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Control and Synchronization of a Memristor-based Hyperchaotic Lorenz System using Sliding Mode Control

Hakan KAYA[1,3*], Yılmaz UYAROĞLU[2]

[1] Electrical and Electronics Engineering, Institute of Natural Sciences, Sakarya University, Türkiye, hakan.kaya16@ogr.sakarya.edu.tr
[2] Electrical and Electronics Engineering Department, Faculty of Engineering, Sakarya University, Türkiye, uyaroglu@sau.edu.tr
[3] Sakarya Regional Directorate, Turasas, Türkiye, hakan.kaya@turasas.gov.tr

ABSTRACT

We studied some new findings on the sliding mode control, that have been derived for the chaos synchronization of memristor – based hyperchaotic Lorenz systems. Nonlinear property has shown that the memristor can be used in chaotic circuits and the latest memristor-based chaotic circuits with different nonlinear equations at times its design attracts quite a lot of attention. The reason why the sliding mode control method is preferred is due to the fact that it is a robust approach and thus less susceptible to the external disturbances. In fact it is affected in a very little range. Numerical simulations of the synchronization of the proposed control methods with the studied system here, have proved to be largely valid.

Keywords: Memristor, lorenz chaos system, sliding mode control

1 Introduction

Inductors, resistors and capacitors are known as circuit elements. These elements are expressed in terms of the relationship between the four fundamental circuit variables: magnetic flux *(φ)*, electric charge *(q)*, current *(i)* and voltage *(v)*. There are six possible combinations of these four fundamental circuit variables. Two of these combinations, *(v-φ)* and *(i-q)*, are familiar from basic circuit theory. The other three relationships are represented by the inductor *(L)*, which describes the relationship between current and magnetic flux *(dφ = Ldi)*; the capacitor *(C)*, which describes the relationship between voltage and charge *(dq = Cdv)*; and the resistor *(R)*, which describes the relationship between voltage and current *(dv = Rdi)*. As understood from these relationships, there are six different combinations of the fundamental circuit variables. However, according to these definitions, only five combinations are specified.

* Corresponding Author's email: hakan.kaya16@ogr.sakarya.edu.tr

In 1971, Leon Chua came with a new definition of a fourth circuit element to represent the recently undefined relationship between charge and magnetic flux, thus completing the sixth combination. Chua named this proposed element the memristor [1]. In 2008, researchers at HP Labs discovered the fundamental *i-v* characteristics of the memristor in a nanoscale device [2]. Memristors, which appear as two-terminal passive circuit elements, exhibit a nonlinear relationship between charge and flux. Due to their characteristic properties resembling synaptic actions, memristors are frequently seen in studies of synapses and artificial neural networks. Other applications include analog circuits, memory elements and sensors [3-8].

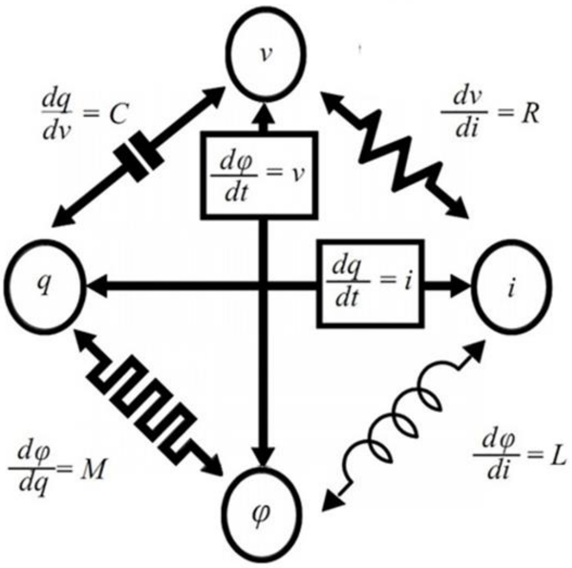


Figure 1. Relationship between current, voltage and electrical charge [2]

The HP memristor, defined by a nonlinear constitutive relation, is expressed in terms of voltage and current as follows [9]:

$$v = M(q)i \tag{1}$$

or

$$i = W(\varphi)v \tag{2}$$

Here, $\varphi = \int v dt$ terminal voltage refers to the relationship between v and i terminal current. The memristance $M(q)$ is expressed as a piecewise function as follows:

Memristance:

$$M(q) = \frac{d\varphi(q)}{dq} = \begin{Bmatrix} \alpha, |q| \le 1 \\ \beta, |q| > 1 \end{Bmatrix} \tag{3}$$

One of the most common applications of chaos theory lies in secure communication, where chaos synchronization plays a critical role. Synchronization, in this context, involves the matching and overlaying of two distinct chaotic signals, enabling the synchronization of a chaotic receiver and transmitter system. If we can use a chaotic signal in the form of a large masked chaotic signal, we can resend it using a chaotic mask. Because chaotic systems are highly sensitive to initial conditions, even a slight change in initial conditions can lead to completely different trajectories. The first study on synchronization was conducted by Pecora and Carroll, who proposed that it is possible to synchronize two chaotic systems with different initial conditions under certain conditions [10]. In this paper, we present new control results for memristor-based hyperchaotic Lorenz systems using the sliding mode control method. This method is widely favored due to its inherent advantages, including insensitivity to parameter uncertainties and external disturbances, ease of implementation, fast response, and strong transient performance.

2 Memristor-based Hyperchaotic Lorenz System

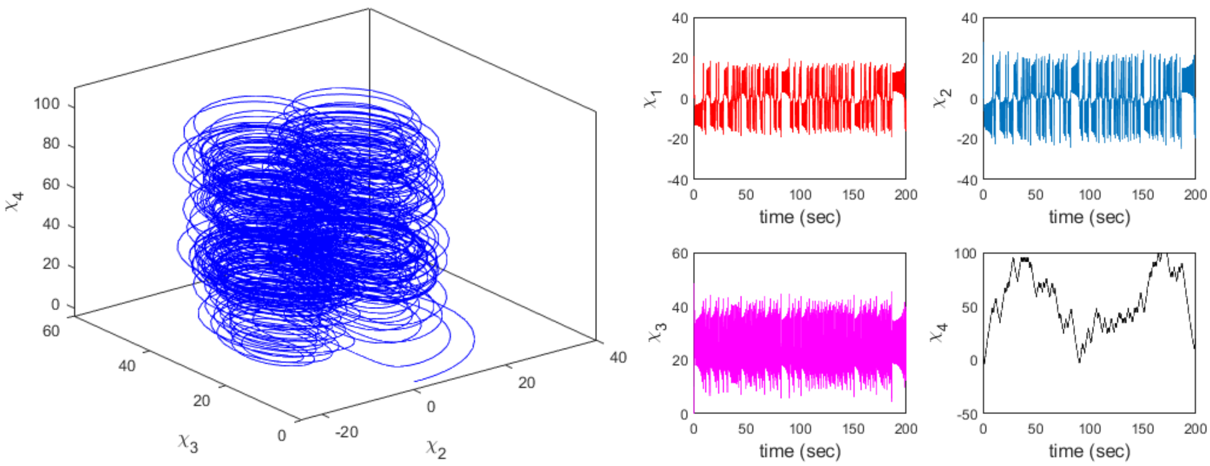
The state equations of a new type of memristor-based Lorenz system are provided below. The memristor-based Lorenz system can be defined as follows [11]:

$$\begin{cases} \dot{\chi}_1 = -\omega_1\chi_1 - W(\chi_4)\chi_1 + \omega_2\chi_2 \\ \dot{\chi}_2 = \omega_3\chi_1 - \chi_2 - \chi_1\chi_3 \\ \dot{\chi}_3 = \chi_1\chi_2 - \omega_4\chi_3 \\ \dot{\chi}_4 = -\chi_1 \end{cases} \tag{4}$$

Here $\chi_1, \chi_2, \chi_3, \chi_4$ are the state variables and $\omega_1, \omega_2, \omega_3, \omega_4$ are the constant parameters of the system. We can define the piecewise linear function $W(\chi_4)$ as follows:

$$W(\chi_4) = \begin{cases} \alpha, |\chi_4| \le 1; \\ \beta, |\chi_4| > 1. \end{cases} \tag{5}$$

As shown in Figure 2, when the parameters are chosen as $\omega_1 = 8, \omega_2 = 15, \omega_3 = 28, \omega_4 = 8/3,$ $\alpha = 5\, and\ \beta = 8$ the system Equation (4) exhibits chaotic behaviour under the initial conditions of $(10^4, 0, 0, 0)$.



(a) (b)

Figure 2. Memristor attractor: (a) State trajectories of memristor-based hyperchaotic lorenz system and dynamical behaviors: (b) Phase portraits of memristor-based hyperchaotic lorenz system

The Lyapunov exponents of the system, whose temporal variations are presented in Figure 3, have been calculated as $L_1 = 1.768023, L_2 = -0.004860, L_3 = 0.005835, L_4 = -25.789058$. The presence of at least two positive exponents ($L_1 and\ L_3$) indicates that the system is hyperchaotic.
The Lyapunov fractal dimension (Kaplan-Yorke) of the memristor-based hyperchaotic Lorenz system has been calculated using Equation (6).

$$D_{KY} = j + \frac{\sum_{i=1}^{j} L_i}{|L_{j+1}|} = 3 + \frac{L_1 + L_2 + L_3}{|L_4|} = 3.07 \quad (6)$$

As can be seen, the fact that the result is fractional indicates that the system exhibits chaotic behaviour.

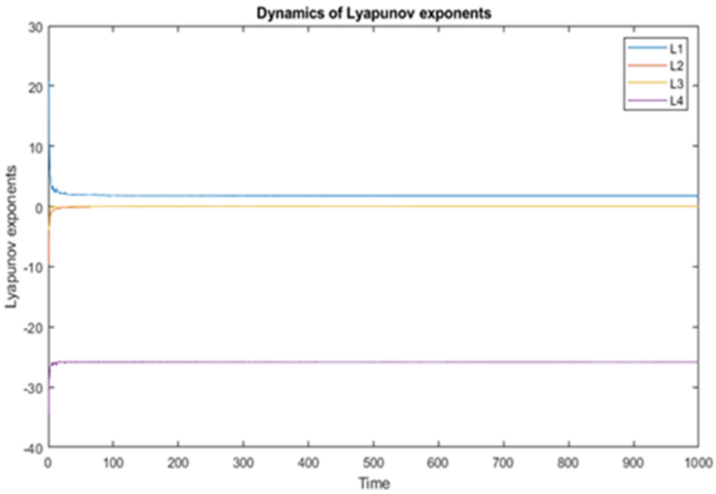


Figure 3. Graph of lyapunov exponents of the system

3 Results and Discussion

In this section, we will examine the synchronization of two identical memristor-based hyperchaotic Lorenz systems.

Taking Equation (4) as the master system [11],

$$\begin{cases} \dot{\chi}_1 = -\omega_1\chi_1 - W(\chi_4)\chi_1 + \omega_2\chi_2 \\ \dot{\chi}_2 = \omega_3\chi_1 - \chi_2 - \chi_1\chi_3 \\ \dot{\chi}_3 = \chi_1\chi_2 - \omega_4\chi_3 \\ \dot{\chi}_4 = -\chi_1 \end{cases}$$

the slave system is given as follows,

$$\begin{cases}\dot{\gamma}_1 = -\omega_1\gamma_1 - W(\gamma_4)\gamma_1 + \omega_2\gamma_2 + u_1, \\ \dot{\gamma}_2 = \omega_3\gamma_1 - \gamma_2 - \gamma_1\gamma_3 + u_2, \\ \dot{\gamma}_3 = \gamma_1\gamma_2 - \omega_4\gamma_3 + u_3, \\ \dot{\gamma}_4 = -\gamma_1 + u_4, \end{cases} \tag{7}$$

Here, $\gamma_1, \gamma_2, \gamma_3, \gamma_4$ are the state variables and u_1, u_2, u_3, u_4 are the controllers.

The chaos synchronization error is defined as follows,

$$e = \gamma - \chi \tag{8}$$

The error dynamics equations can be easily obtained as follows,

$$\begin{cases}\dot{e}_1 = -\omega_1 e_1 + \omega_2 e_2 - W(\gamma_4)\gamma_1 + W(\chi_4)\chi_1 + u_1 \\ \dot{e}_2 = \omega_3 e_1 - e_2 - \gamma_1\gamma_3 + \chi_1\chi_3 + u_2 \\ \dot{e}_3 = -\omega_4 e_3 + \gamma_1\gamma_2 - \chi_1\chi_2 + u_3 \\ \dot{e}_4 = -e_1 + u_4 \end{cases} \tag{9}$$

We write the matrix representations of the error dynamics Equation (9) as follows.

$$\dot{e} = Ke + \eta(\chi, \gamma) + u \tag{10}$$

here

$$K = \begin{bmatrix} -\omega_1 & \omega_2 & 0 & 0 \\ \omega_3 & -1 & 0 & 0 \\ 0 & 0 & -\omega_4 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}, \eta(\chi, \gamma) = \begin{bmatrix} -W(\gamma_4)\gamma_1 + W(\chi_4)\chi_1 \\ -\gamma_1\gamma_3 + \chi_1\chi_3 \\ \gamma_1\gamma_2 - \chi_1\chi_2 \\ 0 \end{bmatrix} \text{and } u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} \tag{11}$$

Firstly, we set *u* as follows,

$$u = -\eta(\chi, \gamma) + Lv \tag{12}$$

Here, *L* is chosen such that *(K, L)* is controllable.

We select *L* as follows:

$$L = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \tag{13}$$

When the sliding mode variable is chosen as follows:

$$s = Ce = \left[-2 - 3 - 1\,4 \right] e = -2e_1 - 3e_2 - e_3 + 4e_4 \tag{14}$$

And when the sliding mode variable is chosen as follows. Here *C* is a constant vector.

We choose the sliding mode gains as $k = 5$ ve $q = 0.2$.

According to the sliding mode control method's characteristic, the control signal is given as follows [12-18].

$$v(t) = -(CL)^{-1} \left[C(kI + K)e + q \,\mathrm{sign}(s) \right] \tag{15}$$

Here, *k* and *q* are the sliding mode control parameters. A high value of *k* tends to create problems which resut in chattering. An acceptable value of *q* suppresses chattering and shortens the time to reach the sliding surface. In this paper, the constants *k* and *q* for sliding mode control are determined by the author.

Now, the control signal *v(t)* becomes as follows,

$$v(t) = -41e_1 - 21e_2 - 1.1670e_3 + 10e_4 - 0.10\,\mathrm{sign}(s) \tag{16}$$

Therefore, the sliding mode controller is as follows:

$$u = -\eta(\chi, \gamma) + Lv \tag{17}$$

Here, $\eta(\chi, \gamma)$, *L* and *v(t)* are written as in equations Equations (11), (13) and (16).

The initial values for the master system Equation (4) are taken as follows:

$$\chi_1(0) = -5,\ \ \chi_2(0) = 30,\ \ \chi_3(0) = 8,\ \ \chi_4(0) = 20.$$

The initial values for the slave system Equation (7) are taken as follows:

$$\gamma_1(0) = -8,\ \gamma_2(0) = 12,\ \gamma_3(0) = -38,\ \gamma_4(0) = 16.$$

Figure 4 illustrates the complete synchronization of two identical memristor-based hyperchaotic Lorenz systems Equations (4) and (7).

Figure 5 shows the error states approaching zero for $e_1(t),\ e_2(t),\ e_3(t),\ e_4(t)$, $t \to \infty$.

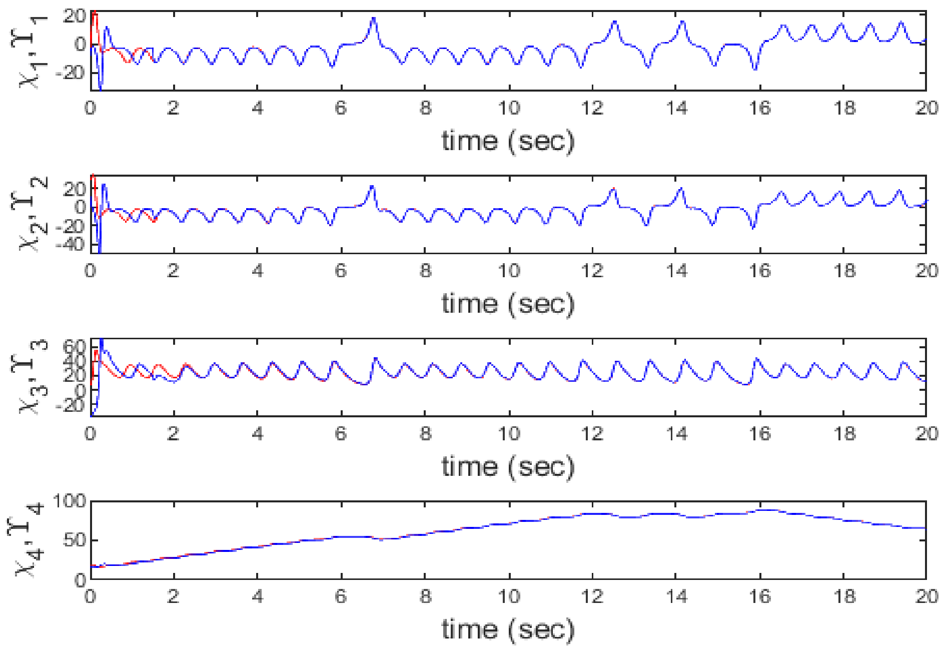


Figure 4 Synchronization of memristor-based hyperchaotic lorenz system

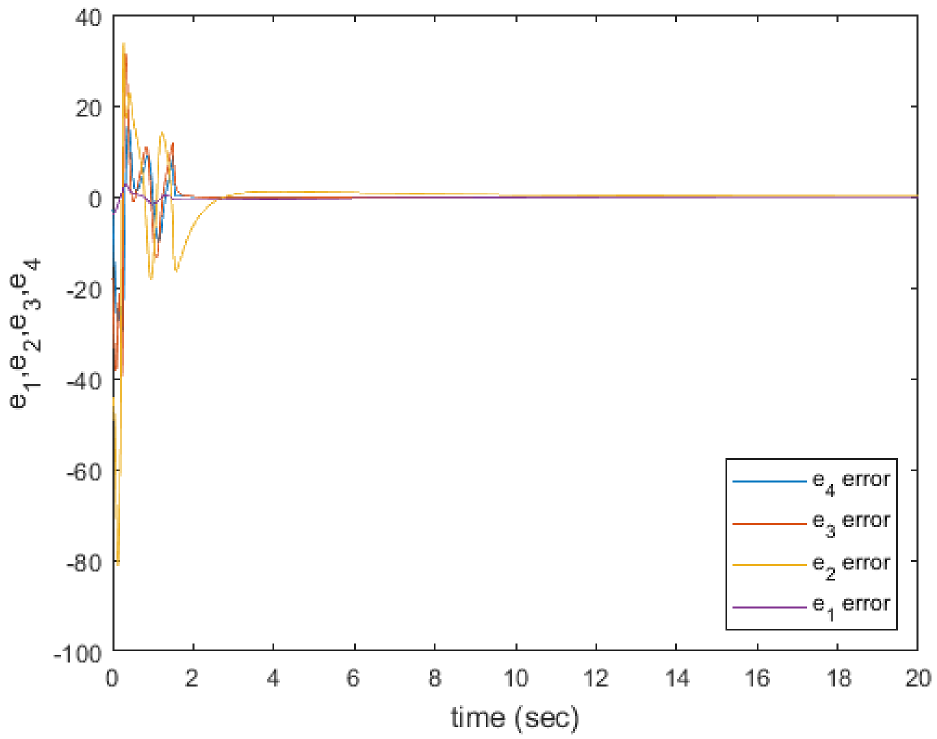


Figure 5. The error responses of the state variables

4 Conclusions

This study aims to explore chaos synchronization in memristor-based hyperchaotic Lorenz systems. The synchronization process for a memristor-based Lorenz system has been introduced, and its effectiveness has been validated through MATLAB simulations. The results of the simulations for the controlled memristor-based Lorenz circuit are presented to demonstrate the efficacy of the proposed synchronization method. Simulation results of the controlled memristor-based Lorenz circuit are provided to demonstrate the effectiveness of the proposed method. The simulation results demonstrate that $(\chi_1,\gamma_1),(\chi_2,\gamma_2),(\chi_3,\gamma_3),(\chi_4,\gamma_4)$ synchronize in as short as 2 seconds. The error parameters approach zero in the $e_1(t)$, $e_2(t)$, $e_3(t)$, $e_4(t)$ formula within 2 seconds. Sliding mode control is a robust approach that is less affected by external disturbances, enabling the system to operate with successful performance, as evidenced in various studies. It provides a strong defence against external influences, ensuring minimal impact and system effectiveness. In this paper, the effectiveness of synchronization results achieved using sliding mode control for memristor-based hyperchaotic Lorenz systems is demonstrated through numerical simulations. The Lyapunov exponents of the system have been examined, showing the presence of two positive exponents.

5 Declarations

5.1 Competing Interests

There is no conflict of interest in this study.

5.2 Authors' Contributions

1. Hakan KAYA: Design and implementation of the research, analysis of the result, writing of the manuscript.

2. Yılmaz UYAROĞLU: Review, editing, supervision.

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