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DEDUCING INDIRECT AGGREGATE FACTOR DEMAND, SUPPLY AND PROFIT CURVES

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ABSTRACT

The aim of this research is to derive the equations of indirect aggregate factor demand, aggregate supply (forming a new growth model) and aggregate profit functions. In the study, based on the perfectly competitive firm profit maximization model, the macroeconomic growth model based on two, three and four factors of production, the equations of aggregate factor demand and total profit were inferred in terms of prices of goods and factors of production using optimization, derivative and consolidation formulas. A new macroeconomic model of growth, factor demand, and profit can be expressed, explained, and modeled in terms of the average commodity and factor prices of the economy. Theoretically, inferences are formed according to price variables linked to four factors of production. On the other hand, in practice, the corresponding variables may not be found in appropriate formations. The study will be leading to establishing and estimating macroeconometric indirect aggregate supply, factor demand and profit models. Additionally, the macroeconomic equations reached in the study can be developed into applied panel data models.

Keywords: Modeling, firm and aggregate, indirect factor demand, supply, profit.

DOLAYLI TOPLAM FAKTÖR TALEP, ARZ VE KAR EĞRİLERİNİN TÜRETİLMESİ

ÖZET

Bu araştırmanın amacı, herhangi bir ekonomiye ait dolaylı toplam faktör talebi, toplam arz (yeni bir büyüme modeli biçimlendirmesi) ve toplam kâr denklemlerini türetmektir. Çalışmada tam rekabetçi firma kâr maksimizasyonu modeli esas alınarak iki, üç ve dört üretim faktörüne dayalı makroekonomik büyüme modeli, toplam faktör talebi ve toplam kâr denklemleri optimizasyon, türev ve toplulaştırma formülleri kullanılarak mal ve üretim faktörü fiyatları cinsinden çıkarsanmıştır. Nihai denklem ve fonksiyonel ilişkilerin nasıl çıkarsandığı aşamalarıyla gösterilmiştir. Yeni bir makroekonomik büyüme, faktör talebi ve kâr modeli ekonominin veya sektörün ortalama mal ve faktör fiyatları ile ifade edilebilir, açıklanabilir ve modellenebilir. *Teorik olarak çıkarsamaların dört üretim faktörüne bağlı fiyat değişkenlerine göre oluşturulmuş olmasıdır. Diğer taraftan, uygulamada karşılık gelen değişkenler yerli yerince bulunamayabilir. Makroekonometrik toplam arz, faktör talebi ve kâr tahmin mdellerinin tahminlemesinde öncülük etmesidir. Çalışmada ulaşılan makroekonomik denklemler uygulamalı panel veri modellerine de genişletilip uygulanabilir.*

Anahtar Kelimeler: Dolaylı faktör talebi, arz ve kâr, firma ve toplam, modelleme.

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1. Introduction

Gross domestic product (GDP) is the aggregate supply of firms operating in a country in a year. We assume each firm decides how much will supply a good under profit maximizing goal in a period. Hence, the firm can derive its demand for a factor of production, its supply and profit curves in terms of price of goods, prices of production factors and technical progress. And, from these profit maximizing curves demand for aggregate production factors, aggregate supply and aggregate profit can be derived in terms of all prices and technology level. This study mainly develop how to reach aggregate curves from firms' profit maximization goals.

Profit-maximizing factor demand curves, indirect supply and profit curves can be obtained from the firm's profit maximization analysis. Depending on the structure of the production function, these curves of the firm can be obtained as structural models. However, When creating patterns, the parameters are misleading in the case of two production factors in the production function. Therefore, the structural models referring to the indirect factor demands, supply and profit are also obtained under three production factors in the production function. From these curves, the inferences of the four production factors demand, indirect supply and profit curves as patterns or formulas are shown. Macroeconomic indirect factor demand curves, indirect supply and profit curves or equations were inferenced from the obtained firm's indirect factor demand, supply and profit equations. These inferences give reasoning and insight to researchers working both theoretically and practically. Inferences will be made through the Cobb-Douglas type production function with an homogeneous structure. Because with the Cobb-Douglas type production function, analysis facilitates the understanding of economic phenomena. In the study, a technical writing method was followed both as a result of the nature of the study and the need not to take up too much space with unnecessary explanations.

In microeconomics textbooks, the expression of firm demand, supply and profit curves in terms of prices of goods and factors of production is included in profit maximization analysis in terms of factor utilization (Hendersen & Quandt 1980; Nicholson 1989; Silberberg 1990). Accordingly, in the literature search, there were no studies that inferenced, determined and estimated the aggregate indirect supply curve in terms of prices. For this reason, the relevant literature could not be included in the study. However, Epple et al. (2010) developed and estimated the home production function in terms of prices. Hilmer & Holt (2005) showed that the U.S. aggregate agricultural supply based on product maximization can be explained by factor prices and firms' total cost.

Davidson (2012) reminds that Keynes only accepts Say's Law assumption on the existence of a full employment output level in the long run, but, in fact, Keynes says that Say's Law does not apply to a monetary, production economy because there will be underemployment equilibrium. However, followers of these two schools; both New Classicals and New Keynesians accept AS and AD equilibrium under the "natural rate" of unemployment, including Moneterists. Felipe & Fisher (2003) do not believe and take the attention of researchers on the applicability of firm or institutional-based aggregate neoclassical production functions. Felipe & Fisher (2008) "They imply that intuitions based on micro variables and micro production functions will often be false when applied to aggregates". However, instead; there shall be a rationality of macroeconomic aggregate supply models which links to microeconomic analyses by relaxing assumptions in aggregation. This rationality is outlined in this study. Otherwise, one can not reach a macroeconomic model and run a macroeconometric model and make inferences. Stoker (2010:1) "The econometrics of aggregation is about modeling the relationship between individual (micro) behavior and aggregate (macro) statistics so that data from both levels can be used for estimation and inference about economic parameters". On the other hand, Salter (202) emphasizes a disciplined way of thinking about the interaction between the nominal economy and the real economy to understand economic fluctuations and business cycles because of the aggregate demand and aggregate supply model. In the long run output growth is determined by physical increases in the labor supply, capital availability, new ideas and technology, and improvements in regulations and institutions. The aggregate supply also links to the prices of these factors as shown in this study. This study shows a rationality and a disciplined way of thinking about the interaction between the nominal and the real economies as contributions.

Empirically, Paravastu et al. (2021) used physical capital and labor amounts in the multi-input single-output production model. Kim (2024) modeled and estimated the production of American manufacturing industry from the assumption of firm profit maximization with the translog function using the physical input variables of capital, labor, energy and non-energy, and found results that were in line with the expectations of producer theory. Petrin et al. (2004) stated that to include the use of a production factor in the profit maximization model, the correlation relationship between production and input levels should be taken into account first, and after confirming this relationship, they estimated the production (gross value added) function with blue- and white-collar workers, electricity and capital inputs for the period 1986-1997 for Chile.

2. Firm's Profit Maximization Model and Inferences

Here, the inferences of macroeconomic models are based on the microeconomic model of firm profit maximization.

2.1. Basic Assumptions

The following assumptions are made for the profit maximization model of a representative firm:

-1. The firm is in profit maximization behavior for each good in each period "t".
-2. The firm operates in a perfectly competitive market in both the factor and goods markets in each period "t".
-3. The firm uses the same production factors in each period "t" and pays only for the production factors it uses.
-4. The firm faces a Cobb-Douglas *type* production equation and output shows diminishing returns to the production factors.
-5. The technology level of the firm is different in each period and the firm ensures profit maximization under the technology level of the relevant period.
-6. Intertemporal profit maximization is out of the question.

-7. If the firm produces more than one good, it uses each input at the marginal product value of each production factor and aquals the marginal purchasing cost of each input.

On the other hand, for the macroeconomic model;
-8. Aggregation problems are ignored.
-9. The aggregate production factor demand for each period “t” consists of the sum of the firms’ production factor demands for the relevant period.
-10. The aggregate supply for each period “t” consists of the sum of the production of the firms’ supply in the relevant period.
-11. The aggregate profit for each period “t” consists of the sum of the profits of the firms for the relevant period.
-12. The technology level is defined outside of the model, and it may differ over time.

2.2. Situation With Two Factors of Production

According to neo-classical theory, it is accepted that companies use only physical labor and capital as production factors, the firm’s total cost consists of fixed cost and total variable cost that include the payments made to these two factors, and the technology level of each firm is different, and given to the firm in a period.

In this section, the firm’s demands for two (for example; capital and labor) factors, profit-maximizing supply and profit curves will be derived from the firm’s profit maximization model. Then, ignoring aggregation problems, indirect aggregate demands for factors, aggregate supply, and aggregate profit equations will be deduced from the firms’ profit maximization model.

2.2.1. Firm’s profit maximization model

Based upon the assumption that the firm's supply equation is $q = aI_1^{\beta_1} I_2^{\beta_2}$, a>0 and $\beta_1 + \beta_2 < 1$; showing decreasing returns to scale, and defining the total cost as $TC = w_1 I_1 + w_2 I_2$; then the firm's profit maximization problem can be stated as in equation (eq)1:

$$\underset{I_1, I_2}{Mak.}\ \pi(I_1, I_2) = p(aI_1^{\beta_1} I_2^{\beta_2}) - (w_1 I_1 + w_2 I_2),\ \beta_1 + \beta_2 < 1 \tag{1}$$

Through optimization process, the firm’s indirect factor demand equations can be obtained by substituting this correlation;

$$p = \frac{w_1}{MP_1} = \frac{w_2}{MP_2} \Leftrightarrow \frac{w_1}{\beta_1 a I_1^{\beta_1 - 1} I_2^{\beta_2}} = \frac{w_2}{\beta_2 a I_1^{\beta_1} I_2^{\beta_2 - 1}} \Rightarrow I_2 = \frac{\beta_2}{\beta_1} \frac{w_1}{w_2} I_1 \tag{2}$$

in one of the equations of the first order conditions; as in[2]

$$\beta_1 ap I_1^{\beta_1-1} I_2^{\beta_2} - w_1 = 0 \Rightarrow \beta_1 ap I_1^{\beta_1-1} \left(\frac{\beta_2}{\beta_1}\frac{w_1}{w_2} I_1\right)^{\beta_2} - w_1 = 0 \qquad (3)$$

then one reaches firm’s indirect factor demand equations as follow (eq.4):

$$\begin{aligned} I_1(p,a,w_1,w_2) &= \beta_1^{(1-\beta_2)/(1-\beta_1-\beta_2)} \beta_2^{\beta_2/(1-\beta_1-\beta_2)} w_1^{(\beta_2-1)/(1-\beta_1-\beta_2)} w_2^{-\beta_2/(1-\beta_1-\beta_2)} p^{1/(1-\beta_1-\beta_2)} a^{1/(1-\beta_1-\beta_2)} \\ I_2(p,a,w_1,w_2) &= \beta_1^{\beta_1/(1-\beta_1-\beta_2)} \beta_2^{(1-\beta_1)/(1-\beta_1-\beta_2)} w_1^{-\beta_1/(1-\beta_1-\beta_2)} w_2^{(\beta_1-1)/(1-\beta_1-\beta_2)} p^{1/(1-\beta_1-\beta_2)} a^{1/(1-\beta_1-\beta_2)} \end{aligned} \qquad (4)$$

If these profit-maximizing factor demand equations are substituted in the supply equation, the indirect supply equation ($q = f(p, \mathrm{a}, w_1, w_2)$) can be obtained as follows (eq.5)[3]:

$$\begin{aligned} q(p,a,w_1,w_2) &= \beta_1^{\beta_1/(1-\beta_1-\beta_2)} \beta_2^{\beta_2/(1-\beta_1-\beta_2)} w_1^{-\beta_1/(1-\beta_1-\beta_2)} w_2^{-\beta_2/(1-\beta_1-\beta_2)} p^{(\beta_1+\beta_2)/(1-\beta_1-\beta_2)} a^{1/(1-\beta_1-\beta_2)} \\ q(p,a,w_1,w_2) &= b\ w_1^{-\beta_1/(1-\beta_1-\beta_2)} w_2^{-\beta_2/(1-\beta_1-\beta_2)} p^{(\beta_1+\beta_2)/(1-\beta_1-\beta_2)} a^{1/(1-\beta_1-\beta_2)},\ b = \beta_1^{\beta_1/(1-\beta_1-\beta_2)} \beta_2^{\beta_2/(1-\beta_1-\beta_2)} \end{aligned} \qquad (5)$$

If these profit-maximizing factor demand equations are substituted in the profit equation, the indirect profit equation can be obtained as follows (eq.6)[4]:

$$\begin{aligned} \pi(p,a,w_1,w_2) &= (1-\beta_1-\beta_2)\beta_1^{\beta_1/(1-\beta_1-\beta_2)} \beta_2^{\beta_2/(1-\beta_1-\beta_2)} w_1^{-\beta_1/(1-\beta_1-\beta_2)} w_2^{-\beta_2/(1-\beta_1-\beta_2)} p^{1/(1-\beta_1-\beta_2)} a^{1/(1-\beta_1-\beta_2)} \\ \pi(p,a,w_1,w_2) &= b'\, w_1^{-\beta_1/(1-\beta_1-\beta_2)} w_2^{-\beta_2/(1-\beta_1-\beta_2)} p^{1/(1-\beta_1-\beta_2)} a^{1/(1-\beta_1-\beta_2)},\ b' = (1-\beta_1-\beta_2)\beta_1^{\beta_1/(1-\beta_1-\beta_2)} \beta_2^{\beta_2/(1-\beta_1-\beta_2)}. \end{aligned} \qquad (6)$$

Here, the firm's certain level of maximum profit can be achieved only under $\beta_1 + \beta_2 < 1$ condition; and thus the demand for a factor and a level of supply that maximize profit can be

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[2] or $\left\{\begin{aligned} I_1 &= \beta_1 b\ w_1^{(\beta_2-1)/(1-\beta_1-\beta_2)} w_2^{-\beta_2/(1-\beta_1-\beta_2)} p^{1/(1-\beta_1-\beta_2)} a^{1/(1-\beta_1-\beta_2)} \\ I_2 &= \beta_2 b\ w_1^{-\beta_1/(1-\beta_1-\beta_2)} w_2^{(\beta_1-1)/(1-\beta_1-\beta_2)} p^{1/(1-\beta_1-\beta_2)} a^{1/(1-\beta_1-\beta_2)} \end{aligned}\right\}, b = \beta_1^{\beta_1/(1-\beta_1-\beta_2)} \beta_2^{\beta_2/(1-\beta_1-\beta_2)}$

[3] By substituting these obtained $I_1(p,a,w_1,w_2)$ and $I_2(p,a,w_1,w_2)$ into the supply;

$$\begin{aligned} q = a(&\beta_1^{(1-\beta_2)/(1-\beta_1-\beta_2)} \beta_2^{\beta_2/(1-\beta_1-\beta_2)} w_1^{(\beta_2-1)/(1-\beta_1-\beta_2)} w_2^{-\beta_2/(1-\beta_1-\beta_2)} a^{1/(1-\beta_1-\beta_2)} p^{1/(1-\beta_1-\beta_2)})^{\beta_1} \\ &(\beta_1^{\beta_1/(1-\beta_1-\beta_2)} \beta_2^{(1-\beta_1)/(1-\beta_1-\beta_2)} w_1^{-\beta_1/(1-\beta_1-\beta_2)} w_2^{(\beta_1-1)/(1-\beta_1-\beta_2)} a^{1/(1-\beta_1-\beta_2)} p^{1/(1-\beta_1-\beta_2)})^{\beta_2} \end{aligned}$$

$$q = b\ w_1^{-\beta_1/(1-\beta_1-\beta_2)} w_2^{-\beta_2/(1-\beta_1-\beta_2)} p^{(\beta_1+\beta_2)/(1-\beta_1-\beta_2)} a^{1/(1-\beta_1-\beta_2)},\ b = \beta_1^{\beta_1/(1-\beta_1-\beta_2)} \beta_2^{\beta_2/(1-\beta_1-\beta_2)}$$

[4] $\pi(p, \mathrm{a}, w_1, w_2) = pq - (w_1 I_1 + w_2 I_2)$

$$\begin{aligned} \pi = p(&\beta_1^{\beta_1/(1-\beta_1-\beta_2)} \beta_2^{\beta_2/(1-\beta_1-\beta_2)} w_1^{-\beta_1/(1-\beta_1-\beta_2)} w_2^{-\beta_2/(1-\beta_1-\beta_2)} p^{(\beta_1+\beta_2)/(1-\beta_1-\beta_2)} a^{1/(1-\beta_1-\beta_2)}) \\ -w_1(&\beta_1^{(1-\beta_2)/(1-\beta_1-\beta_2)} \beta_2^{\beta_2/(1-\beta_1-\beta_2)} w_1^{(\beta_2-1)/(1-\beta_1-\beta_2)} w_2^{-\beta_2/(1-\beta_1-\beta_2)} p^{1/(1-\beta_1-\beta_2)} a^{1/(1-\beta_1-\beta_2)}) \\ -w_2(&\beta_1^{\beta_1/(1-\beta_1-\beta_2)} \beta_2^{(1-\beta_1)/(1-\beta_1-\beta_2)} w_1^{-\beta_1/(1-\beta_1-\beta_2)} w_2^{(\beta_1-1)/(1-\beta_1-\beta_2)} p^{1/(1-\beta_1-\beta_2)} a^{1/(1-\beta_1-\beta_2)}) \end{aligned}$$

$$\pi = w_1^{-\beta_1/(1-\beta_1-\beta_2)} w_2^{-\beta_2/(1-\beta_1-\beta_2)} p^{1/(1-\beta_1-\beta_2)} a^{1/(1-\beta_1-\beta_2)} (\beta_1^{\beta_1/(1-\beta_1-\beta_2)} \beta_2^{\beta_2/(1-\beta_1-\beta_2)} - \beta_1^{(1-\beta_2)/(1-\beta_1-\beta_2)} \beta_2^{\beta_2/(1-\beta_1-\beta_2)} - \beta_1^{\beta_1/(1-\beta_1-\beta_2)} \beta_2^{(1-\beta_1)/(1-\beta_1-\beta_2)})$$

$$\begin{aligned} \pi = &\left[\beta_1^{\beta_1/(1-\beta_1-\beta_2)} \beta_2^{\beta_2/(1-\beta_1-\beta_2)}\right] w_1^{-\beta_1/(1-\beta_1-\beta_2)} w_2^{-\beta_2/(1-\beta_1-\beta_2)} p^{1/(1-\beta_1-\beta_2)} a^{1/(1-\beta_1-\beta_2)} - \\ &\left[\beta_1^{\beta_1/(1-\beta_1-\beta_2)} \beta_2^{(1-\beta_1)/(1-\beta_1-\beta_2)} + \beta_1^{(1-\beta_2)/(1-\beta_1-\beta_2)} \beta_2^{\beta_2/(1-\beta_1-\beta_2)}\right] w_1^{-\beta_1/(1-\beta_1-\beta_2)} w_2^{-\beta_2/(1-\beta_1-\beta_2)} p^{1/(1-\beta_1-\beta_2)} a^{1/(1-\beta_1-\beta_2)} \end{aligned}$$

$$\pi = w_1^{-\beta_1/(1-\beta_1-\beta_2)} w_2^{-\beta_2/(1-\beta_1-\beta_2)} p^{1/(1-\beta_1-\beta_2)} a^{1/(1-\beta_1-\beta_2)} \left[\beta_1^{\beta_1/(1-\beta_1-\beta_2)} \beta_2^{\beta_2/(1-\beta_1-\beta_2)} - \beta_1^{(1-\beta_2)/(1-\beta_1-\beta_2)} \beta_2^{\beta_2/(1-\beta_1-\beta_2)} - \beta_1^{\beta_1/(1-\beta_1-\beta_2)} \beta_2^{(1-\beta_1)/(1-\beta_1-\beta_2)}\right]$$

$$\pi = w_1^{-\beta_1/(1-\beta_1-\beta_2)} w_2^{-\beta_2/(1-\beta_1-\beta_2)} P^{1/(1-\beta_1-\beta_2)} a^{1/(1-\beta_1-\beta_2)} (\beta_1^{\beta_1/(1-\beta_1-\beta_2)} \beta_2^{\beta_2/(1-\beta_1-\beta_2)} - \beta_1^{(1-\beta_2)/(1-\beta_1-\beta_2)} \beta_2^{\beta_2/(1-\beta_1-\beta_2)} - \beta_1^{\beta_1/(1-\beta_1-\beta_2)} \beta_2^{(1-\beta_1)/(1-\beta_1-\beta_2)})$$

$$\pi = (1-\beta_1-\beta_2)\beta_1^{\beta_1/(1-\beta_1-\beta_2)} \beta_2^{\beta_2/(1-\beta_1-\beta_2)} w_1^{-\beta_1/(1-\beta_1-\beta_2)} w_2^{-\beta_2/(1-\beta_1-\beta_2)} p^{1/(1-\beta_1-\beta_2)} a^{1/(1-\beta_1-\beta_2)}$$

$$\pi = b'\, w_1^{-\beta_1/(1-\beta_1-\beta_2)} w_2^{-\beta_2/(1-\beta_1-\beta_2)} p^{1/(1-\beta_1-\beta_2)} a^{1/(1-\beta_1-\beta_2)}, b' = (1-\beta_1-\beta_2)\beta_1^{\beta_1/(1-\beta_1-\beta_2)} \beta_2^{\beta_2/(1-\beta_1-\beta_2)}$$

$$b' = \beta_1^{\beta_1/(1-\beta_1-\beta_2)} \beta_2^{\beta_2/(1-\beta_1-\beta_2)} - \beta_1^{(1-\beta_2)/(1-\beta_1-\beta_2)} \beta_2^{\beta_2/(1-\beta_1-\beta_2)} - \beta_1^{\beta_1/(1-\beta_1-\beta_2)} \beta_2^{(1-\beta_1)/(1-\beta_1-\beta_2)} \Rightarrow$$

$$b' = (1-\beta_1-\beta_2)\beta_1^{\beta_1/(1-\beta_1-\beta_2)} \beta_2^{\beta_2/(1-\beta_1-\beta_2)}$$

obtained[5]. Otherwise, the production of goods will increase as factor demand increases for profits endless. Therefore; a certain level of demand and supply will not be achieved.

If the production function $q = aI_1^{\beta_1} I_2^{\beta_2}$ includes the technical coefficient a>0, as in this study, it will be included in the factor demand (I_1 and I_2) equations as $a^{1/(1-\beta_1-\beta_2)}$ similar to $p^{1/(1-\beta_1-\beta_2)}$ and its values will be positive as $1-\beta_1-\beta_2 > 0$ *and* $1/(1-\beta_1-\beta_2) > 0$ under the assumption of decreasing returns to scales; $\beta_1+\beta_2<1$. In the indirect supply equation, "p" will be in the form of $p^{(\beta_1+\beta_2)/1-\beta_1-\beta_2}$ and "a" will be in the form of $a^{1/(1-\beta_1-\beta_2)}$. Technical progress will be included in the profit equation as $a^{1/(1-\beta_1-\beta_2)}$ similar to the form of commodity prices; $p^{1/(1-\beta_1-\beta_2)}$; and technical progress will increase profitability. And, price increases motivate both supply and profit, *c.p.*

Hence, the technical progress coefficient "a" will affect the demand for factor, supply and profit of the firm affirmatively. The more the factor productivity the more the demand for the factor, and thus, supply and profits improve.

2.2.2. Inferencing aggregate indirect factor demand, supply and profit curves

In this section, ignoring aggregation problems, indirect aggregate factor demand, supply and profit functions will be inferenced by summing the aggregates of firms' factor demand, supply and proofit in the case of two factors of production firm's profit maximization model.

2.2.2.1. Aggregate factor demand curves

In the analysis so far, the price faced by a firm "i" for the good sold in period "t" is defined by small p_{it}, the interest it pays is defined by small w_{1it}, and the wage is defined by small w_{2it}. For the aggregate values, let's define the country's factor demand in period "t" by I_t, factor prices by W_t, average price by P_t. Let the technology level for the whole economy be A_t.

The aggregate factor demands for the whole economy will be deduced from the sum of the factor demands of the firms as follow (eq.7):

$$\begin{aligned} I_{1t} &= \sum_{i=1}^{N} \beta_{1it}^{(1-\beta_{2it})/(1-\beta_{1it}-\beta_{2it})} \beta_{2it}^{\beta_{2it}/(1-\beta_{1it}-\beta_{2it})} w_{1it}^{(\beta_{2it}-1)/(1-\beta_{1it}-\beta_{2it})} w_{2it}^{-\beta_2/(1-\beta_{1it}-\beta_{2it})} p_{it}^{1/(1-\beta_{1it}-\beta_{2it})} a_{it}^{1/(1-\beta_{1it}-\beta_{2it})} \\ I_{2t} &= \sum_{i=1}^{N} \beta_{1it}^{\beta_{1it}/(1-\beta_{1it}-\beta_{2it})} \beta_{2it}^{(1-\beta_{1it})/(1-\beta_{1it}-\beta_{2it})} w_{1it}^{-\beta_{1it}/(1-\beta_{1it}-\beta_{2it})} w_{2it}^{(\beta_{1it}-1)/(1-\beta_{1it}-\beta_{2it})} p_{it}^{1/(1-\beta_{1it}-\beta_{2it})} a^{1/(1-\beta_{1it}-\beta_{2it})} \end{aligned} \quad (7)$$

[5] For example, When β_1=0.25, β_2=0.25 values are given, B'=0.25 is obtained as positive. This profit function inferred for the firm is homogeneous of degree one in terms of commodity and factor prices.

If these demand equations are estimated, the autonomous factors are respectively; autonomous demand for factor of production number one is $B_1 = \frac{\sum_{t=1}^{T}\sum_{i=1}^{N} \beta_{1it}^{(1-\beta_{2it})/(1-\beta_{1it}-\beta_{2it})} \beta_{2it}^{\beta_{2it}/(1-\beta_{1it}-\beta_{2it})}}{T}$ and autonomous demand for factor of production number two is $B_2 = \frac{\sum_{t=1}^{T}\sum_{i=1}^{N} \beta_{1it}^{\beta_{1it}/(1-\beta_{1it}-\beta_{2it})} \beta_{2it}^{(1-\beta_{1it})/(1-\beta_{1it}-\beta_{2it})}}{T}$.

Here, due to aggregation problems these autonomous factors are formulated firstly by decomposing the relationship corresponding to the variables of the firms; then, by averaging the total of firms' autonomous factor demands over time. As a result, the aggregate factor demand curves over the years can be expressed as follows (eq.8):

$$\begin{aligned} I_{1t}(W_{1t}, W_{2t}, P_t, A_t) &= B_1 W_{1t}^{(\beta_2-1)/(1-\beta_1-\beta_2)} W_{2t}^{\beta_2/(1-\beta_1-\beta_2)} P_t^{1/(1-\beta_1-\beta_2)} A_t^{1/(1-\beta_1-\beta_2)} \\ I_{2t}(W_{1t}, W_{2t}, P_t, A_t) &= B_2 W_{1t}^{-\beta_1/(1-\beta_1-\beta_2)} W_{2t}^{(\beta_1-1)/(1-\beta_1-\beta_2)} P_t^{1/(1-\beta_1-\beta_2)} A_t^{1/(1-\beta_1-\beta_2)} \end{aligned} \tag{8}$$

Here, in the double-sided logarithmic equation, LnB_1 and LnB_2 are constant autonomous factor demands. When they are found to be statistically significant, they show the total systematic effect of other explanatory variables not used in the model on factor demands.

2.2.2.2. Aggregate supply curve

On the other hand; let the country's supply in period "t" be defined by large Q_t. In this case, the indirect aggregate supply equation is $Q_t = A_t I_{1t}^{\beta 1} I_{2t}^{\beta 2}$, and the indirect aggregate supply function to be estimated is

$Q_t = \sum_{i=1}^{N} q_{ti}(p_{1it}, a_{1it}, \mathrm{w}_{1it}, \mathrm{w}_{2it}) \Rightarrow Q_t = \mathrm{f}(P_t, A_t, W_{1t}, W_{2t})$. Here, the aggregate supply of the firm(s) producing in the country in the period "t" will be the gross domestic product of the firms in terms of their value added or the aggregate domestic product of the sector[6]. If consolidation is carried out, based on microeconomic fundamentals, the aggregate indirect production for the period "t" can be expressed in equational form as follows (eq.9)[7]:

$$Q_t(P_t, A_t, W_{1t}, W_{2t}) = B\ W_{1t}^{-\beta_1/(1-\beta_1-\beta_2)} W_{2t}^{-\beta_2/(1-\beta_1-\beta_2)} P_t^{(\beta_1+\beta_2)/(1-\beta_1-\beta_2)} A_t^{1/(1-\beta_1-\beta_2)} \tag{9}$$

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$$Q_t = \sum_{i=1}^{N} \beta_{1it}^{\beta_{1it}/(1-\beta_{1it}-\beta_{2it})} \beta_{2it}^{\beta_{2it}/(1-\beta_{1it}-\beta_{2it})} w_{1it}^{-\beta_{1it}/(1-\beta_{1it}-\beta_{2it})} w_{2it}^{-\beta_{2it}/(1-\beta_{1it}-\beta_{2it})} p_{it}^{(\beta_{1it}+\beta_{2it})/(1-\beta_{1it}-\beta_{2it})} a_{it}^{1/(1-\beta_{1it}-\beta_{2it})}$$

$$Q_t(\mathrm{P_t}, \mathrm{A_t}, W_{1t}, W_{2t}) = \sum_{i=1}^{N} b_{it}\ w_{1it}^{-\beta_{1it}/(1-\beta_{1it}-\beta_{2it})} w_{2it}^{-\beta_{2it}/(1-\beta_{1it}-\beta_{2it})} p_{it}^{(\beta_{1it}+\beta_{2it})/(1-\beta_{1it}-\beta_{2it})} a_{it}^{1/(1-\beta_{1it}-\beta_{2it})}$$

$$b_{it} = \beta_{1it}^{\beta_{1it}/(1-\beta_{1it}-\beta_{2it})} \beta_{2it}^{\beta_{2it}/(1-\beta_{1it}-\beta_{2it})}$$

$$B = \frac{\sum_{i=1}^{T} B_t}{T} = \frac{\sum_{t=1}^{T}\sum_{i=1}^{N} b_{it}}{T} = \frac{\sum_{t=1}^{T}\sum_{i=1}^{N} \beta_{1it}^{\beta_{1it}/(1-\beta_{1it}-\beta_{2it})} \beta_{2it}^{\beta_{2it}/(1-\beta_{1it}-\beta_{2it})}}{T} = \beta_1^{\beta_1/(1-\beta_1-\beta_2)} \beta_2^{\beta_2/(1-\beta_1-\beta_2)}.$$

Here, considering that the firm aims to maximize its profit for each period, firm-specific β_i's are accepted separately for each "t" period, and it is accepted that β_1 and β_2 are the average parameters of the period in the aggregate supply curve. In this case, B will be the autonomous average supply over the entire period.

7 However, on the basis of the firm, the technology level of the companies, the input payments they have made, their productivity, factor prices and the prices of the goods they sell may differ. It is not possible to find this data for each firm. The important thing here is how the country's supply is obtained in terms of firm supplies.

$$B = \beta_1^{\beta_1/(1-\beta_1-\beta_2)}\beta_2^{\beta_2/(1-\beta_1-\beta_2)}$$

Here, B or in the double-sided logarithmic equation LnB is the logarithmic value of the autonomous supply. When found to be statistically significant, it shows the total systematic effect of other factors on supply that are not included in the model.

2.2.2. 3. Aggregate profit curve

The profit function of a firm i[th] in period "t" is $\pi_{it}(p_{it}, a_{it}, w_{1it}, w_{2it})$. On the other hand; let us define the country's profit in period "t" with the notation π_t. In this case, the aggregate direct profit curve is $\pi_t(I_{1t}, I_{2t}) = \sum_{i=1}^{N}\pi_{it}(I_{1it}, I_{2it}) = \sum_{i=1}^{N}\left\{p_{it}(a_{it}I_{1it}^{\beta_{1it}}I_{2it}^{\beta_{2it}}) - (w_{1it}I_{1it} + W_{2it}I_{2it})\right\}$.

On the other hand, the indirect aggregate profit function to be estimated is[8];

$\pi_t(P_t, A_t, W_{1t}, W_{2t}) = \sum_{i=1}^{N}\pi_{it}(p_{it}, a_{it}, w_{1it}, w_{2it})$. So it will be inferenced as $\pi_t = f(P_t, A_t, W_{1t}, W_{2t})$.

This profit is the aggregate profits of the firms that have made profit maximization in each period "t". If consolidation is made, the aggregate indirect profit equation for the period "t" based on microeconomic fundamentals can be represented as[9,10,11] follows (eq.10):

$$\pi_t(P_t, A_t, W_t, W_{2t}) = B'\ W_{1t}^{-\beta_1/(1-\beta_1-\beta_2)}W_{2t}^{-\beta_2/(1-\beta_1-\beta_2)}P_t^{1/(1-\beta_1-\beta_2)}A_t^{1/(1-\beta_1-\beta_2)} \quad (10)$$

$$\mathbf{B'} = (1-\beta_1-\beta_2)\beta_1^{\beta_1/(1-\beta_1-\beta_2)}\beta_2^{\beta_2/(1-\beta_1-\beta_2)}$$

8

$$\pi_t = \sum_{i=1}^{N}\left\{\left(\beta_{1it}^{\beta_{1it}/(1-\beta_{1it}-\beta_{2it})}\beta_{2it}^{\beta_{2it}/(1-\beta_{1it}-\beta_{2it})} - \beta_{1it}^{(1-\beta_{2it})/(1-\beta_{1it}-\beta_{2it})}\beta_{2i}^{\beta_{2it}/(1-\beta_{1it}-\beta_{2it})} - \beta_{1it}^{\beta_{1it}/(1-\beta_{1it}-\beta_{2it})}\beta_{2it}^{(1-\beta_{1it})/(1-\beta_{1it}-\beta_{2it})}\right)w_{1it}^{-\beta_{1it}/(1-\beta_{1it}-\beta_{2it})}w_{2it}^{-\beta_{2it}/(1-\beta_{1it}-\beta_{2it})}p_{it}^{1/(1-\beta_{1it}-\beta_{2it})}a_{it}^{1/(1-\beta_{1it}-\beta_{2it})}\right\}$$

9 Here, profit maximization between periods is not taken into account.

10 In note; however, provided that it performs the same function as the symbol $\sum_{i=1}^{N}$ in the profit equation; the sum sign; $\sum_{i=1}^{N}$ may be used in the expression of B', provided that it corresponds to the prices faced by the i[th] firm and the partial factor elasticities. Then, B' can be derived as follows:

$$B'_t = \sum_{t=1}^{N} b_{it}' = \sum_{i=1}^{N}\left(\beta_{1it}^{\beta_{1it}/(1-\beta_{1it}-\beta_{2it})}\beta_{2it}^{\beta_{2it}/(1-\beta_{1it}-\beta_{2it})} - \beta_{1it}^{(1-\beta_{2it})/(1-\beta_{1it}-\beta_{2it})}\beta_{2i}^{\beta_{2i}/(1-\beta_{1it}-\beta_{2it})} - \beta_{1it}^{\beta_{1it}/(1-\beta_{1it}-\beta_{2it})}\beta_{2it}^{(1-\beta_{1it})/(1-\beta_{1it}-\beta_{2it})}\right) \text{ olup}$$

$$B' = \frac{\sum_{t=1}^{T}\sum_{i=1}^{N} b_{it}'}{T} = \frac{\sum_{t=1}^{T}\sum_{i=1}^{N}(1-\beta_{1it}-\beta_{2it})\beta_{1it}^{\beta_{1it}/(1-\beta_{1it}-\beta_{2it})}\beta_{2it}^{\beta_{2it}/(1-\beta_{1it}-\beta_{2it})}}{T}$$

$$B' = (1-\beta_1-\beta_2)\beta_1^{\beta_1/(1-\beta_1-\beta_2)}\beta_2^{\beta_2/(1-\beta_1-\beta_2)}$$

Here, considering that the firm aims to maximize profit for each period, the firm-specific β_i's are considered separately for each "t" period. B' will represent the autonomous profit in the model prediction.

11 However, on the basis of the firm, the technology level of the companies, the input payments they have made, their productivity, factor prices and the prices of the goods they sell may differ. It is not possible to find this data for each firm. What matters is how the aggregate profit is generated in terms of firm profits.

Here, B′ (or LnB′ in the double-sided logarithmic equation) is the constant autonomous aggregate profit value. When found to be statistically significant, it shows the total systematic effect of other factors on profit that are not included in the model.

In model estimation, the values of these variables might not be found promptly to explain factor demand, profit and supply. When there is a measurement problem with the data, for example, when the capital price cannot be found, it is necessary to use the cash interest payment instead. Again, there might exist problems in measuring the technology dimension level variable or which one to use among alternatives. And one shall use a price index as prices of goods on the average. Some firms show decreasing, constant, or increasing returns to scale rather than constant returns to scale all opposing the assumption of constant returns to scale. On the other hand, efficiency differences among companies are visible. Calculations are also required in the determination of wages in models to be established. Representing the technology level promptly is also important in the estimation model. Technology level A can be represented by the number of patents approved in year "t" in the country or by the share of R&D. Problems like these exist.

2.3. Situation with Three Factors of Production

In this section, a profit maximization analysis of a firm with a Cobb-Douglas type production function, which uses three production factors with technology level (a) in a certain period, is obtained. From this profit maximization model, firm’s demand for factor, supply and profit equations will be obtained. These equation structures will help (i) to obtain the firm’s factor demand, supply and profit equations corresponding to Cobb-Douglas type production functions, (ii) to obtain the equations of aggregate supply and profit at the macro level in the use of more than three factors of production.

Assuming a Cobb-Douglas type production function with the use of three factors, the indirect supply and profit equations of the perfectly competitive firm will be obtained[12]. Then, the transitions from the microeconomic basis to the macro model; firms’ demand for production factors, supplies and firm profits will be summed.

2.3.1. Firm’s profit maximization model

When supply equation is stated as $q = aI_1^{\beta_1} I_2^{\beta_2} I_3^{\beta_3}, \sum_{i=1}^{3} \beta_i < 1$, the profit maximization problem of a firm can be writen as follow (eq.11):

$$\underset{I_1, I_2, I_3}{Mak.}\ \pi(I_1, I_2, I_3) = p(aI_1^{\beta_1} I_2^{\beta_2} I_3^{\beta_3}) - (w_1 I_1 + w_2 I_2 + w_3 I_3) \quad (11)$$

Then, from profit maximization first order conditions (eq.12);

[12] The reason for choosing profit maximization with three in addition to the profit maximization with two factors to present the reader a pattern in generalization is that the parameters are misleading in results of two factors case.

$$\left.\begin{cases} \dfrac{\partial\pi}{\partial I_1} = \beta_1 paI_1^{\beta_1-1} I_2^{\beta_2} I_3^{\beta_3} - w_1 = 0 \Leftrightarrow paMP_1 = w_1 \\ \dfrac{\partial\pi}{\partial I_2} = \beta_2 paI_1^{\beta_1} I_2^{\beta_2-1} I_3^{\beta_3} - w_2 = 0 \Leftrightarrow paMP_2 = w_2 \\ \dfrac{\partial\pi}{\partial I_3} = \beta_3 paI_1^{\beta_1} I_2^{\beta_2} I_3^{\beta_3-1} - w_3 = 0 \Leftrightarrow paMP_3 = w_3 \end{cases}\right\} \Rightarrow p = \frac{w_1}{MP_1} = \frac{w_2}{MP_2} = \frac{w_3}{MP_3} \tag{12}$$

the following correlations are obtained (eq.13):

$$\begin{aligned} p &= \frac{w_1}{\beta_1 aI_1^{\beta_1-1} I_2^{\beta_2} I_3^{\beta_3}} = \frac{w_2}{\beta_2 aI_1^{\beta_1} I_2^{\beta_2-1} I_3^{\beta_3}} \Rightarrow I_2 = \frac{\beta_2}{\beta_1}\frac{w_1}{w_2} I_1 \\ p &= \frac{w_1}{\beta_1 aI_1^{\beta_1-1} I_2^{\beta_2} I_3^{\beta_3}} = \frac{w_3}{\beta_3 aI_1^{\beta_1} I_2^{\beta_2} I_3^{\beta_3-1}} \Rightarrow I_3 = \frac{\beta_3}{\beta_1}\frac{w_1}{w_3} I_1 \end{aligned} \tag{13}$$

If these relations are substituted in the first or other equations, the profit-maximizing demand curves are found in terms of prices as follow (eq.14):

$$\beta_1 aPI_1^{\beta_1-1}\left(\frac{\beta_2}{\beta_1}\frac{w_1}{w_2}I_1\right)^{\beta_2}\left(\frac{\beta_3}{\beta_1}\frac{w_1}{w_3}I_1\right)^{\beta_3} = w_1 \Rightarrow$$

$$\begin{aligned} I_1 &= \beta_1^{(1-\beta_2-\beta_3)/(1-\beta_1-\beta_2-\beta_3)} \beta_2^{\beta_2/(1-\beta_1-\beta_2-\beta_3)} \beta_3^{\beta_3/(1-\beta_1-\beta_2-\beta_3)} w_1^{(\beta_2+\beta_3-1)/(1-\beta_1-\beta_2-\beta_3)} w_2^{-\beta_2/(1-\beta_1-\beta_2-\beta_3)} w_3^{-\beta_3/(1-\beta_1-\beta_2-\beta_3)} p^{1/(1-\beta_1-\beta_2-\beta_3)} a^{1/(1-\beta_1-\beta_2-\beta_3)} \\ I_2 &= \beta_1^{\beta_1/(1-\beta_1-\beta_2-\beta_3)} \beta_2^{(1-\beta_1-\beta_3)/(1-\beta_1-\beta_2-\beta_3)} \beta_3^{\beta_3/(1-\beta_1-\beta_2-\beta_3)} w_1^{-\beta_1/(1-\beta_1-\beta_2-\beta_3)} w_2^{(\beta_1+\beta_3-1)/(1-\beta_1-\beta_2-\beta_3)} w_3^{-\beta_3/(1-\beta_1-\beta_2-\beta_3)} p^{1/(1-\beta_1-\beta_2-\beta_3)} a^{1/(1-\beta_1-\beta_2-\beta_3)} \\ I_3 &= \beta_1^{\beta_1/(1-\beta_1-\beta_2-\beta_3)} \beta_2^{\beta_2/(1-\beta_1-\beta_2-\beta_3)} \beta_3^{(1-\beta_1-\beta_2)/(1-\beta_1-\beta_2-\beta_3)} w_1^{-\beta_1/(1-\beta_1-\beta_2-\beta_3)} w_2^{-\beta_2/(1-\beta_1-\beta_2-\beta_3)} w_3^{(\beta_1+\beta_2-1)/(1-\beta_1-\beta_2-\beta_3)} p^{1/(1-\beta_1-\beta_2-\beta_3)} a^{1/(1-\beta_1-\beta_2-\beta_3)} \end{aligned} \tag{14}$$

Each factor demand of the profit-maximizing firm is negatively related to each of factor price, including price itself. For example, for $I_1(p,a,w_1,w_2,w_3)$ demand, that is $\forall\ \dfrac{\partial I_i}{\partial w_i} < 0, i = 1,2,3$ and $\forall\ \dfrac{\partial I_i}{\partial w_j} < 0,\ i \neq j, i = 1,2,3,\ j = 1,2,3.$ And $\forall\ \dfrac{\partial I_i}{\partial a} > 0,\ \ \forall \dfrac{\partial I_i}{\partial p} > 0$ as both $\beta_2+\beta_3<1$ and $\beta_1+\beta_2+\beta_3<1$ are assumed to be.

Each factor demand of the profit-maximizing firm is homogeneous of degree zero in terms of good and factor prices. If the price of the good in question and the factor prices used in its production increase at the same level, the factor demand will not change. Therefore, the supply of good is also homogeneous of degree zero in terms of good and factor prices.

If these profit-maximizing factor demands are substituted for the demand curves in the supply equation, the firm's indirect supply curve is obtained as follow (eq.15)[13]:

$$q = \beta_1^{\beta_1/(1-\beta_1-\beta_2-\beta_3)} \beta_2^{\beta_2/(1-\beta_1-\beta_2-\beta_3)} \beta_3^{\beta_3/(1-\beta_1-\beta_2-\beta_3)} w_1^{-\beta_1/(1-\beta_1-\beta_2-\beta_3)} w_2^{-\beta_2/(1-\beta_1-\beta_2-\beta_3)} w_3^{-\beta_3/(1-\beta_1-\beta_2-\beta_3)} p^{(\beta_1+\beta_2+\beta_3)/(1-\beta_1-\beta_2-\beta_3)} a^{1/(1-\beta_1-\beta_2-\beta_3)}$$

$$q = bw_1^{-\beta_1/(1-\beta_1-\beta_2-\beta_3)} w_2^{-\beta_2/(1-\beta_1-\beta_2-\beta_3)} w_3^{-\beta_3/(1-\beta_1-\beta_2-\beta_3)} p^{(\beta_1+\beta_2+\beta_3)/(1-\beta_1-\beta_2-\beta_3)} a^{1/(1-\beta_1-\beta_2-\beta_3)} \tag{15}$$

$$b = \beta_1^{\beta_1/(1-\beta_1-\beta_2-\beta_3)} \beta_2^{\beta_2/(1-\beta_1-\beta_2-\beta_3)} \beta_3^{\beta_3/(1-\beta_1-\beta_2-\beta_3)}$$

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$$\begin{aligned} q = a&\left[\beta_1^{(1-\beta_2-\beta_3)/(1-\beta_1-\beta_2-\beta_3)} \beta_2^{\beta_2/(1-\beta_1-\beta_2-\beta_3)} \beta_3^{\beta_3/(1-\beta_1-\beta_2-\beta_3)} w_1^{(\beta_2+\beta_3-1)/(1-\beta_1-\beta_2-\beta_3)} w_2^{-\beta_2/(1-\beta_1-\beta_2-\beta_3)} w_3^{-\beta_3/(1-\beta_1-\beta_2-\beta_3)} p^{1/(1-\beta_1-\beta_2-\beta_3)} a^{1/(1-\beta_1-\beta_2-\beta_3)}\right]^{\beta_1} \\ &\left[\beta_1^{\beta_1/(1-\beta_1-\beta_2-\beta_3)} \beta_2^{(1-\beta_1-\beta_3)/(1-\beta_1-\beta_2-\beta_3)} \beta_3^{\beta_3/(1-\beta_1-\beta_2-\beta_3)} w_1^{-\beta_1/(1-\beta_1-\beta_2-\beta_3)} w_2^{(\beta_1+\beta_3-1)/(1-\beta_1-\beta_2-\beta_3)} w_3^{-\beta_3/(1-\beta_1-\beta_2-\beta_3)} p^{1/(1-\beta_1-\beta_2-\beta_3)} a^{1/(1-\beta_1-\beta_2-\beta_3)}\right]^{\beta_2} \\ &\left[\beta_1^{\beta_1/(1-\beta_1-\beta_2-\beta_3)} \beta_2^{\beta_2/(1-\beta_1-\beta_2-\beta_3)} \beta_3^{(1-\beta_1-\beta_2)/(1-\beta_1-\beta_2-\beta_3)} w_1^{-\beta_1/(1-\beta_1-\beta_2-\beta_3)} w_2^{-\beta_2/(1-\beta_1-\beta_2-\beta_3)} w_3^{(\beta_1+\beta_2-1)/(1-\beta_1-\beta_2-\beta_3)} p^{1/(1-\beta_1-\beta_2-\beta_3)} a^{1/(1-\beta_1-\beta_2-\beta_3)}\right]^{\beta_3} \end{aligned}$$

Here, $\frac{\partial q}{\partial p} > 0, \frac{\partial q}{\partial w_1} < 0, \frac{\partial q}{\partial w_2} < 0, \frac{\partial q}{\partial w_3} < 0, \frac{\partial q}{\partial a} > 0$, assuming $\beta_1 + \beta_2 + \beta_3 < 1$. If the production function obtains "a" representing technology dimension factor as Solow (1957) introduced, the relationship is formed as $\frac{\partial q}{\partial a} > 0$. That is, the supply of firm increases at the same level of factor use as a level of the firm's technology progresses.

On the other hand, if this profit-maximizing factor demand equations are substituted in the profit equation (eq.16), the indirect profit curve is obtained in terms of good and factor prices as follows (eq.17)[14]:

$$\pi(p,w) = pq(p,w) - TC(p,w),$$

$$\pi(p,w) = p\left[aI_1^{\beta_1} I_2^{\beta_2} I_3^{\beta_3}\right] - \left[w_1 I_1 + w_2 I_2 + w_3 I_3\right] \tag{16}$$

$$\pi = (1-\beta_1-\beta_2-\beta_3)\beta_1^{\beta_1/(1-\beta_1-\beta_2-\beta_3)}\beta_2^{\beta_2/(1-\beta_1-\beta_2-\beta_3)}\beta_3^{\beta_3/(1-\beta_1-\beta_2-\beta_3)} w_1^{-\beta_1/(1-\beta_1-\beta_2-\beta_3)} w_2^{-\beta_2/(1-\beta_1-\beta_2-\beta_3)} w_3^{-\beta_3/(1-\beta_1-\beta_2-\beta_3)} p^{1/(1-\beta_1-\beta_2-\beta_3)} a^{1/(1-\beta_1-\beta_2-\beta_3)}$$

$$\pi = b' w_1^{-\beta_1/(1-\beta_1-\beta_2-\beta_3)} w_2^{-\beta_2/(1-\beta_1-\beta_2-\beta_3)} w_3^{-\beta_3/(1-\beta_1-\beta_2-\beta_3)} p^{1/(1-\beta_1-\beta_2-\beta_3)} a^{1/(1-\beta_1-\beta_2-\beta_3)} \tag{17}$$

$$b' = (1-\beta_1-\beta_2-\beta_3)\beta_1^{\beta_1/(1-\beta_1-\beta_2-\beta_3)}\beta_2^{\beta_2/(1-\beta_1-\beta_2-\beta_3)}\beta_3^{\beta_3/(1-\beta_1-\beta_2-\beta_3)}$$ [15].

This curve called the indirect profit equation or function, is first-order homogeneous in terms of good and factor prices. This indicates that if the prices of good and factors increase together at the same level, the profit of the firm will double. In this profit equation, assuming $\beta_1 + \beta_2 + \beta_3 < 1$, if the production function contains a technology dimension factor, such as "a", the relationship is formed like $\frac{\partial \pi}{\partial a} > 0$. In other words, as a result of technical progress, production increases, cost decreases and firm profit increases.

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$$\pi = pa\left[\beta_1^{(1-\beta_2-\beta_3)/(1-\beta_1-\beta_2-\beta_3)}\beta_2^{\beta_2/(1-\beta_1-\beta_2-\beta_3)}\beta_3^{\beta_3/(1-\beta_1-\beta_2-\beta_3)} w_1^{(\beta_2+\beta_3-1)/(1-\beta_1-\beta_2-\beta_3)} w_2^{-\beta_2/(1-\beta_1-\beta_2-\beta_3)} w_3^{-\beta_3/(1-\beta_1-\beta_2-\beta_3)} p^{1/(1-\beta_1-\beta_2-\beta_3)} a^{1/(1-\beta_1-\beta_2-\beta_3)}\right]^{\beta_1}$$

$$\left[\beta_1^{\beta_1/(1-\beta_1-\beta_2-\beta_3)}\beta_2^{(1-\beta_1-\beta_3)/(1-\beta_1-\beta_2-\beta_3)}\beta_3^{\beta_3/(1-\beta_1-\beta_2-\beta_3)} w_1^{-\beta_1/(1-\beta_1-\beta_2-\beta_3)} w_2^{(\beta_1+\beta_3-1)/(1-\beta_1-\beta_2-\beta_3)} w_3^{-\beta_3/(1-\beta_1-\beta_2-\beta_3)} p^{1/(1-\beta_1-\beta_2-\beta_3)} a^{1/(1-\beta_1-\beta_2-\beta_3)}\right]^{\beta_2}$$

$$\left[\beta_1^{\beta_1/(1-\beta_1-\beta_2-\beta_3)}\beta_2^{\beta_2/(1-\beta_1-\beta_2-\beta_3)}\beta_3^{(1-\beta_1-\beta_2)/(1-\beta_1-\beta_2-\beta_3)} w_1^{-\beta_1/(1-\beta_1-\beta_2-\beta_3)} w_2^{-\beta_2/(1-\beta_1-\beta_2-\beta_3)} w_3^{(\beta_1+\beta_2-1)/(1-\beta_1-\beta_2-\beta_3)} p^{1/(1-\beta_1-\beta_2-\beta_3)} a^{1/(1-\beta_1-\beta_2-\beta_3)}\right]^{\beta_3}$$

$$-$$

$$\begin{bmatrix} w_1\beta_1^{(1-\beta_2-\beta_3)/(1-\beta_1-\beta_2-\beta_3)}\beta_2^{\beta_2/(1-\beta_1-\beta_2-\beta_3)}\beta_3^{\beta_3/(1-\beta_1-\beta_2-\beta_3)} w_1^{(\beta_2+\beta_3-1)/(1-\beta_1-\beta_2-\beta_3)} w_2^{-\beta_2/(1-\beta_1-\beta_2-\beta_3)} w_3^{-\beta_3/(1-\beta_1-\beta_2-\beta_3)} p^{1/(1-\beta_1-\beta_2-\beta_3)} a^{1/(1-\beta_1-\beta_2-\beta_3)} \\ +w_2\beta_1^{\beta_1/(1-\beta_1-\beta_2-\beta_3)}\beta_2^{(1-\beta_1-\beta_3)/(1-\beta_1-\beta_2-\beta_3)}\beta_3^{\beta_3/(1-\beta_1-\beta_2-\beta_3)} w_1^{-\beta_1/(1-\beta_1-\beta_2-\beta_3)} w_2^{(\beta_1+\beta_3-1)/(1-\beta_1-\beta_2-\beta_3)} w_3^{-\beta_3/(1-\beta_1-\beta_2-\beta_3)} p^{1/(1-\beta_1-\beta_2-\beta_3)} a^{1/(1-\beta_1-\beta_2-\beta_3)} \\ +w_3\beta_1^{\beta_1/(1-\beta_1-\beta_2-\beta_3)}\beta_2^{\beta_2/(1-\beta_1-\beta_2-\beta_3)}\beta_3^{(1-\beta_1-\beta_2)/(1-\beta_1-\beta_2-\beta_3)} w_1^{-\beta_1/(1-\beta_1-\beta_2-\beta_3)} w_2^{-\beta_2/(1-\beta_1-\beta_2-\beta_3)} w_3^{(\beta_1+\beta_2-1)/(1-\beta_1-\beta_2-\beta_3)} p^{1/(1-\beta_1-\beta_2-\beta_3)} a^{1/(1-\beta_1-\beta_2-\beta_3)} \end{bmatrix}$$

$$\pi = pa\left[\beta_1^{\beta_1/(1-\beta_1-\beta_2-\beta_3)}\beta_2^{\beta_2/(1-\beta_1-\beta_2-\beta_3)}\beta_3^{\beta_3/(1-\beta_1-\beta_2-\beta_3)} w_1^{-\beta_1/(1-\beta_1-\beta_2-\beta_3)} w_2^{-\beta_2/(1-\beta_1-\beta_2-\beta_3)} w_3^{-\beta_3/(1-\beta_1-\beta_2-\beta_3)} p^{(\beta_1+\beta_2+\beta_3)/(1-\beta_1-\beta_2-\beta_3)} a^{(\beta_1+\beta_2+\beta_3)/(1-\beta_1-\beta_2-\beta_3)}\right]$$

$$-\left[\beta_1^{(1-\beta_2-\beta_3)/(1-\beta_1-\beta_2-\beta_3)}\beta_2^{\beta_2/(1-\beta_1-\beta_2-\beta_3)}\beta_3^{\beta_3/(1-\beta_1-\beta_2-\beta_3)} + \beta_1^{\beta_1/(1-\beta_1-\beta_2-\beta_3)}\beta_2^{(1-\beta_1-\beta_3)/(1-\beta_1-\beta_2-\beta_3)}\beta_3^{\beta_3/(1-\beta_1-\beta_2-\beta_3)} + \beta_1^{\beta_1/(1-\beta_1-\beta_2-\beta_3)}\beta_2^{\beta_2/(1-\beta_1-\beta_2-\beta_3)}\beta_3^{(1-\beta_1-\beta_2)/(1-\beta_1-\beta_2-\beta_3)}\right]$$

$$\left(w_1^{-\beta_1/(1-\beta_1-\beta_2-\beta_3)} w_2^{-\beta_2/(1-\beta_1-\beta_2-\beta_3)} w_3^{-\beta_3/(1-\beta_1-\beta_2-\beta_3)} p^{1/(1-\beta_1-\beta_2-\beta_3)} a^{1/(1-\beta_1-\beta_2-\beta_3)}\right)$$

15 *Note*: When $\beta_1^{(1-\beta_2-\beta_3)/(1-\beta_1-\beta_2-\beta_3)}$ *is* multiplied and divided by β_1^{-1}, $\beta_2^{(1-\beta_1-\beta_3)/(1-\beta_1-\beta_2-\beta_3)}$ *is* multiplied and divided by β_2^{-1} *and* $\beta_3^{(1-\beta_1-\beta_2)/(1-\beta_1-\beta_2-\beta_3)}$ *is* multiplied and divided by β_3^{-1}, one reaches b'. *In addition* b' is related to b as $b'=(1-\beta_1-\beta_2-\beta_3)\, b$.

2.3.2. Inferencing aggregate indirect factor demand, supply and profit curves

In this section, ignoring aggregation problems, indirect aggregate factor demand, supply and profit functions will be inferenced as the aggregate of firms' factors demands, supplies and profits curves in the case of *three* factors of production firm’s profit maximization model.

2.3.2.1. Aggregate factor demand curves

Aggregate factor demand curves are obtained as follow (eq.18):

$$\begin{aligned}
I_{1t} &= \sum_{i=1}^{N} \beta_{1it}^{(1-\beta_{2it}-\beta_{3it})/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} \beta_{2it}^{\beta_{2it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} \beta_{3it}^{\beta_{3it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} w_{1it}^{(\beta_{2it}+\beta_{3it}-1)/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} w_{2it}^{-\beta_{2it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} w_{3it}^{-\beta_{3it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} p_{it}^{1/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} a_{it}^{1/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} \\
I_{2t} &= \sum_{i=1}^{N} \beta_{1it}^{\beta_{1it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} \beta_{2it}^{(1-\beta_{1it}-\beta_{3it})/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} \beta_{3it}^{\beta_{3it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} w_{1it}^{-\beta_{1it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} w_{2it}^{(\beta_{1it}+\beta_{3it}-1)/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} w_{3it}^{-\beta_{3it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} p_{it}^{1/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} a_{it}^{1/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} \qquad (18) \\
I_{3t} &= \sum_{i=1}^{N} \beta_{1it}^{\beta_{1it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} \beta_{2it}^{\beta_{2it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} \beta_{3it}^{(1-\beta_{1it}-\beta_{2it})/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} w_{1it}^{-\beta_{1it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} w_{2it}^{-\beta_{2it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} w_{3it}^{(\beta_{1it}+\beta_{2it}-1)/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} p_{it}^{1/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} a_{it}^{1/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})}
\end{aligned}$$

If these demand equations are to be estimated, the autonomous factors are the autonomous demands for production factors one, two and three, are formulized respectively as (eq.19)[16]:

$$B_1 = \frac{\sum_{t=1}^{T}\sum_{i=1}^{N}\beta_{1it} b_{it}}{T}, B_2 = \frac{\sum_{t=1}^{T}\sum_{i=1}^{N}\beta_{2it} b_{it}}{T}, B_3 = \frac{\sum_{t=1}^{T}\sum_{i=1}^{N}\beta_{3it} b_{it}}{T},\; b_{it} = \beta_{1it}^{\beta_{1it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} \beta_{2it}^{\beta_{2it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} \beta_{3it}^{\beta_{3it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} \qquad (19)$$

It can be formulated by separating it from the relationship corresponding to the variables of each firm due to aggregation problems according to the time dimension. As a result, aggregate factor demand curves respectivelly to be used in macroeconometric model are as follow (eq.20):

$$\begin{aligned}
I_{1t}(W_t, P_t, A_t) &= B_1 W_{1t}^{(\beta_2+\beta_3-1)/(1-\beta_1-\beta_2-\beta_3)} W_{2t}^{-\beta_2/(1-\beta_1-\beta_2-\beta_3)} W_{3t}^{-\beta_3/(1-\beta_1-\beta_2-\beta_3)} P_t^{1/(1-\beta_1-\beta_2-\beta_3)} A_t^{1/(1-\beta_1-\beta_2-\beta_3)} \\
I_{2t}(W_t, P_t, A_t) &= B_2 W_{1t}^{-\beta_1/(1-\beta_1-\beta_2-\beta_3)} W_{2t}^{(\beta_1+\beta_3-1)/(1-\beta_1-\beta_2-\beta_3)} W_{3t}^{-\beta_3/(1-\beta_1-\beta_2-\beta_3)} P_t^{1/(1-\beta_1-\beta_2-\beta_3)} A_t^{1/(1-\beta_1-\beta_2-\beta_3)} \qquad (20) \\
I_{3t}(W_t, P_t, A_t) &= B_3 W_{1t}^{-\beta_1/(1-\beta_1-\beta_2-\beta_3)} W_{2t}^{-\beta_2/(1-\beta_1-\beta_2-\beta_3)} W_{3t}^{(\beta_1+\beta_2-1)/(1-\beta_1-\beta_2-\beta_3)} P_t^{1/(1-\beta_1-\beta_2-\beta_3)} A_t^{1/(1-\beta_1-\beta_2-\beta_3)}
\end{aligned}$$

The expected signs of the relationships between the aggregate factor demands and each explanatory variable is as follow:

$$\forall\ \frac{\partial I_i}{\partial W_i} < 0, i = 1,2,3 \text{ and } \forall\ \frac{\partial I_i}{\partial W_j} < 0,\ \text{i} \neq \text{j}, i = 1,2,3,\ j = 1,2,3. \text{ Ve } \forall \frac{\partial I_i}{\partial P} > 0 \,, \forall\ \frac{\partial I_i}{\partial A} > 0 \,.$$

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$$\begin{aligned}
B_1 &= \frac{\sum_{t=1}^{T}\sum_{i=1}^{N}\beta_{1it}^{(1-\beta_{2it}-\beta_{3it})/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})}\beta_{2it}^{\beta_{2it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})}\beta_{3it}^{\beta_{3it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})}}{T} \\
B_2 &= \frac{\sum_{t=1}^{T}\sum_{i=1}^{N}\beta_{1it}^{\beta_{1it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})}\beta_{2it}^{(1-\beta_{1it}-\beta_{3it})/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})}\beta_{3it}^{\beta_{3it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})}}{T} \\
B_3 &= \frac{\sum_{t=1}^{T}\sum_{i=1}^{N}\beta_{1it}^{\beta_{1it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})}\beta_{2it}^{\beta_{2it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})}\beta_{3it}^{(1-\beta_{1it}-\beta_{2it})/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})}}{T}
\end{aligned}$$

If the aggregate factor demand curves are to be estimated macroeconometrically, for example, the indirect aggregate factor demad equation of the second factor can be written as follows (eq.21):

$$LnI_{2t} = LnB_2 - \frac{\beta_1}{1-\beta_1-\beta_2-\beta_3} LnW_{1t} + \frac{\beta_1+\beta_3-1}{1-\beta_1-\beta_2-\beta_3} LnW_{2t} - \frac{\beta_3}{1-\beta_1-\beta_2-\beta_3} LnW_{3t} + \frac{1}{1-\beta_1-\beta_2-\beta_3} LnP_t + \frac{1}{1-\beta_1-\beta_2-\beta_3} LnA_t + \varepsilon_t \quad (21)$$

Here, the coefficients in front of the variables give the coefficients of elasticity of factor demand with respect to factor prices, price of good, and technology accordingly. The good price and technology elasticities of factor demand are positive, and the factor price elasticities of the factor of production are negative.

2.3.2.2. Aggregate supply curve

The aggregate supply and profit equations are obtained based on the assumption that the firm maximizes its profit for each "t" period. Namely;

$Q_t = \sum_{i=1}^{N} q_{it}(a_{it}, p_{tit}, w_{1it}, w_{2it}, ..., w_{nit}) \rightarrow GDP_t$ =f(P_t,W_t): It is the aggregate supply of firms producing in the country in period "t". This indirect supply curve is obtained as a function of the prices of the goods that firms sell and the factors of production they use. When the technology dimension coefficient is added, the functional structure takes the form of GDP_t=f(P_t,W_t,A_t) as in equation 22[17];

$$Q = B\ W_1^{-\beta_1/(1-\beta_1-\beta_2-\beta_3)} W_2^{-\beta_2/(1-\beta_1-\beta_2-\beta_3)} W_3^{-\beta_3/(1-\beta_1-\beta_2-\beta_3)}\ P^{(\beta_1+\beta_2+\beta_3)/(1-\beta_1-\beta_2-\beta_3)} A^{1/(1-\beta_1-\beta_2-\beta_3)} \quad (22)$$

$$B = \beta_1^{\beta_1/(1-\beta_1-\beta_2-\beta_3)}\ \beta_2^{\beta_2/(1-\beta_1-\beta_2-\beta_3)}\ \beta_3^{\beta_3/(1-\beta_1-\beta_2-\beta_3)}$$

If the natural logarithm values of both parts of the equation are taken, the aggregate supply equation will be as follows (eq.23):

$$LnQ_t = LnB - \frac{\beta_1}{1-\beta_1-\beta_2-\beta_3} LnW_{1t} - \frac{\beta_2}{1-\beta_1-\beta_2-\beta_3} LnW_{2t} - \frac{\beta_3}{1-\beta_1-\beta_2-\beta_3} LnW_{3t} + \frac{\beta_1+\beta_2+\beta_3}{1-\beta_1-\beta_2-\beta_3} LnP_t + \frac{1}{1-\beta_1-\beta_2-\beta_3} LnA_t + \varepsilon_t \quad (23)$$

LnB is the constant autonomous value in natural logarithmic form. When it is found to be statistically significant, it shows the total systematic effect of other factors on aggregate supply that are not used in the model. Here, the coefficients in front of each variable give the factor

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$$Q_t = \sum_{i=1}^{N} \beta_{1it}^{\beta_{1it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} \beta_{2it}^{\beta_{2i}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} \beta_{3it}^{\beta_{3it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} w_{1it}^{-\beta_{1it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} w_{2it}^{-\beta_{2i}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})}$$
$$w_{3it}^{-\beta_{3it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} p_{it}^{(\beta_{1it}+\beta_{2it}+\beta_{3it})/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} a_{it}^{1/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})}$$

$$Q_t(A_t, P_t, W_{1t}, W_{2t}) = \sum_{i=1}^{N} b_{it}\ w_{1it}^{-\beta_{1it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} w_{2it}^{-\beta_{2i}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} w_{3it}^{-\beta_{3it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} p_{it}^{(\beta_{1it}+\beta_{2it}+\beta_{3it})/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} a_{it}^{1/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})}$$

$$b_{it} = \beta_{1it}^{\beta_{1it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} \beta_{2it}^{\beta_{2i}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} \beta_{3it}^{\beta_{3it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})}$$

$$B = \frac{\sum_{i=1}^{T} B_t}{T} = \frac{\sum_{t=1}^{T}\sum_{i=1}^{N} b_{it}}{T} = \frac{\sum_{t=1}^{T}\sum_{i=1}^{N} \beta_{1it}^{\beta_{1it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} \beta_{2it}^{\beta_{2it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})} \beta_{3it}^{\beta_{3it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})}}{T}$$

$$B = \beta_1^{\beta_1/(1-\beta_1-\beta_2-\beta_3)}\ \beta_2^{\beta_2/(1-\beta_1-\beta_2-\beta_3)}\ \beta_3^{\beta_3/(1-\beta_1-\beta_2-\beta_3)}$$

and commodity price and technology elasticities of the aggregate supply respectively. These elasticity coefficients and their theortically expected signs are as follow:

$$\frac{\partial LnQ}{\partial LnW_1}=-\frac{\beta_1}{1-\beta_1-\beta_2-\beta_3}<0,\frac{\partial LnQ}{\partial LnW_2}=-\frac{\beta_2}{1-\beta_1-\beta_2-\beta_3}<0,\frac{\partial LnQ}{\partial LnW_3}=-\frac{\beta_3}{1-\beta_1-\beta_2-\beta_3}<0,\frac{\partial LnQ}{\partial LnP}=\frac{\beta_1+\beta_2+\beta_3}{1-\beta_1-\beta_2-\beta_3}>0,\frac{\partial LnQ}{\partial LnA}=\frac{1}{1-\beta_1-\beta_2-\beta_3}>0$$

2.3.2.3. Aggregate profit curve

Aggregate profit is the sum of the profits that firms make from the goods they produce. It is assumed that the firm that produces more than one good maximizes its profit on each good. Aggregate profit; $\pi_t=\sum_{i=1}^{N}\pi_{tit}(a_{it},p_{it},\mathrm{w}_{1it},w_{2it},...,w_{nit})\rightarrow\pi_t$ =f(P_t,W_t): The aggregate profits of the firm(s) that maximize profits in each period are obtained as a function of the prices of the goods they sell and the factors of production they use. When A_t Technology dimension coefficient is added, the functional structure takes the form of π_t=f(P_t,W_t,A_t) as in equation 24[18];

$$\pi=B'W_1^{-\beta_1/(1-\beta_1-\beta_2-\beta_3)}W_2^{-\beta_2/(1-\beta_1-\beta_2-\beta_3)}W_3^{-\beta_3/(1-\beta_1-\beta_2-\beta_3)}P^{1/(1-\beta_1-\beta_2-\beta_3)}A^{1/(1-\beta_1-\beta_2-\beta_3)} \qquad (24)$$

$$B'=(1-\beta_1-\beta_2-\beta_3)\beta_1^{\beta_1/(1-\beta_1-\beta_2-\beta_3)}\beta_2^{\beta_2/(1-\beta_1-\beta_2-\beta_3)}\beta_3^{\beta_3/(1-\beta_1-\beta_2-\beta_3)}$$

From this, the equation of aggregate profit for macroeconometric estimation purposes is inferenced as follows (eq.25)[19]:

$$Ln\pi_t=LnB'-\frac{\beta_1}{1-\beta_1-\beta_2-\beta_3}LnW_{1t}-\frac{\beta_2}{1-\beta_1-\beta_2-\beta_3}LnW_{2t}-\frac{\beta_3}{1-\beta_1-\beta_2-\beta_3}LnW_{3t}+\frac{1}{1-\beta_1-\beta_2-\beta_3}LnP_t+\frac{1}{1-\beta_1-\beta_2-\beta_3}LnA_t+\varepsilon_t \qquad (25)$$

LnB′ is the constant autonomous value. When found to be statistically significant, it shows the aggregate systematic effect of other factors on profit that are not used in the model.

Here, the coefficients in front of each variable give the price elasticities of the production factor of the aggregate profit and the elasticities of goods prices and technology respectively.

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[18] $$\pi_t=\sum_{i=1}^{N}\left\{a_{it}^{1/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})}w_{1it}^{-\beta_{1it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})}w_{2it}^{-\beta_{2it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})}w_{3it}^{-\beta_{3it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})}p_{it}^{1/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})}b_{it}'\right\}$$

$$\begin{aligned}b_{it}'=(&\beta_{1it}^{\beta_{1it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})}\beta_{2it}^{\beta_{2i}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})}\beta_{3it}^{\beta_{3i}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})}\\ &-\beta_{1it}^{\beta_{1it}/(1-\beta_{1i}-\beta_{2it}-\beta_{3it})}\beta_{2it}^{(1-\beta_{1it}-\beta_{3it})/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})}\beta_{3it}^{(1-\beta_{1it}-\beta_{2it})/(1-\beta_{1i}-\beta_{2it}-\beta_{3it})}\\ &-\beta_{1it}^{(1-\beta_{2it}-\beta_{3it})/(1-\beta_{1i}-\beta_{2it}-\beta_{3it})}\beta_{2it}^{\beta_{2it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})}\beta_{3it}^{(1-\beta_{1it}-\beta_{2it})/(1-\beta_{1i}-\beta_{2it}-\beta_{3it})}\\ &-\beta_{1it}^{(1-\beta_{2it}-\beta_{3it})/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})}\beta_{2it}^{(1-\beta_{1it}-\beta_{3it})/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})}\beta_{3it}^{\beta_{3it}/(1-\beta_{1i}-\beta_{2it}-\beta_{3it})})\end{aligned}$$

$$\pi=B'W_1^{-\beta_1/(1-\beta_1-\beta_2-\beta_3)}W_2^{-\beta_2/(1-\beta_1-\beta_2-\beta_3)}W_3^{-\beta_3/(1-\beta_1-\beta_2-\beta_3)}P^{1/(1-\beta_1-\beta_2-\beta_3)}A^{1/(1-\beta_1-\beta_2-\beta_3)}$$

$$B'=\frac{\sum_{i=1}^{T}B_t'}{T}=\frac{\sum_{i=1}^{T}\sum_{i=1}^{N}b_{it}'}{T}=\frac{\sum_{t=1}^{T}\sum_{i=1}^{N}(1-\beta_{1it}-\beta_{2it}-\beta_{3it})\beta_{1it}^{\beta_{1it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})}\beta_{2it}^{\beta_{2it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})}\beta_{3it}^{\beta_{3it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it})}}{T}$$

$$B'=(1-\beta_1-\beta_2-\beta_3)\beta_1^{\beta_1/(1-\beta_1-\beta_2-\beta_3)}\beta_2^{\beta_2/(1-\beta_1-\beta_2-\beta_3)}\beta_3^{\beta_3/(1-\beta_1-\beta_2-\beta_3)}.$$

[19] $B'=(1-\beta_1-\beta_2-\beta_3)B,\ B=\beta_1^{\beta_1/(1-\beta_1-\beta_2-\beta_3)}\beta_2^{\beta_2/(1-\beta_1-\beta_2-\beta_3)}\beta_3^{\beta_3/(1-\beta_1-\beta_2-\beta_3)}$

These elasticity coefficients and their theortically expected signs are: $\frac{\partial Ln\pi}{\partial LnW_1}=-\frac{\beta_1}{1-\beta_1-\beta_2-\beta_3}<0, \frac{\partial Ln\pi}{\partial LnW_2}=-\frac{\beta_2}{1-\beta_1-\beta_2-\beta_3}<0, \frac{\partial Ln\pi}{\partial LnW_3}=-\frac{\beta_3}{1-\beta_1-\beta_2-\beta_3}<0, \frac{\partial Ln\pi}{\partial LnP}=\frac{1}{1-\beta_1-\beta_2-\beta_3}>0, \frac{\partial Ln\pi}{\partial LnA}=\frac{1}{1-\beta_1-\beta_2-\beta_3}>0$

Since macro supply and profit models are aggregates, it is necessary to use the annual deflator or the price index of the relevant sector as average prices instead of the price of goods and the annual average price of each factor instead of factor prices of firms paid. However, the assumption of diminishing returns to scale should be made flexible in aggregation because tehere exist diferent types economies of scales depending on the sector. And one shall also relax the assumtion of the fully competitive market because we observe mostly imperfect competition in reality.

3. Inferences in the Case of Four Factors of Production

This section explains how to infer the factor demand, supply and profit equations mentioned above when the number of production factors is increased; in other words; it is discussed to show how to expand the patterns. However, in the case of four factors of production, the profit maximization problem of the firm can be stated as follows (eq. 26):

$$\underset{I_1, I_2, I_3, I_4}{Mak.}\ \pi(I_1, I_2, I_3, I_4) = p(aI_1^{\beta_1} I_2^{\beta_2} I_3^{\beta_3} I_4^{\beta_4}) - (w_1 I_1 + w_2 I_2 + w_3 I_3 + w_4 I_4), \sum_{i=1}^{4} \beta_i < 1 \quad (26)$$

From here, the following aggregate curves in relation to four factors of production case will be inferenced based on the derivations with three factors of production profit maximization case. Inferences for both the firm and the aggregate values are shown under subheadings without comments.

3.1. Firm Indirect Factor Demand Curves

In the case with four inputs, firms' indirect factor demand curves are inferenced as follows (eq.27)[20]:

[20]

$$I_1 = B_1 b\, w_1^{(\beta_2+\beta_3+\beta_4-1)/(1-\beta_1-\beta_2-\beta_3-\beta_4)} w_2^{-\beta_2/(1-\beta_1-\beta_2-\beta_3-\beta_4)} w_3^{-\beta_3/(1-\beta_1-\beta_2-\beta_3-\beta_4)} w_4^{-\beta_4/(1-\beta_1-\beta_2-\beta_3-\beta_4)} p^{1/(1-\beta_1-\beta_2-\beta_3-\beta_4)} a^{1/(1-\beta_1-\beta_2-\beta_3-\beta_4)}$$

$$I_2 = B_2 b\, w_1^{-\beta_1/(1-\beta_1-\beta_2-\beta_3-\beta_4)} w_2^{(\beta_1+\beta_3+\beta_4-1)/(1-\beta_1-\beta_2-\beta_3-\beta_4)} w_3^{-\beta_3/(1-\beta_1-\beta_2-\beta_3-\beta_4)} w_4^{-\beta_4/(1-\beta_1-\beta_2-\beta_3-\beta_4)} p^{1/(1-\beta_1-\beta_2-\beta_3-\beta_4)} a^{1/(1-\beta_1-\beta_2-\beta_3-\beta_4)}$$

$$I_3 = B_3 b\, w_1^{(\beta_2+\beta_3+\beta_4-1)/(1-\beta_1-\beta_2-\beta_3-\beta_4)} w_2^{-\beta_2/(1-\beta_1-\beta_2-\beta_3-\beta_4)} w_3^{(\beta_1+\beta_2+\beta_4-1)/(1-\beta_1-\beta_2-\beta_3-\beta_4)} w_4^{-\beta_4/(1-\beta_1-\beta_2-\beta_3-\beta_4)} p^{1/(1-\beta_1-\beta_2-\beta_3-\beta_4)} a^{1/(1-\beta_1-\beta_2-\beta_3-\beta_4)}$$

$$I_4 = B_4 b\, w_1^{-\beta_1/(1-\beta_1-\beta_2-\beta_3-\beta_4)} w_2^{(\beta_1+\beta_3+\beta_4-1)/(1-\beta_1-\beta_2-\beta_3-\beta_4)} w_3^{-\beta_3/(1-\beta_1-\beta_2-\beta_3-\beta_4)} w_4^{(\beta_1+\beta_2+\beta_3-1)/(1-\beta_1-\beta_2-\beta_3-\beta_4)} p^{1/(1-\beta_1-\beta_2-\beta_3-\beta_4)} a^{1/(1-\beta_1-\beta_2-\beta_3-\beta_4)}$$

$$b = \beta_1^{\beta_1/(1-\beta_1-\beta_2-\beta_3-\beta_4)} \beta_2^{\beta_2/(1-\beta_1-\beta_2-\beta_3-\beta_4)} \beta_3^{\beta_3/(1-\beta_1-\beta_2-\beta_3-\beta_4)} \beta_4^{\beta_4/(1-\beta_1-\beta_2-\beta_3-\beta_4)}$$

$$
\begin{aligned}
I_1 &= \beta_1^{(1-\beta_2-\beta_3-\beta_4)/(1-\beta_1-\beta_2-\beta_3-\beta_4)}\beta_2^{\beta_2/(1-\beta_1-\beta_2-\beta_3-\beta_4)}\beta_3^{\beta_3/(1-\beta_1-\beta_2-\beta_3-\beta_4)}\beta_4^{\beta_4/(1-\beta_1-\beta_2-\beta_3-\beta_4)}w_1^{(\beta_2+\beta_3+\beta_4-1)/(1-\beta_1-\beta_2-\beta_3-\beta_4)}w_2^{-\beta_2/(1-\beta_1-\beta_2-\beta_3-\beta_4)}\\
&\quad w_3^{-\beta_3/(1-\beta_1-\beta_2-\beta_3-\beta_4)}w_4^{-\beta_4/(1-\beta_1-\beta_2-\beta_3-\beta_4)}p^{1/(1-\beta_1-\beta_2-\beta_3-\beta_4)}a^{1/(1-\beta_1-\beta_2-\beta_3-\beta_4)}\\
I_2 &= \beta_1^{\beta_1/(1-\beta_1-\beta_2-\beta_3-\beta_4)}\beta_2^{(1-\beta_1-\beta_3-\beta_4)/(1-\beta_1-\beta_2-\beta_3-\beta_4)}\beta_3^{\beta_3/(1-\beta_1-\beta_2-\beta_3-\beta_4)}\beta_4^{\beta_4/(1-\beta_1-\beta_2-\beta_3-\beta_4)}w_1^{-\beta_1/(1-\beta_1-\beta_2-\beta_3-\beta_4)}w_2^{(\beta_1+\beta_3+\beta_4-1)/(1-\beta_1-\beta_2-\beta_3-\beta_4)}\\
&\quad w_3^{-\beta_3/(1-\beta_1-\beta_2-\beta_3-\beta_4)}w_4^{-\beta_4/(1-\beta_1-\beta_2-\beta_3-\beta_4)}p^{1/(1-\beta_1-\beta_2-\beta_3-\beta_4)}a^{1/(1-\beta_1-\beta_2-\beta_3-\beta_4)}\\
I_3 &= \beta_1^{(1-\beta_2-\beta_3-\beta_4)/(1-\beta_1-\beta_2-\beta_3-\beta_4)}\beta_2^{\beta_2/(1-\beta_1-\beta_2-\beta_3-\beta_4)}\beta_3^{(1-\beta_1-\beta_2-\beta_4)/(1-\beta_1-\beta_2-\beta_3-\beta_4)}\beta_4^{\beta_4/(1-\beta_1-\beta_2-\beta_3-\beta_4)}w_1^{(\beta_2+\beta_3+\beta_4-1)/(1-\beta_1-\beta_2-\beta_3-\beta_4)}w_2^{-\beta_2/(1-\beta_1-\beta_2-\beta_3-\beta_4)}\\
&\quad w_3^{(\beta_1+\beta_2+\beta_4-1)/(1-\beta_1-\beta_2-\beta_3-\beta_4)}w_4^{-\beta_4/(1-\beta_1-\beta_2-\beta_3-\beta_4)}p^{1/(1-\beta_1-\beta_2-\beta_3-\beta_4)}a^{1/(1-\beta_1-\beta_2-\beta_3-\beta_4)}\\
I_4 &= \beta_1^{\beta_1/(1-\beta_1-\beta_2-\beta_3-\beta_4)}\beta_2^{\beta_2/(1-\beta_1-\beta_2-\beta_3-\beta_4)}\beta_3^{\beta_3/(1-\beta_1-\beta_2-\beta_3-\beta_4)}\beta_4^{(1-\beta_1-\beta_2-\beta_3)/(1-\beta_1-\beta_2-\beta_3-\beta_4)}w_1^{-\beta_1/(1-\beta_1-\beta_2-\beta_3-\beta_4)}w_2^{(\beta_1+\beta_3+\beta_4-1)/(1-\beta_1-\beta_2-\beta_3-\beta_4)}\\
&\quad w_3^{-\beta_3/(1-\beta_1-\beta_2-\beta_3-\beta_4)}w_4^{(\beta_1+\beta_2+\beta_3-1)/(1-\beta_1-\beta_2-\beta_3-\beta_4)}p^{1/(1-\beta_1-\beta_2-\beta_3-\beta_4)}a^{1/(1-\beta_1-\beta_2-\beta_3-\beta_4)}
\end{aligned}
\tag{27}
$$

The direction of the relationship between each factor demand and explanatory variables that maximizes firm profit is shown as follows: $\forall \frac{\partial I_i}{\partial w_i} < 0, i = 1,2,3,4,\ \forall \frac{\partial I_i}{\partial w_j} < 0,\ \text{i} \neq \text{j}, i = 1,2,3,4,\ j = 1,2,3,4,\ \forall \frac{\partial I_i}{\partial p} > 0,\ \forall \frac{\partial I_i}{\partial a} > 0$

3. 2. Firm Indirect Supply Curve

The firm's indirect supply curve is inferenced as follows (eq.28)[21]:

$$
\begin{aligned}
q &= \beta_1^{\beta_1/(1-\beta_1-\beta_2-\beta_3-\beta_4)}\beta_2^{\beta_2/(1-\beta_1-\beta_2-\beta_3-\beta_4)}\beta_3^{\beta_3/(1-\beta_1-\beta_2-\beta_3-\beta_4)}\beta_4^{\beta_4/(1-\beta_1-\beta_2-\beta_3-\beta_4)}w_1^{-\beta_1/(1-\beta_1-\beta_2-\beta_3-\beta_4)}w_2^{-\beta_2/(1-\beta_1-\beta_2-\beta_3-\beta_4)}\\
&\quad w_3^{-\beta_3/(1-\beta_1-\beta_2-\beta_3-\beta_4)}w_4^{-\beta_4/(1-\beta_1-\beta_2-\beta_3-\beta_4)}p^{(\beta_1+\beta_2+\beta_3+\beta_4)/(1-\beta_1-\beta_2-\beta_3-\beta_4)}a^{1/(1-\beta_1-\beta_2-\beta_3-\beta_4)}
\end{aligned}
\tag{28}
$$

3. 3. Firm Profit Curve

The firm's indirect profit curve is inferenced as follows (eq.29)[22]:

$$
\begin{aligned}
\pi &= (1-\beta_1-\beta_2-\beta_3-\beta_4)\beta_1^{\beta_1/(1-\beta_1-\beta_2-\beta_3-\beta_4)}\beta_2^{\beta_2/(1-\beta_1-\beta_2-\beta_3-\beta_4)}\beta_3^{\beta_3/(1-\beta_1-\beta_2-\beta_3-\beta_4)}\beta_4^{\beta_4/(1-\beta_1-\beta_2-\beta_3-\beta_4)}w_1^{-\beta_1/(1-\beta_1-\beta_2-\beta_3-\beta_4)}\\
&\quad w_2^{-\beta_2/(1-\beta_1-\beta_2-\beta_3-\beta_4)}w_3^{-\beta_3/(1-\beta_1-\beta_2-\beta_3-\beta_4)}w_4^{-\beta_4/(1-\beta_1-\beta_2-\beta_3-\beta_4)}p^{1/(1-\beta_1-\beta_2-\beta_3-\beta_4)}a^{1/(1-\beta_1-\beta_2-\beta_3-\beta_4)}
\end{aligned}
\tag{29}
$$

3.4. Aggregate Factor Demand, Supply and Profit Curves

3.4.1. Aggregate factor demand

In the case with four inputs, aggregate factor demand equations are aggregated as in equation 30;

21

$$q(w,p,a) = b\ w_1^{-\beta_1/(1-\beta_1-\beta_2-\beta_3-\beta_4)}w_2^{-\beta_2/(1-\beta_1-\beta_2-\beta_3-\beta_4)}w_3^{-\beta_3/(1-\beta_1-\beta_2-\beta_3-\beta_4)}w_4^{-\beta_4/(1-\beta_1-\beta_2-\beta_3-\beta_4)}p^{(\beta_1+\beta_2+\beta_3+\beta_4)/(1-\beta_1-\beta_2-\beta_3-\beta_4)}a^{1/(1-\beta_1-\beta_2-\beta_3-\beta_4)}$$

$$b = \beta_1^{\beta_1/(1-\beta_1-\beta_2-\beta_3-\beta_4)}\beta_2^{\beta_2/(1-\beta_1-\beta_2-\beta_3-\beta_4)}\beta_3^{\beta_3/(1-\beta_1-\beta_2-\beta_3-\beta_4)}\beta_4^{\beta_4/(1-\beta_1-\beta_2-\beta_3-\beta_4)}$$

22

$$\pi = b'w_1^{-\beta_1/(1-\beta_1-\beta_2-\beta_3-\beta_4)}w_2^{-\beta_2/(1-\beta_1-\beta_2-\beta_3-\beta_4)}w_3^{-\beta_3/(1-\beta_1-\beta_2-\beta_3-\beta_4)}w_4^{-\beta_4/(1-\beta_1-\beta_2-\beta_3-\beta_4)}p^{1/(1-\beta_1-\beta_2-\beta_3-\beta_4)}a^{1/(1-\beta_1-\beta_2-\beta_3-\beta_4)},\quad \beta_1+\beta_2+\beta_3+\beta_4<1$$

$$b' = (1-\beta_1-\beta_2-\beta_3-\beta_4)\beta_1^{\beta_1/(1-\beta_1-\beta_2-\beta_3-\beta_4)}\beta_2^{\beta_2/(1-\beta_1-\beta_2-\beta_3-\beta_4)}\beta_3^{\beta_3/(1-\beta_1-\beta_2-\beta_3-\beta_4)}\beta_4^{\beta_4/(1-\beta_1-\beta_2-\beta_3-\beta_4)}$$

$$b' = (1-\beta_1-\beta_2-\beta_3-\beta_4)b$$

$$
\begin{aligned}
I_{1t} &= \sum_{i=1}^{N} \beta_{1it}^{(1-\beta_{2it}-\beta_{3it}-\beta_{4it})/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} \beta_{2it}^{\beta_{2it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} \beta_{3it}^{\beta_{3it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} \beta_{4it}^{\beta_{4it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} w_{1it}^{(\beta_{2it}+\beta_{3it}+\beta_{4it}-1)/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} \\
&\quad w_{2it}^{-\beta_{2it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} w_{3it}^{-\beta_{3it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} w_{4it}^{-\beta_{4it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} p_{it}^{1/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} a_{it}^{1/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} \\
I_{2t} &= \sum_{i=1}^{N} \beta_{1it}^{\beta_{1it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} \beta_{2it}^{(1-\beta_{1it}-\beta_{3it}-\beta_{4it})/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} \beta_{3it}^{\beta_{3it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} \beta_{4it}^{\beta_{4it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} w_{1it}^{-\beta_{1it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} \\
&\quad w_{2it}^{(\beta_{1it}+\beta_{3it}+\beta_{4it}-1)/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} w_{3it}^{-\beta_{3it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} w_{4it}^{-\beta_{4it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} p_{it}^{1/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} a_{it}^{1/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} \\
I_{3t} &= \sum_{i=1}^{N} \beta_{1it}^{\beta_{1it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} \beta_{2it}^{\beta_{2it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} \beta_{3it}^{(1-\beta_{1it}-\beta_{2it}-\beta_{4it})/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} \beta_{4it}^{\beta_{4it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} w_{1it}^{-\beta_{1it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} \\
&\quad w_{2it}^{-\beta_{2it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} w_{3it}^{(\beta_{1it}+\beta_{2it}+\beta_{4it}-1)/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} w_{4it}^{-\beta_{4it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} p_{it}^{1/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} a_{it}^{1/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} \\
I_{4t} &= \sum_{i=1}^{N} \beta_{1it}^{\beta_{1it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} \beta_{2it}^{\beta_{2it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} \beta_{3it}^{\beta_{3it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} \beta_{4it}^{(1-\beta_{1it}-\beta_{2it}-\beta_{3it})/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} w_{1it}^{-\beta_{1it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} \\
&\quad w_{2it}^{-\beta_{2it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} w_{3it}^{-\beta_{3it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} w_{4it}^{(\beta_{1it}+\beta_{2it}+\beta_{3it}-1)/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} p_{it}^{1/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} a_{it}^{1/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})}
\end{aligned}
\tag{30}
$$

and the autonomous demands for production factors one, two, three, and four are determined as follows respectively in equation 31[23];

$$
B_1 = \frac{\sum_{t=1}^{T}\sum_{i=1}^{N} \beta_{1it} b_{it}}{T},\; B_2 = \frac{\sum_{t=1}^{T}\sum_{i=1}^{N} \beta_{2it} b_{it}}{T},\; B_3 = \frac{\sum_{t=1}^{T}\sum_{i=1}^{N} \beta_{3it} b_{it}}{T},\; B_4 = \frac{\sum_{t=1}^{T}\sum_{i=1}^{N} \beta_{4it} b_{it}}{T}, \tag{31}
$$

$$
b_{it} = \beta_{1it}^{\beta_{1it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} \beta_{2it}^{\beta_{2it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} \beta_{3it}^{\beta_{3it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} \beta_{4it}^{\beta_{4it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})}
$$

Then, each aggregate factor demand equation is deduced as follows (eq.32):

$$
\begin{aligned}
I_{1t}(W_t, P_t, A_t) &= B_1 W_{1t}^{(\beta_2+\beta_3+\beta_4-1)/(1-\beta_1-\beta_2-\beta_3-\beta_4)} W_{2t}^{-\beta_2/(1-\beta_1-\beta_2-\beta_3-\beta_4)} W_{3t}^{-\beta_3/(1-\beta_1-\beta_2-\beta_3-\beta_4)} W_{4t}^{-\beta_4/(1-\beta_1-\beta_2-\beta_3-\beta_4)} P_t^{1/(1-\beta_1-\beta_2-\beta_3-\beta_4)} A_t^{1/(1-\beta_1-\beta_2-\beta_3-\beta_4)} \\
I_{2t}(W_t, P_t, A_t) &= B_2 W_{1t}^{-\beta_1/(1-\beta_1-\beta_2-\beta_3-\beta_4)} W_{2t}^{(\beta_1+\beta_3+\beta_4-1)/(1-\beta_1-\beta_2-\beta_3-\beta_4)} W_{3t}^{-\beta_3/(1-\beta_1-\beta_2-\beta_3-\beta_4)} W_{4t}^{-\beta_4/(1-\beta_1-\beta_2-\beta_3-\beta_4)} P_t^{1/(1-\beta_1-\beta_2-\beta_3-\beta_4)} A_t^{1/(1-\beta_1-\beta_2-\beta_3-\beta_4)} \\
I_{3t}(W_t, P_t, A_t) &= B_3 W_{1t}^{-\beta_1/(1-\beta_1-\beta_2-\beta_3-\beta_4)} W_{2t}^{-\beta_2/(1-\beta_1-\beta_2-\beta_3-\beta_4)} W_{3t}^{(\beta_1+\beta_2+\beta_4-1)/(1-\beta_1-\beta_2-\beta_3-\beta_4)} W_{4t}^{-\beta_4/(1-\beta_1-\beta_2-\beta_3-\beta_4)} P_t^{1/(1-\beta_1-\beta_2-\beta_3-\beta_4)} A_t^{1/(1-\beta_1-\beta_2-\beta_3-\beta_4)} \\
I_{4t}(W_t, P_t, A_t) &= B_4 W_{1t}^{-\beta_1/(1-\beta_1-\beta_2-\beta_3-\beta_4)} W_{2t}^{-\beta_2/(1-\beta_1-\beta_2-\beta_3-\beta_4)} W_{3t}^{-\beta_3/(1-\beta_1-\beta_2-\beta_3-\beta_4)} W_{4t}^{(\beta_1+\beta_2+\beta_3-1)/(1-\beta_1-\beta_2-\beta_3-\beta_4)} P_t^{1/(1-\beta_1-\beta_2-\beta_3-\beta_4)} A_t^{1/(1-\beta_1-\beta_2-\beta_3-\beta_4)}
\end{aligned}
\tag{32}
$$

And each one of them can be used for a macroeconometric model. The expected directions of the relationship between profit-maximizing aggregate factor demands and explanatory variables are as follow:

$\forall\ \frac{\partial I_i}{\partial W_i} < 0, i = 1,2,3,4$ and $\forall\ \frac{\partial I_i}{\partial W_j} < 0,\ \text{i} \neq \text{j}, i = 1,2,3,4,\ j = 1,2,3,4.$ And $\forall \frac{\partial I_i}{\partial P} > 0$, $\forall\ \frac{\partial I_i}{\partial A} > 0$.

3.4.2. Aggregate supply

23

$$
B_1 = \frac{\sum_{t=1}^{T}\sum_{i=1}^{N} \beta_{1it}^{(1-\beta_{2it}-\beta_{3it}-\beta_{4it})/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} \beta_{2it}^{\beta_{2it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} \beta_{3it}^{\beta_{3it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} \beta_{4it}^{\beta_{4it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})}}{T}
$$

$$
B_2 = \frac{\sum_{t=1}^{T}\sum_{i=1}^{N} \beta_{1it}^{\beta_{1it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} \beta_{2it}^{(1-\beta_{1it}-\beta_{3it}-\beta_{4it})/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} \beta_{3it}^{\beta_{3it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} \beta_{4it}^{\beta_{4it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})}}{T}
$$

$$
B_3 = \frac{\sum_{t=1}^{T}\sum_{i=1}^{N} \beta_{1it}^{\beta_{1it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} \beta_{2it}^{\beta_{2it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} \beta_{3it}^{(1-\beta_{1it}-\beta_{2it}-\beta_{4it})/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} \beta_{4it}^{\beta_{4it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})}}{T}
$$

$$
B_4 = \frac{\sum_{t=1}^{T}\sum_{i=1}^{N} \beta_{1it}^{\beta_{1it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} \beta_{2it}^{\beta_{2it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} \beta_{3it}^{\beta_{3it}/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})} \beta_{4it}^{(1-\beta_{1it}-\beta_{2it}-\beta_{3it})/(1-\beta_{1it}-\beta_{2it}-\beta_{3it}-\beta_{4it})}}{T}
$$

The indirect aggregate supply equation is deduced as follows (eq.33 and eq.34):

$$Q_t = \beta_1^{\beta_1/(1-\beta_1-\beta_2-\beta_3-\beta_4)} \beta_2^{\beta_2/(1-\beta_1-\beta_2-\beta_3-\beta_4)} \beta_3^{\beta_3/(1-\beta_1-\beta_2-\beta_3-\beta_4)} \beta_4^{\beta_4/(1-\beta_1-\beta_2-\beta_3-\beta_4)} W_{1t}^{-\beta_1/(1-\beta_1-\beta_2-\beta_3-\beta_4)} W_{2t}^{-\beta_2/(1-\beta_1-\beta_2-\beta_3-\beta_4)} W_{3t}^{-\beta_3/(1-\beta_1-\beta_2-\beta_3-\beta_4)} W_{4t}^{-\beta_4/(1-\beta_1-\beta_2-\beta_3-\beta_4)} P_t^{(\beta_1+\beta_2+\beta_3+\beta_4)/(1-\beta_1-\beta_2-\beta_3-\beta_4)} A_t^{1/(1-\beta_1-\beta_2-\beta_3-\beta_4)} \quad (33)$$

$$Q_t(W_t, P_t, A_t) = B\, W_{1t}^{-\beta_1/(1-\beta_1-\beta_2-\beta_3-\beta_4)} W_{2t}^{-\beta_2/(1-\beta_1-\beta_2-\beta_3-\beta_4)} W_{3t}^{-\beta_3/(1-\beta_1-\beta_2-\beta_3-\beta_4)} W_4^{-\beta_4/(1-\beta_1-\beta_2-\beta_3-\beta_4)} P_t^{(\beta_1+\beta_2+\beta_3+\beta_4)/(1-\beta_1-\beta_2-\beta_3-\beta_4)} A_t^{1/(1-\beta_1-\beta_2-\beta_3-\beta_4)}$$

$$B = \beta_1^{\beta_1/(1-\beta_1-\beta_2-\beta_3-\beta_4)} \beta_2^{\beta_2/(1-\beta_1-\beta_2-\beta_3-\beta_4)} \beta_3^{\beta_3/(1-\beta_1-\beta_2-\beta_3-\beta_4)} \beta_4^{\beta_4/(1-\beta_1-\beta_2-\beta_3-\beta_4)} \quad (34)$$

3.4.3. Aggregate profit

The indirect aggregate profit equation is deduced as follows (eq.35-36):

$$\pi_t = (1-\beta_1-\beta_2-\beta_3-\beta_4)\beta_1^{\beta_1/(1-\beta_1-\beta_2-\beta_3-\beta_4)} \beta_2^{\beta_2/(1-\beta_1-\beta_2-\beta_3-\beta_4)} \beta_3^{\beta_3/(1-\beta_1-\beta_2-\beta_3-\beta_4)} \beta_4^{\beta_4/(1-\beta_1-\beta_2-\beta_3-\beta_4)} W_{1t}^{-\beta_1/(1-\beta_1-\beta_2-\beta_3-\beta_4)} W_{2t}^{-\beta_2/(1-\beta_1-\beta_2-\beta_3-\beta_4)} W_{3t}^{-\beta_3/(1-\beta_1-\beta_2-\beta_3-\beta_4)} W_{4t}^{-\beta_4/(1-\beta_1-\beta_2-\beta_3-\beta_4)} P_t^{1/(1-\beta_1-\beta_2-\beta_3-\beta_4)} A_t^{1/(1-\beta_1-\beta_2-\beta_3-\beta_4)} \quad (35)$$

$$\pi_t(W_t, P_t, A_t) = B' W_{1t}^{-\beta_1/(1-\beta_1-\beta_2-\beta_3-\beta_4)} W_{2t}^{-\beta_2/(1-\beta_1-\beta_2-\beta_3-\beta_4)} W_{3t}^{-\beta_3/(1-\beta_1-\beta_2-\beta_3-\beta_4)} W_{4t}^{-\beta_4/(1-\beta_1-\beta_2-\beta_3-\beta_4)} P_t^{1/(1-\beta_1-\beta_2-\beta_3-\beta_4)} A_t^{1/(1-\beta_1-\beta_2-\beta_3-\beta_4)} \quad (36)$$

$$B' = (1-\beta_1-\beta_2-\beta_3-\beta_4)\beta_1^{\beta_1/(1-\beta_1-\beta_2-\beta_3-\beta_4)} \beta_2^{\beta_2/(1-\beta_1-\beta_2-\beta_3-\beta_4)} \beta_3^{\beta_3/(1-\beta_1-\beta_2-\beta_3-\beta_4)} \beta_4^{\beta_4/(1-\beta_1-\beta_2-\beta_3-\beta_4)}$$

$$B' = (1-\beta_1-\beta_2-\beta_3-\beta_4)B$$

4. Directions of Functional Relationships

In this section, the relationships between the firm's and the aggregate factor demand, supply, and profit curves and their explanatory variables will be stated collectively.

4.1. Factor Demand

Factor demands that maximize firm profit; the direction of the relationship between $I_i(w,p,a)$ and each explanatory variable is as follows:

$$\forall \frac{\partial I_i}{\partial w_i} < 0,\ i = 1,2,3,4,...,\ \forall \frac{\partial I_i}{\partial w_j} < 0,\ i \neq j,\ i = 1,2,3,4,...,\ j = 1,2,3,4,...,\ \forall \frac{\partial I_i}{\partial p} > 0,\ \forall \frac{\partial I_i}{\partial a} > 0\ .$$

Profit maximizing aggregate factor demands; the expected direction of the relationship between $I_i(W,P,A)$ and each explanatory variable is as follows:

$$\forall \frac{\partial I_i}{\partial W_i} < 0,\ i = 1,2,3,4,...,\ \forall \frac{\partial I_i}{\partial W_j} < 0,\ i \neq j,\ i = 1,2,3,4,...,\ j = 1,2,3,4,...,\ \forall \frac{\partial I_i}{\partial P} > 0,\ \forall \frac{\partial I_i}{\partial A} > 0\ .$$

4.2. Supply

The supply that maximizes firm profit; the direction of the relationship between q(w,p,a) and each explanatory variable is as follows: $\forall \frac{\partial q}{\partial w_i} < 0, i = 1,2,3,4,...,\ and\ \frac{\partial q}{\partial p} > 0, \frac{\partial q}{\partial a} > 0$.

The expected direction of the relationship between aggregate supply; Q(W, P, A), and each explanatory variable is as follows: $\forall \frac{\partial Q}{\partial W_i} < 0, i = 1,2,3,4,...,\ and\ \frac{\partial Q}{\partial P} > 0, \frac{\partial Q}{\partial A} > 0$.

4.3. Profit

The expected direction of the relationship between the firm's profit; π(w,p,a), and each explanatory variable is as follows: $\forall \frac{\partial \pi}{\partial w_i} < 0, i = 1,2,3,4,..., \; and \; \frac{\partial \pi}{\partial p} > 0, \frac{\partial \pi}{\partial a} > 0$

The expected direction of the relationship between aggregate profit; π(W,P,A) and each explanatory variable is as follows: $\forall \frac{\partial \pi}{\partial W_i} < 0, i = 1,2,3,4,..., \; and \; \frac{\partial \pi}{\partial P} > 0, \frac{\partial \pi}{\partial A} > 0 \cdot$

5. Conclusion

Inferring aggregate factor demand, supply and profit functions for a sector or the whole of the economy from micro-foundations is identical to the profit maximization behavior of firms. In line with this idea, in this study, through firm profit maximization analysis, (i) How to relate macroeconomics to microeconomics issues, (ii) How to reach macroeconomic equations from microeconomic equations, (iii) How to deduce equation patterns and expand, (iv) How to create micro and macro-econometric model equations based on the inferences patterns, What the expected direction of the relationships will be and What the parameters mean are revealed.

In the monetary economy, economic actors are more concerned with the prices of goods and factors of production rather than physical values when making decisions. Models that take these monetary factors into account will help one understand the economic logical relationships. Therefore, from profit maximization analysis based on the Cobb-Douglas type production function, micro and macro factor demand, supply and profit functions or equations can be estimated by the average prices of factors and goods and technical progress variables. Then, the aggregated economic models deduced can be tested. Especially in macro growth models, prices can be used instead of physical labor and capital. As a result of the concentration of studies in this direction, solutions are sought to the measurement problems of the variables to be used to represent goods and factor prices.

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