmatrix} 9 & 20 \\ 1 & 16 \end{bmatrix} \\
 B_9 &= \begin{bmatrix} 15 & 13 \\ 21 & 8 \end{bmatrix}
 \end{aligned}$$

**Step8.** We obtain the message matrix  $M$ :

$$M = \begin{bmatrix} 18 & 9 & 11 & 13 & 10 & 0 \\ 1 & 16 & 9 & 20 & 8 & 20 \\ 2 & 11 & 9 & 0 & 8 & 10 \\ 20 & 13 & 11 & 19 & 17 & 22 \\ 15 & 8 & 9 & 20 & 15 & 13 \\ 26 & 17 & 1 & 16 & 21 & 8 \end{bmatrix} = \begin{bmatrix} J & A & C & O & B & S \\ T & H & A & L & 0 & L \\ U & C & A & S & 0 & B \\ L & O & C & K & I & N \\ G & 0 & A & L & G & O \\ R & I & T & H & M & 0 \end{bmatrix}$$

**Step9.** End of algorithm.

### Conclusions

In this article, we developed a new algorithm in coding theory with the help of  $J$ -matrix and  $C$ -matrix. In this algorithm, we divide the desired message

into blocks and transform the message matrix into an encoded message matrix. In the decryption process, we converted the encrypted message matrix to the desired message matrix by using the J-matrix and C-matrix.

## References

- [1] Horadam, A. F., Jacobsthal representation numbers, *The Fibonacci Quarterly*, 37(2), (1996), 40-52.
- [2] Stakhov, A., Massingue, V. and Sluchenkov, A., *Introduction into Fibonacci coding and cryptography*. Osnova, Kharkov, 1999.
- [3] Stakhov, A. P., Fibonacci matrices, a generalization of the Cassini formula and a new coding theory, *Chaos Solitons Fractals* 30(1), (2006), 56-66.
- [4] Basu, M., Prasad, B., The generalized relations among the code elements for Fibonacci coding theory. *Chaos Solitons Fractals* 41(5), (2009), 2517-2525.
- [5] Wang, F., Ding, J., Dai, Z., Peng, Y., An application of mobile phone encryption based on Fibonacci structure of chaos. 2010 Second WRIWorld Congress on Software Engineering.
- [6] Tahghighi, M., Jafaar, A. R. Mahmood, Generalization of Golden Cryptography based on k-Fibonacci Numbers in (ICINC) at 2010.
- [7] Prajapat, S., Jain, A. and Thakur, R. S., A novel approach for information security with automatic variable key using Fibonacci Q-matrix. *IJCCT* 3 (2012), 3, 54-57.
- [8] Esmaeili, M., Esmaeili, M., Fibonacci-polynomial based coding method with error detection and correction, *Comput. Math. Appl.* 60(10), (2010), 2738-2752.
- [9] Gong, Y., Jiang Z., Gao, Y., On Jacobsthal and Jacobsthal-Lucas circulant type matrices, *Abstract and Applied Analysis*, Article ID 418293, 11, (2015).
- [10] Prasad, B., Coding theory on Lucas p-numbers. *Discrete Math. Algorithms Appl.* 8 (2016), no.4, 17 pages.
- [11] Tas, N., Ucar, S., Ozgur, N. Y., Kaymak, O. O , A new coding/decoding algorithm using Fibonacci numbers, *Discrete Mathematics, Algorithms and Applications*, 2 (2018), 1850028.
- [12] Ucar, S., Ozgur, N. Y., Right circulant matrices with generalized Fibonacci and Lucas polynomials and coding theory, *J. BAUN Inst. Sci. Technol.* 21(1), (2019), 306-322.

- [13] Uçar, S., Taş N., Özgür N.Y., A New Application to Coding Theory via Fibonacci and Lucas Numbers, *Mathematical Sciences and Applications e-Notes*,7(1), (2019), 62-70.