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COMPACT ANALYSIS OF THE NECESSITY OF PADÉ APPROXIMATION FOR DELAYED CONTINUOUS-TIME MODELS IN LQR, H-INFINITY AND ROOT LOCUS CONTROL STRATEGIES

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Abstract: This paper presents a comprehensive analysis of the need for the Padé approximation for continuous-time models with delays, focusing on its critical role in addressing the control challenges posed by time delays. Time delays, often referred to as dead times, transport delays or time lags, are inherent in a wide range of industrial and engineering processes. These delays introduce phase shifts that degrade control performance by reducing control bandwidth and threatening the stability of closed-loop systems. Accurate modelling and compensation of these delays is essential to maintain system stability and ensure effective control. This paper highlights the difficulties that arise when using advanced control techniques such as root locus (RL), linear quadratic regulator (LQR) and H-infinity (H_∞) control in systems with delays. Representing delays in exponential form leads to an infinite number of state problems, complicating the design and analysis of controllers in such systems. To address these challenges, the Padé approximation is proposed as an effective method for approximating time delays with rational polynomials of appropriate order. This approach allows for more accurate simulation, system analysis and controller design, thereby mitigating the problems caused by delays. The paper also provides a detailed comparative analysis between the Padé approximation and Taylor polynomials, demonstrating the superiority of the former in achieving accurate delay modelling and control performance. The results show that the use of Padé approximation not only improves the accuracy of system models, but also improves the robustness and stability of control strategies such as RL, LQR, and H_∞. These results highlight the importance of the Padé approximation as a valuable tool in the design of delay-affected control systems, offering significant advantages for both theoretical and practical applications.

Keywords: Padé approximation, Time delay systems, Continuous-time models, Infinite number of state problem, Rational polynomial approximation, Dead time compensation

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1. Introduction

Time delays, both distorting and non-distorting, are an integral part of many engineering applications, particularly in the design and analysis of advanced control systems (Pujol-Vazquez et al., 2020; Zhang et al., 2020; Zhang et al., 2024). In these systems, delays occur naturally due to physical phenomena, communication delays or processing times, and they significantly complicate both the design and performance of control strategies. Ignoring the effects of these delays can have serious consequences, such as reduced system performance or even instability, since time delays tend to degrade the control process by introducing phase shifts and reducing the control bandwidth (Mondié et al., 2022; Shangguan et al., 2020; Wu et al., 2023). Therefore, the development of appropriate control principles for time-delayed dynamic systems with uncertainties has been an important research topic (Li et al., 2020; Abbasspour et al., 2020; Belhamel et al., 2020).

In classical control theory, root locus (RL) analysis is a widely used graphical method to investigate how the roots of a system change in response to variations in system parameters, in particular the feedback gain (Luyben, 2020; Werth et al., 2020). However, when dealing with delayed systems, RL analysis faces significant challenges. Time delays introduce transcendental terms into the system's transfer function, creating an infinite number of poles. These poles make it virtually impossible to plot the RL diagram and complicate closed loop stability analysis.

In modern control theory, approaches such as linear quadratic regulator (LQR) and H-infinity (H_∞) control are popular because of their ability to optimize system performance and stability while dealing with uncertainties and disturbances (Handaya and Fauziah, 2021; Priyambodo et al., 2020; Menezes and Araújo, 2023; Anh, 2020). LQR seeks to minimize a quadratic cost function while controlling a dynamic system with linear dynamics (Fridovich-Keil et al., 2020; Khamies et al., 2021; Yang et al., 2021). Similarly, H_∞ control focuses on optimizing system performance by solving a mathematical optimization problem to achieve stability with guaranteed robustness (Yang et al., 2021; Zhou et al., 2020). Despite their wide applicability and advantages such as optimality, computational efficiency and robustness - both LQR and H_∞ face significant difficulties when applied to continuous-time systems with delays due to the infinite number of state problems introduced by time delays (Kanokmedhakul et al., 2024). To overcome these challenges, it is essential to convert the time delay term into a rational function form, which simplifies the analysis of the system and the design of the controller (De Persis and Tesi, 2021; Chen et al., 2020; Abdullah, 2021; Maghfiroh et al., 2022). The Padé approximation provides a practical solution to this conversion by approximating the time delay as a rational polynomial (Wei et al., 2016; Hu et al., 2024). This method is widely favoured for its flexibility in adjusting accuracy, preserving system dynamics, and ease of implementation. In addition, it significantly improves frequency domain analysis and provides more manageable system representations for control design (Gluzman, 2020).

This paper presents a detailed investigation of the necessity of the Padé approximation for delayed continuous-time models, with a focus on its application to RL, LQR, and H_∞ control strategies. The main contributions of this paper are as follows:

i. The study provides a clear methodology for converting time delays from exponential form to rational polynomials using the Padé approximation, which simplifies the analysis of control systems with delays.

ii. By implementing the Padé approximation, the infinite pole problem in delayed systems is addressed, allowing for more accurate RL analysis and improved system stability when using LQR and H_∞ techniques.

iii. A comprehensive comparison with other polynomial approximations, such as Taylor series, shows that Padé offers superior accuracy and preservation of system behaviour, making it ideal for practical control system design.

iv. The study highlights the relevance of these results in real engineering applications where the use of delayed models is inevitable, providing practical solutions for improving the performance of modern control systems.

2. Materials and Methods

2.1. Padé Approximations of Time Delay

Approximations can be derived by determining the numerator and denominator coefficients and expressing a function as the ratio of two power series, known as a rational polynomial. When functions contain poles, Padé approximations offer a significant advantage over Taylor series and Taylor polynomials, which are among the most commonly used approximation methods. This advantage arises because Padé approximations use rational functions, which allow a more accurate representation of functions with poles than traditional power series expansions (Pinheiro and Colón, 2024).

The Padé approximation $R_{L/M} \equiv [L/M]$ to any power series is given by (equation 1)

$$A(x) = \sum_{j=0}^{\infty} a_j\, x^j \tag{1}$$

Considering that $A(x)$ is a transcendental function (e.g. e^x), as the time delay approximation is the basis of this study, each term of the expansion of equation 1 is given by the Taylor series about x_0 (equation 2)

$$a_n = \frac{1}{n!} A^{(n)}(x_0) \tag{2}$$

Substituting in equation 2, the coefficients are as follows (equation 3):

$$A(x) - \frac{P_L(x)}{Q_M(x)} = 0 \tag{3}$$

An additional constraint can be enforced since $Q_M(x)$ can be multiplied by any constant, which will rescale the other coefficients. The standard scaling method is defined as $Q_M(0) = 1$. The expansion of equation 3 gives (equations 4 and 5)

$$P_L(x) = p_0 + p_1 x + p_2 x^2 + \cdots p_L x^L \tag{4}$$

$$Q_M(x) = 1 + q_1 x + q_2 x^2 + \cdots q_M x^M \tag{5}$$

The set of equations based on the equations 3-5 are given by (equation 6)

$$\begin{gathered} a_0 = p_0 \\ a_1 + a_0 q_1 = p_1 \\ a_2 + a_1 q_1 + a_0 q_2 = p_2 \\ \vdots \\ a_L + a_{L-1} q_1 + \cdots + a_0 q_L = p_L \\ a_{L+1} + a_L q_1 + \cdots + a_{L-M+1} q_M = 0 \\ \vdots \\ a_{L+M} + a_{L+M-1} q_1 + \cdots + a_L q_M = 0 \end{gathered} \tag{6}$$

where $q_j = 0$ for $j > M$ and $a_n = 0$ for $n < 0$. Directly solving these yields (equation 7):

$$\frac{L}{M} = \frac{\begin{bmatrix} a_{L-m+1} & a_{L-m+2} & \cdots & a_{L+1} \\ \vdots & \vdots & \ddots & a_{L+M} \\ a_L & a_{L+1} & \cdots & a_{L+M} \\ \sum_{j=M}^{L} a_{j-M} x^j & \sum_{j=M-1}^{L} a_{j-M+1} x^j & \cdots & \sum_{j=0}^{L} a_j x^j \end{bmatrix}}{\begin{bmatrix} a_{L-M+1} & a_{L-M+2} & \cdots & a_{L+1} \\ \vdots & \vdots & \ddots & \vdots \\ a_L & a_{L+1} & \cdots & a_{L+M} \\ x^M & x^{M-1} & \cdots & 1 \end{bmatrix}} \tag{7}$$

If the lower index is greater than the upper, the sums are replaced by a zero. Alternative forms are shown as (equations 8 and 9)

$$\frac{L}{M} = \sum_{j=0}^{L-M} a_j x^j + x^{L-M+1} \boldsymbol{w^T}_{L/M}\, W^{-1}{}_{L/M} \boldsymbol{w}_{L/M} \tag{8}$$

$$\frac{L}{M} = \sum_{j=0}^{L+n} a_j x^j + x^{L+n+1} \boldsymbol{w^T}_{(L+M)/M}\, W^{-1}{}_{L/M} \boldsymbol{w}_{(L+n)/M} \tag{9}$$

where (equations 10 and 11)

$$W_{L/M} = \begin{bmatrix} a_{L-M+1} - x a_{L-M+2} & \cdots & a_L - x a_{L+1} \\ \vdots & \ddots & \vdots \\ a_L - x a_{L+1} & \cdots & a_{L+M-1} - x a_{L+M} \end{bmatrix} \tag{10}$$

$$\boldsymbol{w}_{L/M} = \begin{bmatrix} a_{L-M+1} \\ a_{L-M+2} \\ \vdots \\ a_L \end{bmatrix} \tag{11}$$

and the relation between n and M satisfies the criterion of $0 \leq n \leq M$. The Padé approximation of e^{-x} with a numerator order $L = 3$ and denominator order $M = 3$ is given in Table 1.

Table 1. Padé approximation of e^{-x} with numerator order $L = 3$ and denominator order $M = 3$

L M	0	1	2	3
0	$\frac{1}{1}$	$\frac{1-x}{1}$	$\frac{1-x+\frac{1}{2}x^2}{1}$	$\frac{1-x+\frac{1}{2}x^2-\frac{1}{6}x^3}{1}$
1	$\frac{1}{1+x}$	$\frac{1-\frac{1}{2}x}{1+\frac{1}{2}x}$	$\frac{1-\frac{2}{3}x+\frac{1}{6}x^2}{1+\frac{1}{3}x}$	$\frac{1-\frac{3}{4}x+\frac{1}{4}x^2-\frac{1}{24}x^3}{1+\frac{1}{4}x^2}$
2	$\frac{1}{1+x+\frac{1}{2}x^2}$	$\frac{1-\frac{1}{3}x}{1+\frac{2}{3}x+\frac{1}{6}x^2}$	$\frac{1-\frac{1}{2}x+\frac{1}{12}x^2}{1+\frac{1}{2}x+\frac{1}{12}x^2}$	$\frac{1-\frac{3}{5}x+\frac{3}{20}x^2-\frac{1}{60}x^3}{1+\frac{2}{5}x+\frac{1}{20}x^2}$
3	$\frac{1}{1+x+\frac{1}{2}x^2+\frac{1}{6}x^3}$	$\frac{1-\frac{1}{4}x}{1+\frac{3}{4}x+\frac{1}{4}x^2+\frac{1}{24}x^3}$	$\frac{1-\frac{2}{5}x+\frac{1}{20}x^2}{1+\frac{3}{5}x+\frac{3}{20}x^2+\frac{1}{60}x^3}$	$\frac{1-\frac{1}{2}x+\frac{1}{10}x^2-\frac{1}{120}x^3}{1+\frac{1}{2}x+\frac{1}{10}x^2+\frac{1}{120}x^3}$

2.2. A Comparative Analysis using Taylor Polynomials

An analogy with the Taylor series expansion of e^{-x} around $x = 0$, as presented in equation 12, helps to highlight the significance of the Padé approximation.

$$e^{-x} = \sum_{n=0}^{\infty} \frac{(-x)^n}{n!} = 1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3 + \cdots \tag{12}$$

To transform equation 12 into the Padé approximation, the desired order of the rational polynomial - specifically, the choice of numerator and denominator orders as shown in Table 1 - must be determined (Conca et al., 2024). For example, if both the numerator order L and the denominator order M are chosen as first order, the Padé approximation of order M over L is defined by the rational function shown in equation 13.

$$R(x) = \frac{\sum_{j=0}^{M} a_j\, x^j}{1 + \sum_{k=1}^{L} b_k x^k} \tag{13}$$

Solving equation 13 for the Padé approximation of order 1/1 gives the expression provided in equation 14.

$$R(x) = \frac{\sum_{j=0}^{1} a_j\, x^j}{1 + \sum_{k=1}^{1} b_k x^k} = \frac{a_0 + a_1 x}{1 + b_1 x} \tag{14}$$

The coefficients a_0, a_1, and a_2 are determined by equating equation 14 to the Taylor polynomial of order $M + L$. Since both the numerator and denominator are of order 1, this corresponds to equating the expression to the second-order Taylor polynomial, as shown in equation 15.

$$R(x) = \frac{a_0 + a_1 x}{1 + b_1 x} = 1 - x + \frac{1}{2}x^2 \tag{15}$$

Extending equation 15 to solve for the coefficients yields the following results:

$$a_0 + a_1 x = 1 - x + \frac{1}{2}x^2 + b_1 x - b_1 x^2 + \frac{1}{2} b_1 x^3 \tag{16}$$

$$a_0 = 1, \quad a_1 = -\frac{1}{2}, \quad b_1 = \frac{1}{2} \tag{17}$$

An important aspect to note here is the presence of a residual term in the equation. In this example, when considering up to order $M + L$, the x^3 term is discarded during the process. This distinction highlights a key difference between the Padé approximation and the Taylor polynomial, as the higher-order term is omitted in the Padé approach. By excluding this term, the resulting approximation provides a closer fit to e^{-x} compared to the second-order Taylor polynomial, as demonstrated in Figure 1.

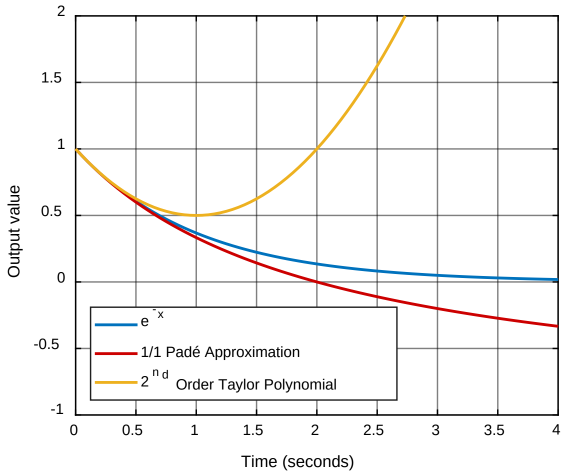

Figure 1. Comparison of the Padé approximation and the second-order Taylor polynomial e^{-x}.

Looking at these results in the context of transfer functions used for mathematical modelling in control systems, it is clear that for the same number of states, a transfer function with both poles and zeros exhibits more complex behaviour than one with only zeros.

2.3. Determining the Order of Approximation

One of the key considerations in approximation applications is the choice of order for both the numerator and denominator polynomials, particularly when deciding between equal-order and mixed-order approximations. Each of the approximation in Table 1 can be chosen to represent the e^{-x}. A comparison between the 2/2 and 2/3 Padé approximations is shown in Figure 2.

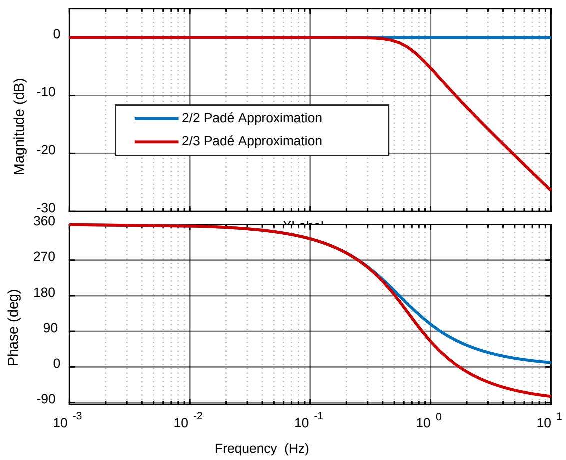

Figure 2. Comparison of 2/2 and 2/3 Padé approximations for e^{-x}.

However, only the equal-order Padé approximations (terms along the diagonal in Table 1) affect the phase without affecting the gain. These approximations behave like all-pass filters, a type of signal processing filter that adjusts the phase relationship between different frequencies while maintaining a uniform gain across all frequencies.

The 2/2 Padé approximation results in a gain of 0 decibels at all frequencies, affecting only the phase of the system, while the gain remains constant. In contrast, the 2/3 approximation shows a drop in gain at higher frequencies. The preference for an all-pass filter is due to the fact that a time delay only affects the phase of a signal without affecting its gain. The choice of the appropriate order of approximation is critical and depends on both the magnitude of the delay and the speed of the system. Figure 3 illustrates a comparison of the step and phase responses between the delay-free approximation and the original time-delayed system.

As shown in Figure 3, higher-order approximations give a closer phase match to the actual function over a wider frequency range. Specifically, the 2/2 approximation is accurate up to approximately 2 rad/sec, the 3/3 approximation up to 4 rad/sec, the 4/4 approximation up to 6 rad/sec, and the 5/5 approximation up to 10 rad/sec. These results highlight that the choice of approximation order should be guided by the critical frequencies of the system. In control system design, the most critical frequency is often the cut-off frequency where the gain falls below -3 dB, as this point has a significant impact on system performance in a closed-loop configuration.

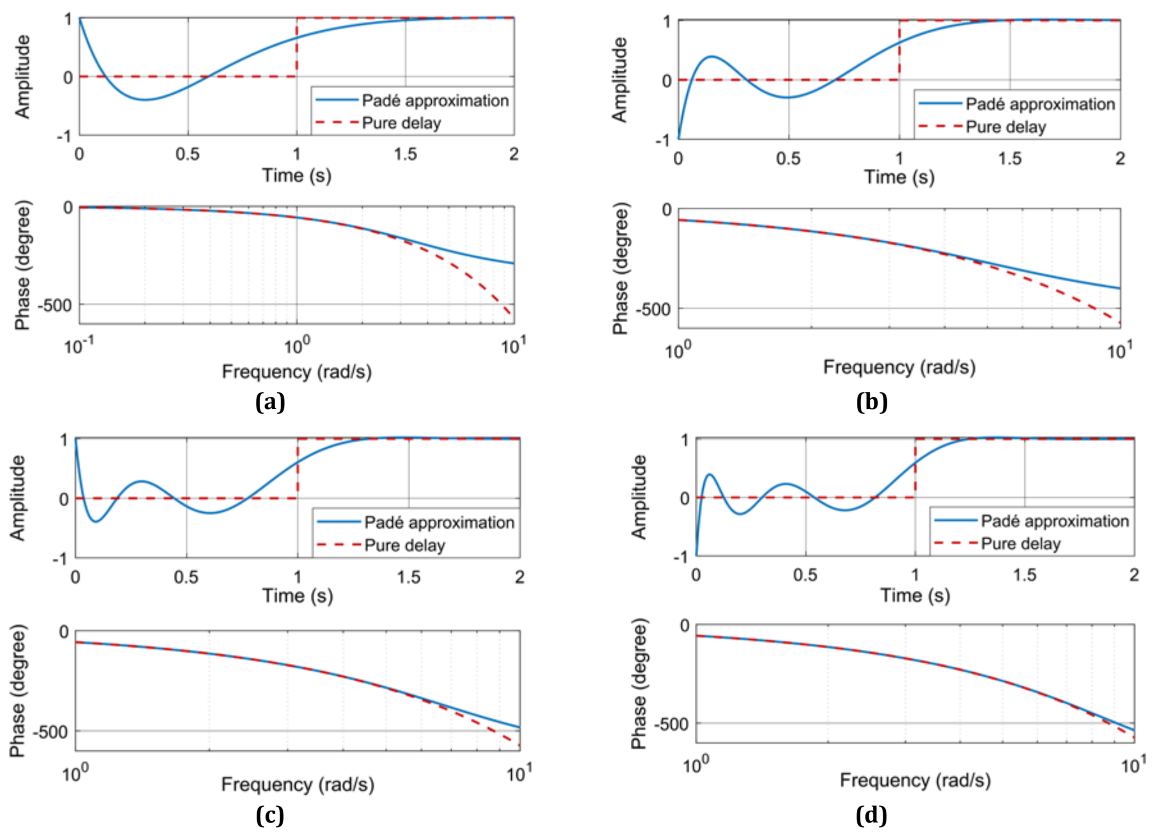

Figure 3. Comparison of the phase response of the Padé approximations: a) 2/2, b) 3/3, c) 4/4, and d) 5/5.

2.4. Time Delay in Control Systems

Dead times, also referred to as time lags or transport delays, are a common feature of processes in feedback control systems. These delays pose significant challenges to control, as they introduce linear phase shifts that reduce the control bandwidth and compromise the stability of the control system. The generic form of the time delay differential equation for $x(t) \in \mathbb{R}^n$ is given as follows:

$$\frac{d}{dt}x(t) = f(t, x(t), x_t) \tag{18}$$

where $x_t = \{x(\tau): \tau \leq t\}$ is the past trajectory of the solution. f acts as a functional operator from $\mathbb{R} \times \mathbb{R}^n \times C^1(\mathbb{R}, \mathbb{R}^n)$ to $\mathbb{R}^n$ in this equation.

Consider a signal $f = x(t)$, which is zero for negative time but undergoes a change at $t = 0$ seconds. The delayed function, $x(t - T)$,, represents this signal being delayed by T seconds. The Laplace transform of such a delayed function is expressed as

$$g(t) = \begin{cases} 0, 0 \leq t \leq T \\ f(t-T), t \geq T \end{cases} \tag{19}$$

$$G(s) = \mathcal{L}\{g(t)\} = \mathcal{L}\{f(t-T)\} = \int_0^\infty e^{-st} g(t) dt \tag{20}$$

$$G(s) = \int_T^\infty e^{-st} f(t-T) dt = \int_0^\infty e^{-s(\tau+T)} f(\tau) d\tau \tag{21}$$

$$G(s) = e^{-sT} F(s) \tag{22}$$

where T, the delay, is expressed in terms of seconds. Thus, an element delaying T seconds has a transfer function of $TF(s) = e^{-sT}$.

3. Results and Discussion

Accurately capturing time delays in system models is essential for reliable simulation, system analysis and effective controller design because delays are inherent in many dynamic systems. Failure to model them properly can lead to performance degradation and inaccurate predictions of system behaviour. Tools such as the Bode plot, widely used in control theory, provide an effective means of analyzing systems with delays, providing valuable insight into frequency response, stability margins and overall system robustness. These tools are particularly important in controller design, where ensuring stability and achieving desired performance are key objectives. However, when using advanced control techniques such as RL, LQR, and H_∞ synthesis, the presence of time delays introduces complexity. These methods, traditionally designed for systems without delays, are challenged by the infinite-dimensional nature of time-delay systems. In such cases, an approximation of the time-delay transfer function by rational polynomials is often required to make these methods viable. While this approximation can simplify the design process, it comes with trade-offs in accuracy and fidelity of system behaviour. Overall, this study highlights the importance of considering time delays in control system design and the need for careful approximation techniques when using advanced control methods. Future research should focus on refining these approximations and developing novel strategies for directly addressing time delays in more complex control frameworks. By doing so, we can improve the robustness and accuracy of controllers, especially in applications where delays play a significant role in system dynamics. This will contribute to more reliable and efficient control systems in various engineering domains.

Author Contributions

The percentages of the authors' contributions are presented below. The authors reviewed and approved the final version of the manuscript.

	C.Y.	A.A.
C	50	50
D	50	50
S	50	50
DCP	50	50
DAI	50	50
L	50	50
W	50	50
CR	50	50
SR	50	50

C=Concept, D= design, S= supervision, DCP= data collection and/or processing, DAI= data analysis and/or interpretation, L= literature search, W= writing, CR= critical review, SR= submission and revision.

Conflict of Interest

The authors declared that there is no conflict of interest.

Ethical Consideration

Ethics committee approval was not required for this study because of there was no study on animals or humans.

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