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Araştırma Makalesi / Research Article

Application of Chaotic Maps to Economic Load Dispatch Problem

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Abstract

This paper aims to solve the economic load dispatch problem (ELD) by using random numbers generated by chaotic maps with particle swarm optimization (PSO). The randomly generated coefficients r1 and r2 in the velocity equation of the PSO algorithm are generated by three different chaotic map methods namely logistic map, gaussian map, and tent map. As a result, three different methods are proposed: PSO with logistic map (LMPSO), PSO with Gaussian map (GMSPO), and PSO with tent map (TMPSO). These algorithms are applied to a 40-unit test system that includes transmission line losses, and the results are compared with the standard PSO algorithm. Each algorithm was run 50 times, and the maximum, minimum, and average values were recorded. All the proposed methods found lower costs than the standard PSO algorithm. Although the lowest cost was achieved with the GMPSO algorithm, the LMPSO algorithm was observed to be more successful on average.

Keywords: Economic Load Dispatch, Particle Swarm Optimization, Chaotic Maps, Optimization.

Kaotik Haritaların Ekonomik Yük Dağıtımı Problemine Uygulanması

Öz

Bu çalışmada kaotik haritalar ile üretilen rassal sayıların parçacık sürü optimizasyonu (PSO) ile kullanılarak ekonomik yük dağıtımı probleminin (EYD) çözülmesi hedeflenmiştir. PSO algoritmasının hız denkleminde yer alan ve rastgele oluşturulan r1 ve r2 katsayıları Lojistik kaotik harita metodu ile oluşturularak Lojistik haritalı PSO (LMPSO), gauss kaotik harita metodu ile oluşturularak gauss haritalı PSO (GMPSO) ve çadır kaotik harita metodu ile oluşturularak çadır haritalı PSO (TMPSO) metotları oluşturulmuştur. Oluşturulan bu algoritmalar iletim hattı kayıplarının dahil edildiği 40 üniteli test sistemine uygulanmış ve sonuçlar standart PSO algoritması ile karşılaştırılmıştır. Her algoritma 50 defa çalıştırılmış ve maksimum, minimum ve ortalama değerler kaydedilmiştir. Önerilen metotların hepsi standart PSO algoritmasından daha düşük maliyetler bulmuştur. En düşük maliyete GMPSO algoritması ile ulaşılmış olsa da ortalamada LMPSO algoritmasının daha başarılı olduğu gözlenmiştir.

Anahtar Kelimeler: Ekonomik Yük Dağıtımı, Parçacık Sürü Optimizasyonu, Kaotik Haritalar, Optimizasyon.

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1. Introduction

Economic load dispatch (ELD) is one of the most important problems to be solved in the operation and planning of a power system. The main objective of the ELD problem is to determine the optimum values of the power outputs of all generating units to meet the load demand at minimum operating cost while satisfying the system equality and inequality constraints. Therefore, the ELD problem can be defined as a nonlinear, constrained optimization problem (Balamurugan & Subramanian, 2007). Metaheuristic algorithms are used to solve nonlinear optimization problems and natural phenomena have caused different metaheuristic algorithms to be proposed (Onan, 2013). For example, the genetic algorithm (GA) is based on natural selection (Barati & Sadeghi, 2018). The particle swarm optimization (PSO) algorithm is based on the movements of the swarm of birds and fish. (Doğru et al., 2019).

In the literature, it is seen that different metaheuristic algorithms, such as PSO (Sudhakaran et al., 2007) and GA (Walters & Sheble, 1993), are used to solve the ELD problem. New hybrid metaheuristic algorithms have also been created by using metaheuristic algorithms together to solve the ELD problem faster and more optimally (Younes & Benhamida, 2011).

The chaotic maps are used to improve the performance of metaheuristic algorithms. Chaotic maps have also been utilized to obtain the optimum result in the ELD problem. Chaotic maps are used to generate the random numbers needed by metaheuristic algorithms by utilizing chaos theory with simple mathematical equations and easy initial conditions. Chaotic firefly algorithm (Arul et al., 2013), chaotic bat algorithm (Adarsh et al., 2016), and chaotic PSO (Tao & Jin-ding, 2009) are among the works encountered in the literature to solve the ELD problem.

The work by Xu et al. demonstrates that integrating Chaotic Local Search (CLS) into the Grey Wolf Optimizer (GWO) significantly improves the algorithm's performance, with the piecewise linear chaotic map (PWLCM) and Gaussian map identified as particularly effective in enhancing GWO's search capabilities (Xu et al., 2021). Another study introduces the Chaotic Artificial Ecosystem-Based Optimization Algorithm (CAEO) to optimize economic load dispatch and reduce environmental pollution, showing superior results compared to conventional methods (Hassan et al., 2021).In 2018, Rezaie et al. studied to develop the Chaotic Improved Harmony Search Algorithm (CIHSA), which, when applied to the Combined Economic Emission Dispatch (CEED) problem, delivers higher quality and more accurate solutions compared to other existing techniques.

In the literature review, it was observed that different chaotic maps and optimization methods were applied to the ELD problem, but no study was found in which different chaotic maps were added to the same optimization method and applied to the ELD problem. In this study, the randomly generated numbers in the velocity equation of the PSO algorithm are generated by logistic chaotic map, Gaussian chaotic map, and tent chaotic map methods. With these methods, PSO with logistic map, PSO with Gaussian map, and PSO with tent map methods are created.

2. Materials and Methods

2.1. Economic Load Dispatch

In the ELD problem, the cost function of each unit is represented by a quadratic function. The fuel cost corresponding to the power produced by each generator is shown in equation (1) (Zaraki & Othman, 2009).

$$C_i = \sum_{i=1}^{N} a_i P_i^2 + b_i P_i + c_i \tag{1}$$

Where,

Ci: fuel cost for generator i.

N: Number of units

ai, bi, ci: Fuel cost coefficients for generator i.

Pi: Generated power by generator i.

The total cost of the system is calculated by summing the cost of each unit separately. The total power generated by the units must be equal to the demand power. Since transmission line losses are included in the system in this study, the total demand power is obtained by summing the demand power and line losses. This equality constraint is shown in equation (2) (Tao & Jin-ding, 2009).

$$\sum_{i=1}^{N} P_i = P_D + P_L \tag{2}$$

PD= Demand power

PL=Transmission line loss

Line losses are modeled as developed by Kron and adopted by Kirchmayer (Zaraki & Othman, 2009). According to this model, line losses are written in matrix form.

$$P_L = \sum_{i=1}^{N} \sum_{j=1}^{N} P_i B_{ij} P_j + \sum_{i=1}^{N} B_{i0} P_i + B_{00} \tag{3}$$

Bij: NxN dimensional matrix of loss coefficients

Bi0: 1xN dimensional loss coefficients matrix

B00: Dimensionless loss coefficients matrix

In addition, the amount of power to be produced by each unit must be between the maximum and minimum points that the units can produce. This inequality constraint is shown as follows (Zaraki & Othman, 2009).

$$for\ i = 1{:}N \quad P_{i,max} \geq P_i \geq P_{i,min} \tag{4}$$

In this study, other constraints such as ramp rates, prohibited operating zones, valve point effect, and multi-fuel options were neglected.

2.2. Particle Swarm Optimization

In the PSO algorithm, a swarm of particles with predefined constraints is randomly distributed in the search space. The performance of each particle is evaluated by the value of the objective function. Given that minimization is desired, then the particle with the lower value has better performance.

For each particle, the best values in iterations are saved and called the personal best (Pbest). The best Pbest value among particles determines the global best (Gbest). Using these values, the new velocity values of each particle are calculated by equation (5) (Doğru et al., 2019).

$$V_i^{k+1} = w^{k+1} * V_i^k + c1r1\left(X_{pbest} - X_i^k\right) + c2r2(X_{gbest} - X_i^k) \tag{5}$$

V_i^{k+1} : Particle velocity in the current iteration

V_i^{k+1} : Particle velocity at iteration k

X_i^k : Particle position at iteration k

r1, r2: Random values between 0-1

w^{k+1}: Inertia weight value

c1: Personal learning coefficient

c2: Global learning coefficient

The calculated velocity is summed with the particle positions and the new positions of the particles are calculated.

$$X_i^{k+1} = X_i^k + V_i^{k+1} \tag{6}$$

Particles whose positions are updated may exceed the upper and lower limits. In this case, particles exceeding the limits are pulled to the limit values (Doğru et al., 2019). The objective function is recalculated at each iteration with the updated velocity and position vectors. After reaching the maximum iteration, the objective function value of Xgbest is given as the best value.

2.2. Test System Used

For ELD, a power system consisting of 40 generation units with a total demand power of 10,500 MW is considered. The data for the 40-unit system, including line losses, is taken from Barisal & Prusty (2015).

2.3. The Methods Used

Long-period random number sequences play an important role in metaheuristic algorithms. The risk of the algorithm getting stuck in local optima may increase when randomly generated numbers are collected in a certain area or the same values are repeated. By using a chaotic map, it is possible to avoid the optimum points or reduce the risk of getting stuck in the local optimum. The chaotic number sequence is easy and fast to generate and store. With a simple function and a few initial parameters, it is very easy to obtain sequences of any length. It has been theoretically proven that the numbers generated by chaotic maps are unpredictable, have spread spectrum characteristics, and are non-periodic (Tanyıldızı & Cigalı, 2017).

Optimization algorithms by their nature require random numbers. Chaotic maps can be used to generate the random numbers needed by these algorithms with simple mathematical equations and easy initial conditions by utilizing chaos theory.

In this section, the random numbers r1 and r2 in the PSO velocity equation in equation (5) are replaced with numbers obtained from different chaotic map methods and Logistic Map PSO (LMPSO), Gaussian Map PSO (GMPSO) and Tent Map PSO (TMPSO) methods are created. These methods are applied to the ELD problem with 40 units and transmission line losses.

2.3.1. Logistic Map PSO

The logistic map, a chaotic system, is used to generate a sequence of chaotic numbers. In the PSO algorithm, these chaotic numbers are employed to replace the random numbers typically used for updating the velocity and position of particles. This integration enhances the diversity of the search process and helps avoid premature convergence to local minima. The logistic map typically generates a chaotic sequence within the interval [0,1], based on initial conditions, which is then integrated into the PSO algorithm.

The logistic map is often preferred due to its simple structure. The mathematical expression of the logistic map is given in equation (7) (Burak Demir et al., 2019).

$$X_{n+1} = aX_n(1 - X_n), \qquad x_0 \neq \{0.25, 0.5, 0.75\} \tag{7}$$

n: Number of iterations

Xn: Chaotic number in iteration n

a: Logistic map parameter

In this study, the logistic map parameter is taken as 4.

2.3.2. Gaussian Map PSO

The Gaussian map, leveraging chaos theory, can be employed to generate long-term random numbers. In the PSO algorithm, the random numbers in the velocity equation are generated using the Gaussian map, with the aim of enhancing the algorithm's performance.

The Gaussian map is also a frequently preferred chaotic map method in the literature. The Gaussian map is shown in equation (8) (Burak Demir et al., 2019).

$$X_{n+1} = \begin{cases} 0\,, & X_n = 0 \\ \dfrac{1}{X_n} mod(1), & X_n \in (0,1) \end{cases}$$
$$1/X_n\, mod(1) = \frac{1}{X_n} - \left\lfloor \frac{1}{X_n} \right\rfloor \tag{8}$$

n: Number of iterations

Xn: Chaotic number in iteration n

$\left\lfloor \frac{1}{X_n} \right\rfloor$: represents the largest integer less than $\frac{1}{X_n}$ (Alataş, 2007).

2.3.3. Tent Map PSO

The tent map, utilizing chaos theory, can be used to generate long-term random numbers. In the PSO algorithm, the random numbers in the velocity equation are generated using the tent map, with the goal of improving the algorithm's performance.

The Tent map is one of the chaotic map methods used to generate numbers between 0 and 1. The mathematical expression for the Tent map is shown in equation (9). (Arul et al., 2013).

$$X_{n+1} = \begin{cases} 2a_n\,, & a_n \leq 0.5 \\ 2(1-a_n)\,, & a_n \geq 0.5 \end{cases} \qquad x_n \neq \{0, 0.25, 0.5, 0.75, 1\} \tag{9}$$

3. Findings and Discussion

The simulation was performed in MATLAB R2023b program on a personal computer with Intel İ5 1240P 4.40 GHz, 8 Gb RAM. The swarm population in the algorithms was set to 200, and the maximum number of iterations was set to 1000. For each algorithm, 50 trial runs are made, and the maximum, minimum, and average costs are given in Table 1.

Table 1. Maximum, minimum, and average fuel cost table for PSO, LMPSO, GMPSO, and TMPSO ($/h)

Method / Cost	PSO	LMPSO	GMPSO	TMPSO
Max	134,932.1	135,370.28	134,907.303	138,675.23
Average	134,318.4	133,274.42	133,322.436	134,872.03
Min	133,847.6	132,857.54	132,856.657	133,483.30

Since cost minimization is performed in this study, the minimum output of each method is considered the best value. Table 2 shows the generator power outputs, line losses, and fuel costs for the best values. Table 2 also shows the computation times of the algorithms for 50 trial runs.

Table 2. Best Generator Power Outputs

Units	Generator Power Output (MW)			
	PSO	LMPSO	GMPSO	TMPSO
1	106,1541	114	114	65,4914
2	111,5905	114	114	114
3	101,5961	120	120	120
4	190	190	190	190
5	84,3717	97	97	80,3336
6	106,8004	119,9844	116,9314	102,6339
7	300	300	300	300
8	299,2505	300	300	300
9	286,1491	300	300	300
10	268,5757	300	300	300
11	217,4998	94	94	131,8292
12	94	94	94	94
13	471,2866	359,9022	378,5870	365,9508
14	393,2629	440,6596	436,4317	500
15	469,8322	447,3151	448,9715	500
16	500	500	500	500
17	467,9436	500	500	500
18	499,9663	500	480,0221	399,0247
19	544,7558	550	550	550
20	550	550	550	550
21	550	550	550	550
22	550	550	550	550
23	550	550	550	550
24	529,9300	550	550	550
25	550	550	550	550
26	550	550	550	550
27	11,4107	10	10	10
28	10	10	10	10
29	10	10	10	10
30	85,4914	97	97	97
31	183,8706	190	190	190
32	180,1104	190	190	190
33	190	190	190	190
34	200	200	200	200
35	200	200	200	200
36	186,3784	200	200	200
37	96,3703	110	110	110
38	110	110	110	110
39	110	110	110	110
40	550	550	550	550
Total Power	11466,5981	11467,8616	11460,9440	11440,2640
Loss	965,9984	967,8354	960,9434	939,6143
Fuel Cost ($/h)	133,847.6665	132,857.5413	132,856.6574	133,483.3054
Time (s)	207,25	209,53	219,77	194,64

4. Conclusions and Recommendations

Chaotic maps can be used to generate long-term sequences of random, non-repeating numbers (Eke et al., 2023). The non-repeating nature of these sequences and the sensitivity to even the smallest initial changes are reflected in the optimization results. Each chaotic map exhibits its own unique distribution. Thus, it can be argued that the varying power output distributions and different power loss levels observed between algorithms are due to these distinct numerical distributions. Consequently, it can be said that the algorithms demonstrate different distributions to meet the total power demand, thereby forming different production strategies.

The generation cost is different for each unit in the ELD problem. The algorithms created meet the demand power with minimum cost by keeping the production quantities of the units with high production costs low and increasing the production quantities of the cheaper units.

In this study, the goal was to achieve the minimum cost, and in this context, the GMPSO algorithm can be considered the most successful. However, when examining the average values in Table 2, the LMPSO algorithm provided a lower average cost. This suggests that during the 50 iterations of the algorithms, the LMPSO algorithm produced more minimum results, leading to a lower average cost.

In light of all these findings, it can be concluded that the LMPSO algorithm yielded the best overall results among the proposed methods. However, if only the minimum value is considered, the GMPSO algorithm is the most successful in this study, as it achieves the lowest cost. This paper demonstrates the applicability of chaotic maps using chaos theory to the ELD problem.

Authors' Contributions

Ertuğrul Çam identification of conceptual and design processes, critical examination of intellectual content and interpretation of analyses, Mehmet Safa Aydın data collection, data analysis, data interpretation, identification of conceptual and design processes and interpretation of analyses.

Statement of Conflicts of Interest

The authors confirm that there are no known conflicts of interest or common interests with any organization or individual.

Statement of Research and Publication Ethics

The author declares that this study complies with Research and Publication Ethics.

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