*Konuralp Journal of Mathematics*, 6 (1) (2018) 102-109

**Konuralp Journal of Mathematics**

*Journal Homepage:* www.dergipark.gov.tr/konuralpjournalmath
*e-ISSN: 2147-625X*

# A Quantitative Approach to Fractional Option Pricing Problems with Decomposition Series

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## Abstract

In this study, we get a novel identification of Adomian Decomposition Method (ADM) to have an accurate and quick solution for the European option pricing problem by using Black-Scholes equation of time-fractional order (FBSE) with the initial condition and generalized Black-Scholes equation of fractional order (GFBSE). The fractional operator is understood in the Caputo sense. First of all, we redefine the Black-Scholes equation as fractional mean which computes the option price for fractional values. Then we have applied the ADM to the FBSE and GFBSE, so we have obtained accurate and quick approximate analytical solutions for these equations. The results related to the solutions have been presented in figures.

***Keywords:*** *Adomian decomposition method; convergence analysis; fractional Black-Scholes model; option pricing*
***2010 Mathematics Subject Classification:*** *49M27; 26A33; 91B25*

## 1. Introduction

Many studies have been undertaken in the last quarter of a century on fractional derivative operators. Modelling with the fractional derivative operators and numerical-approximate solution methods defined using these operators are extensively used in the solution of real-life problems in economy, fluid flow, control, electrochemistry, physical, chemical and biological processes [1, 6, 11, 16, 18, 19, 20, 23, 25, 29, 31, 33, 39]. Over the past decades, finance has become a highly acclaimed place both personally and institutionally. In addition, the financial system can be viewed as money, capital and derivative markets (options, futures, forwards, swaps, etc.). Options are widely used in global financial markets. An option is a right but not obligation. The most important benefit of the option is the ability to invest in large amounts with a very small capital. In 1973, an option pricing formula which in closed form solution class is derived by Fischer Black and Myron Scholes [9]. This model is developed in order to appreciate the European call and put options that pay no dividend to the holder of the option. Also Black-Scholes model is used to calculate the amount paid or collected when buying or selling an option is "reasonable". In this context, Black-Scholes model is one of the most important pricing models. The solution of this model can be obtained using the heat transfer equation. Therefore, it is very important to convert the Black-Scholes equation (BSE) to the heat equation by making new conversions.
Many powerful numerical and analytical techniques have been presented in literature on finance. Among them, homotopy perturbation method (HPM) with Laplace transform (LT) [36], variational iteration method with Sumudu transform, finite difference method (FDM) [13, 21, 30], homotopy analysis method (HAM) [27, 37], fractional variational iteration method (fVIM) [4, 7, 24], numerical simulation method [41], generalized differential transform method (GDTM) [34], Pade approximation technique [26] are relatively new approaches give an analytical and numerical approximation to the fractional Black-Scholes equation (FBSE). Also, the followings are some other studies related to the solution partial differential equations with Adomian decomposition method: [8, 15, 17, 35, 38, 40]. Many linear and nonlinear fractional PDEs can be solved with this method. Many different merchandises and payment types have been priced by using the Black-Scholes model. This form of the pricing model is one of the most meaningful mathematical equations for a financial staple. This model is defined in order to price an option as [32]:

$$\frac{\partial C}{\partial t}+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 C}{\partial S^2}+r(t)S\frac{\partial C}{\partial S}-r(t)C=0, \quad (S,t)\in \mathrm{R}^+\times(0,T) \tag{1.1}$$

where $C=C(S,t)$ shows the European option price at asset price $S$ and time $t$. $T$ represents the maturity time, $r(t)$ is the risk-free interest rate and $\sigma(S,t)$ is the volatility function according to the underlying asset. In Eq. (1.1), we observe that $C(0,t)=0, C(S,t)\sim S$ as $S\to\infty$ and $C(S,T)=\max(S-E,0)$, such that $E$ is the strike price. Eq. (1.1) resembles the diffusion equation, despite more terms. The closed form

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solution of the Eq. (1.1) can be obtained by using the heat equation. In order to obtain the fractional Black-Scholes equation, we make the following conversions:

$$S = Ee^x, \quad t = T - \frac{2\tau}{\sigma^2}, \quad C = Ev(x,\tau).$$

This holds in the equation [22]

$$\frac{\partial^\alpha v(x,\tau)}{\partial \tau^\alpha} = \frac{\partial^2 v(x,\tau)}{\partial x^2} + (k-1)\frac{\partial v(x,\tau)}{\partial x} - kv(x,\tau), \quad \tau > 0, \; x \in R, \; 0 < \alpha \le 1, \tag{1.2}$$

with initial condition

$$v(x,0) = \max(e^x - 1, 0). \tag{1.3}$$

We can see that this system of equations (1.2)-(1.3) computes the price options even for fractional values. This situation is very important in financial market. In Eq. (1.2), we define $k = 2r/\sigma^2$ where $k$ shows the equilibrium between the interest rates and stock returns variability. The generalized fractional Black-Scholes equation of the classical FBSE (1.2)-(1.3) has been presented by Cen and Le (2011) [12] by taking $r = 0.06$ and $\sigma = 0.4(2+\sin x)$ in Eq. (1.2):

$$\frac{\partial^\alpha v}{\partial \tau^\alpha} + 0.08(2+\sin x)^2 \frac{\partial^2 v}{\partial x^2} + 0.06x\frac{\partial v}{\partial x} - 0.06v = 0, \quad \tau > 0, \; x \in R, \; 0 < \alpha \le 1, \tag{1.4}$$

with the initial condition

$$v(x,0) = \max\left(x - 25e^{-0.06}, 0\right). \tag{1.5}$$

The main purpose of this study is to solve the time-fractional equations (1.2)-(1.3) and (1.4)-(1.5) with Adomian decomposition method (ADM).

## 2. Some Definitions

**Definition 2.1.** *[28] The Riemann-Liouville (RL) fractional integral operator of order $\alpha > 0$, of a function $f \in C_\mu, \mu \ge -1$ is defined as*

$$\begin{aligned} J^\alpha f(x) &= \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} f(t)\,dt, \quad \alpha > 0, \quad x > 0, \\ J^0 f(x) &= f(x). \end{aligned}$$

**Definition 2.2.** *[10] The fractional derivative definition in the Caputo sense (CS) is given as*

$$D_{*t}^\alpha f(x) = J^{m-\alpha} D^m f(x) = \frac{1}{\Gamma(m-\alpha)} \int_0^x (x-t)^{m-\alpha-1} f^{(m)}(t)\,dt,$$

*for $m-1 < \alpha \le m, \; m \in \mathrm{N}, \; x > 0, \; f \in C_{-1}^m$. In addition, we can give the Caputo time-fractional derivative operator as*

$$D_{*t}^\alpha u(x,t) = \begin{cases} \frac{1}{\Gamma(m-\alpha)} \int_0^t (t-\tau)^{m-\alpha-1} \frac{\partial^m u(x,\tau)}{\partial \tau^m} d\tau, & m-1 < \alpha < m, \\ \frac{\partial^m u(x,t)}{\partial t^m}, & m = \alpha \in \mathrm{N}. \end{cases}$$

Note that the LT of the RL derivative include initial conditions that can not be commented. But, the LT of the Caputo derivative implies the initial conditions with non–fractional order. This property makes the Caputo derivative more useful in terms of physical applications.

## 3. Description of the Suggested Method: Adomian Decomposition Method of Fractional Order

Adomian decomposition method which was developed by Adomian [3] needs that the FBSE (1.2) with initial condition (1.3) and GFBSE (1.4) with initial condition (1.5) be regarded as the operator form

$$D_{*t}^\alpha u + f_0(x)u + f_1(x)L_{1x}u + f_2(x)L_{2x}u + \ldots + f_n(x)L_{nx}u = q(x,t), \tag{3.1}$$

where $L_{1x} = \frac{\partial}{\partial x}, \; L_{2x} = \frac{\partial^2}{\partial x^2}, \ldots, L_{nx} = \frac{\partial^n}{\partial x^n}$. We also define: $D_{*t}^\alpha = \frac{\partial^\alpha}{\partial t^\alpha}$. If we apply the inverse operator of $D_{*t}^\alpha$ on both of sides of Eq. (3.1), we obtain

$$\begin{aligned} u(x,t) = &\sum_{\varepsilon=0}^{m-1} \frac{\partial^\varepsilon u}{\partial t^\varepsilon}(x,0^+)\frac{t^\varepsilon}{\varepsilon!} \\ &- J^\alpha\left(f_0(x)u + f_1(x)L_{1x}u + \cdots + f_n(x)L_{nx}u - q(x,t)\right). \end{aligned} \tag{3.2}$$

Then, the ADM [2, 3] supposes $u(x,t)$ solution as

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t), \tag{3.3}$$

such that the components $u_n(x,t)$ are obtained with iteration. Substituting Eq. (3.3) into both sides of Eq. (3.2), we get

$$\begin{aligned}\sum_{n=0}^{\infty} u_n(x,t) &= \sum_{\varepsilon=0}^{m-1} \frac{\partial^{\varepsilon} u}{\partial t^{\varepsilon}}(x,0^+)\frac{t^{\varepsilon}}{\varepsilon!} \\ &\quad -J^{\alpha}\left(f_0(x)\left(\sum_{n=0}^{\infty} u_n(x,t)\right)+\cdots+f_n(x)L_{nx}\left(\sum_{n=0}^{\infty} u_n(x,t)\right)-q(x,t)\right).\end{aligned}$$

Therefore, we obtain the recurrence relations with respect to the decomposition series as

$$\begin{aligned}u_0(x,t) &= \sum_{\varepsilon=0}^{m-1} \frac{\partial^{\varepsilon} u}{\partial t^{\varepsilon}}(x,0^+)\frac{t^{\varepsilon}}{\varepsilon!}+J^{\alpha}(q(x,t)), \\ u_{i+1}(x,t) &= -J^{\alpha}(f_0(x)u_i(x,t)+f_1(x)L_{1x}u_i(x,t)+\cdots+f_n(x)L_{nx}u_i(x,t)),\ i\geq 0.\end{aligned} \tag{3.4}$$

At the end of the solution steps, we have the approximate solution $u(x,t)$

$$\phi_N(x,t)=\sum_{n=0}^{N-1} u_n(x,t), \qquad \lim_{N\to\infty}\phi_N(x,t)=u(x,t).$$

ADM obtains the series expansions which lead to exact solution for linear differential equations. Moreover, for nonlinear fractional equations, ADM gives accurate and effective numerical solutions with high correctness. The convergence of the decomposition series has been discovered in [14].

## 4. Applications and Results

To confirm our suggestion with respect to the method we mentioned above, two special fractional option pricing problems will be solved. The symbolic calculus program Maple is used for all the results and computations.

### 4.1. Example

Let us keep in view the linear time-fractional option pricing equation (1.2) subject to the initial condition (1.3). If we substitute Eq. (1.2) and the initial condition (1.3) into Eq. (3.4) in order to solve the problem using the recommended method, we get the following recurrence relation

$$\begin{aligned}v_0(x,\tau) &= v(x,0)=\max(e^x-1,0) \\ v_{j+1}(x,\tau) &= J^{\alpha}(L_{2x}v_j(x,\tau)+(k-1)L_{1x}v_j(x,\tau)-kv_j(x,\tau))\end{aligned} \tag{4.1}$$

Regarding as the recurrence relations in (4.1), some basic constituents of the decomposition series can be created as follows:

$$\begin{aligned}v_0(x,\tau) &= \max(e^x-1,0), \\ v_1(x,\tau) &= J^{\alpha}(L_{2x}v_0(x,\tau)+(k-1)L_{1x}v_0(x,\tau)-kv_0(x,\tau)) \\ &= J^{\alpha}(e^x+(k-1)e^x-k\max(e^x-1,0)) \\ &= e^x k\frac{\tau^{\alpha}}{\Gamma(\alpha+1)}-\max(e^x-1,0)k\frac{\tau^{\alpha}}{\Gamma(\alpha+1)}, \\ v_2(x,\tau) &= J^{\alpha}(L_{2x}v_1(x,\tau)+(k-1)L_{1x}v_1(x,\tau)-kv_1(x,\tau)) \\ &= J^{\alpha}\left(\max(e^x-1,0)\frac{k^2\tau^{\alpha}}{\Gamma(\alpha+1)}-e^x\frac{k^2\tau^{\alpha}}{\Gamma(\alpha+1)}\right) \\ &= \frac{k^2\tau^{2\alpha}}{\Gamma(2\alpha+1)}\max(e^x-1,0)-\frac{k^2\tau^{2\alpha}}{\Gamma(2\alpha+1)}e^x, \\ v_3(x,\tau) &= J^{\alpha}(L_{2x}v_2(x,\tau)+(k-1)L_{1x}v_2(x,\tau)-kv_2(x,\tau)) \\ &= J^{\alpha}\left(\max(e^x-1,0)\frac{-k^3\tau^{2\alpha}}{\Gamma(2\alpha+1)}+e^x\frac{k^3\tau^{2\alpha}}{\Gamma(2\alpha+1)}\right) \\ &= -\frac{k^3\tau^{3\alpha}}{\Gamma(3\alpha+1)}\max(e^x-1,0)+\frac{k^3\tau^{3\alpha}}{\Gamma(3\alpha+1)}e^x, \\ &\ \ \vdots\end{aligned}$$

and so on, hereby we can obtain the rest of constituents of the decomposition series. Then the series solution of the proposed problem is given by

$$\begin{aligned} v(x,\tau) &= \sum_{n=0}^{\infty} v_n(x,\tau) && (4.2)\\ &= \max(e^x-1,0)+\frac{k\tau^{\alpha}}{\Gamma(\alpha+1)}e^x-\frac{k\tau^{\alpha}}{\Gamma(\alpha+1)}\max(e^x-1,0)\\ &\quad+\frac{k^2\tau^{2\alpha}}{\Gamma(2\alpha+1)}\max(e^x-1,0)-\frac{k^2\tau^{2\alpha}}{\Gamma(2\alpha+1)}e^x\\ &\quad-\frac{k^3\tau^{3\alpha}}{\Gamma(3\alpha+1)}\max(e^x-1,0)+\frac{k^3\tau^{3\alpha}}{\Gamma(3\alpha+1)}e^x\\ &\quad+\frac{k^4\tau^{4\alpha}}{\Gamma(4\alpha+1)}\max(e^x-1,0)-\frac{k^4\tau^{4\alpha}}{\Gamma(4\alpha+1)}e^x+\cdots\\ &= \max(e^x-1,0)\left(1-\frac{k\tau^{\alpha}}{\Gamma(\alpha+1)}+\frac{k^2\tau^{2\alpha}}{\Gamma(2\alpha+1)}-\frac{k^3\tau^{3\alpha}}{\Gamma(3\alpha+1)}+\cdots\right)\\ &\quad+e^x\left(\frac{k\tau^{\alpha}}{\Gamma(\alpha+1)}-\frac{k^2\tau^{2\alpha}}{\Gamma(2\alpha+1)}+\frac{k^3\tau^{3\alpha}}{\Gamma(3\alpha+1)}-\cdots\right)\\ &= \max(e^x-1,0)E_{\alpha}(-k\tau^{\alpha})+e^x(1-E_{\alpha}(-k\tau^{\alpha})). \end{aligned}$$

Taking the special case $\alpha=1$, we obtain the exact solution as

$$v(x,\tau) = \max(e^x-1,0)e^{-k\tau}+e^x\left(1-e^{-k\tau}\right). \tag{4.3}$$

Following surfaces show the solutions $v(x,\tau)$ with respect to Eq. (1.2) with the initial condition (1.3):

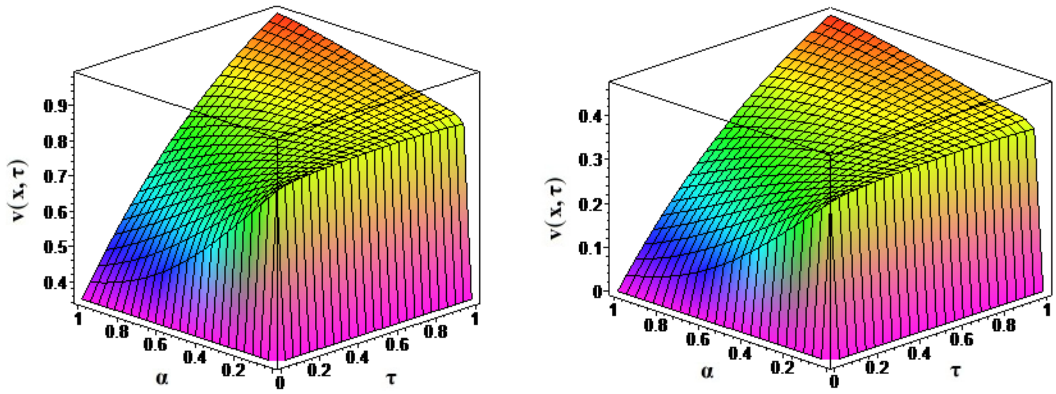


**Figure 4.1:** The solution of Eq. (1.2)-(1.3) for $x=0.3$ (left) and $x=-0.3$ (right)

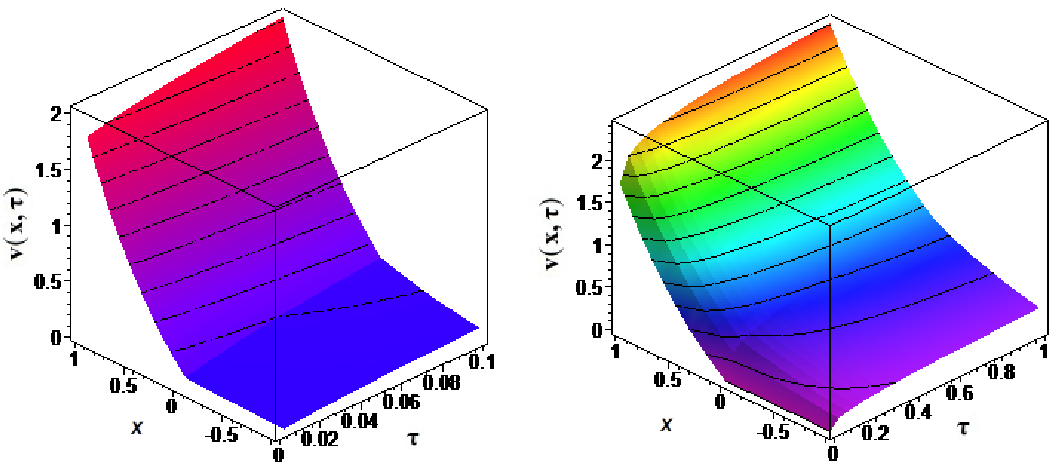


**Figure 4.2:** The solution of Eq. (1.2)-(1.3) for $\alpha=0.75$ (left) and $\alpha=0.5$ (right)

Figure 4.3 shows the vanilla call option prices, which is given in Eq. (1.1) with exercise price $E=70$, for fractional values $\alpha=0.25$, $\alpha=0.50$, $\alpha=0.75$ and $\alpha=1$.. According to Figure 4.3, we can say that the option has the lowest price in exercise time of the option $(\tau=T)$, when $\alpha=1$. As $\alpha$ decrease, the payoff of the option increases. When $\alpha=0.25$, we observe that the option is overpriced.

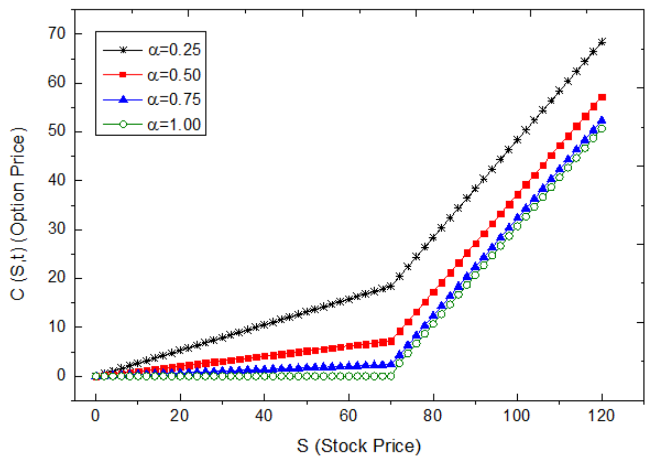


**Figure 4.3:** Option prices $C(S,t)$ with respect to underlying asset $S$ for different $\alpha$ values

## 4.2. Example

Now, we consider the generalized fractional option pricing equation (1.4) subject to the initial condition (1.5). Substituting Eq. (1.4) with its initial condition (1.5) into Eq. (3.4) in order to solve the proposed problem using the mentioned method, we have the following recurrence relation

$$\begin{aligned} v_0(x,\tau) &= v(x,0) = \max\left(x - 25e^{-0.06}, 0\right), \\ v_{j+1}(x,\tau) &= J^{\alpha}\left(0.08\,(2+\sin x)^2 x^2 L_{2x} v_j(x,\tau) + 0.06 x L_{1x} v_j(x,\tau) - 0.06 v_j(x,\tau)\right). \end{aligned} \tag{4.4}$$

Considering the recurrence relations in (4.4), some basic components of the decomposition series can be derived as shown:

$$\begin{aligned} v_0(x,\tau) &= \max\left(x - 25e^{-0.06}, 0\right), \\ v_1(x,\tau) &= J^{\alpha}\left(0.08\,(2+\sin x)^2 x^2 L_{2x} v_0(x,\tau) + 0.06 x L_{1x} v_0(x,\tau) - 0.06 v_0(x,\tau)\right) \\ &= J^{\alpha}\left(-0.06 \max\left(x - 25e^{-0.06}, 0\right) + 0.06x\right) \\ &= \frac{\tau^{\alpha}}{\Gamma(\alpha+1)}\left(-0.06 \max\left(x - 25e^{-0.06}, 0\right) + 0.06x\right) \\ &= \frac{(-0.06)\,\tau^{\alpha}}{\Gamma(\alpha+1)}\left(\max\left(x - 25e^{-0.06}, 0\right) - x\right), \\ v_2(x,\tau) &= J^{\alpha}\left(0.08\,(2+\sin x)^2 x^2 L_{2x} v_1(x,\tau) + 0.06 x L_{1x} v_1(x,\tau) - 0.06 v_1(x,\tau)\right) \\ &= J^{\alpha}\left(0.06\left(\frac{(0.06)\,\tau^{\alpha}}{\Gamma(\alpha+1)}\right)\left(\max\left(x - 25e^{-0.06}, 0\right) - x\right)\right) \\ &= \frac{(-0.06)^2\,\tau^{2\alpha}}{\Gamma(2\alpha+1)}\left(\max\left(x - 25e^{-0.06}, 0\right) - x\right), \\ v_3(x,\tau) &= J^{\alpha}\left(0.08\,(2+\sin x)^2 x^2 L_{2x} v_2(x,\tau) + 0.06 x L_{1x} v_2(x,\tau) - 0.06 v_2(x,\tau)\right) \\ &= J^{\alpha}\left(-0.06\left(\frac{(0.06)^2\,\tau^{2\alpha}}{\Gamma(2\alpha+1)}\right)\left(\max\left(x - 25e^{-0.06}, 0\right) - x\right)\right) \\ &= \frac{(-0.06)^3\,\tau^{3\alpha}}{\Gamma(3\alpha+1)}\left(\max\left(x - 25e^{-0.06}, 0\right) - x\right), \\ &\vdots \end{aligned}$$

and going on this approach, we can obtain the rest of constituents of the decomposition series. Moreover, the series solution of the generalized fractional option pricing problem is presented by

$$\begin{aligned} v(x,\tau) &= \sum_{n=0}^{\infty} v_n(x,\tau) \\ &= \max\left(x-25e^{-0.06},0\right)\frac{(0.06)\,\tau^{\alpha}}{\Gamma(\alpha+1)}\left(\max\left(x-25e^{-0.06},0\right)-x\right) \\ &\quad+\frac{(0.06)^2\,\tau^{2\alpha}}{\Gamma(2\alpha+1)}\left(\max\left(x-25e^{-0.06},0\right)-x\right) \\ &\quad-\frac{(0.06)^3\,\tau^{3\alpha}}{\Gamma(3\alpha+1)}\left(\max\left(x-25e^{-0.06},0\right)-x\right)+\cdots \\ &= \max\left(x-25e^{-0.06},0\right)\left[1-\frac{(0.06)\,\tau^{\alpha}}{\Gamma(\alpha+1)}+\frac{(0.06)^2\,\tau^{2\alpha}}{\Gamma(2\alpha+1)}-\frac{(0.06)^3\,\tau^{3\alpha}}{\Gamma(3\alpha+1)}+\cdots\right] \\ &\quad+x\left[1-\left(1-\frac{(0.06)\,\tau^{\alpha}}{\Gamma(\alpha+1)}+\frac{(0.06)^2\,\tau^{2\alpha}}{\Gamma(2\alpha+1)}-\frac{(0.06)^3\,\tau^{3\alpha}}{\Gamma(3\alpha+1)}+\cdots\right)\right] \\ &= \max\left(x-25e^{-0.06},0\right)E_{\alpha}\left(-0.06\tau^{\alpha}\right)+x\left(1-E_{\alpha}\left(-0.06\tau^{\alpha}\right)\right). \end{aligned} \tag{4.5}$$

The exact solution of generalized BSE for special case $\alpha = 1$ is given by

$$v(x,\tau) = x\left(1-e^{-0.06\tau}\right)+e^{-0.06\tau}\max\left(x-25e^{-0.06},0\right). \tag{4.6}$$

The following surfaces show the solutions $v(x,\tau)$ of Eq. (1.4) with initial condition (1.5) for various values of $\alpha$, $x$ and $\tau$:

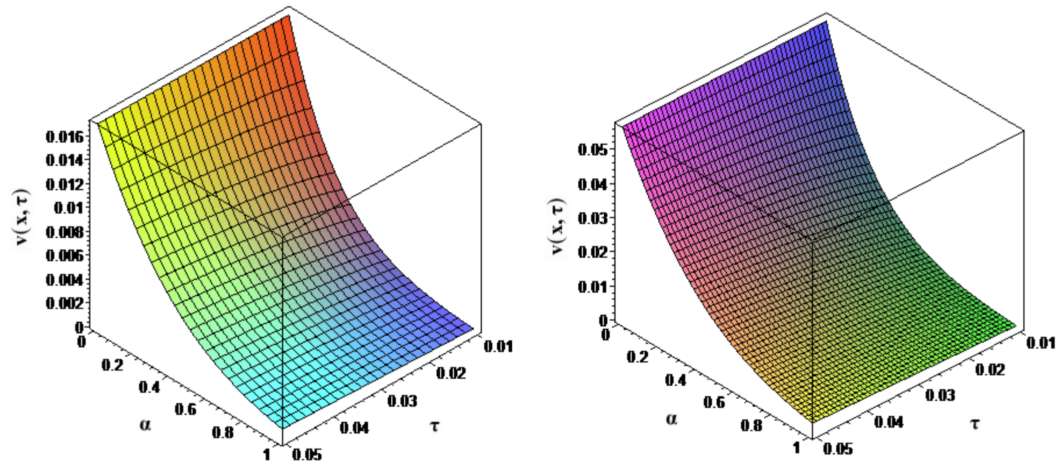


**Figure 4.4:** The solutions of Eq. (1.4)-(1.5) for $x = 0.3$ (left) and $x = 1$ (right)

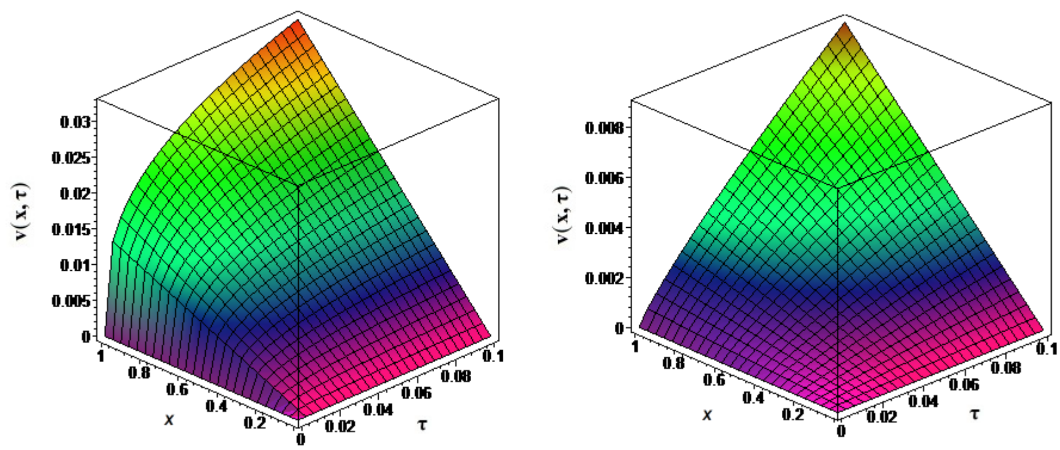


**Figure 4.5:** The solutions of Eq. (1.4)-(1.5) for $\alpha = 0.3$ (left) and $\alpha = 0.85$ (right)

## 5. Obtained Values Compatibility Test Using Stability Analysis

In this part of the study, we have explained that obtained values compatibility test by regarding as the convergence and the stability of the recommended method. Because the series (4.2) and (4.5) converges, these series have to be the solution of initial value problems (1.2)-(1.3) and (1.4)-(1.5), respectively. In addition, the solution results represent that the suggested solution technique is convergent and stable. The method we used in this study provides a good convergence area of the solution. The numerical results found with the decomposition series are good settlement with the exact solutions. For the purpose of understanding the convergent and stability of the fractional ADM defined in Section 3, amount of the absolute error $R_\varepsilon$ for some values of $\alpha$ and $\tau$ have been presented. In Figure 5.1, we have also investigated the error rates based on the numerical and exact solution results. According to the results of this stability analysis, it can be concluded that the ADM in Caputo fractional operator sense is effective and accurate method which is computable the series easily in short time.

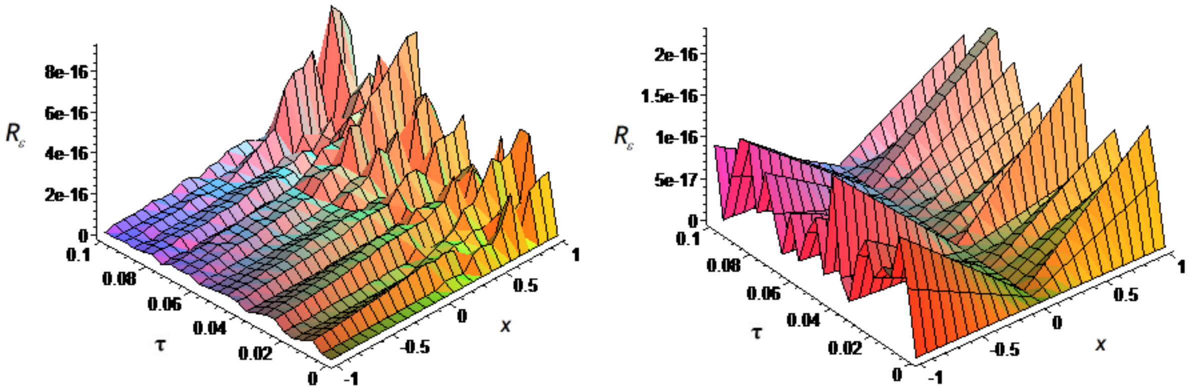


**Figure 5.1:** Absolute error values $R_\varepsilon$ for some values of $x$ and $\tau$ for (1.2)-(1.3) (left) and (1.4)-(1.5) (right)

## 6. Conclusion

In this paper, an approximate analytical solution of FBSE and GFBSE have been achieved with the ADM which is known as a powerful technique for solving many fractional PDEs. In order to specify the correctness and the simplicity of the ADM, we have used two examples from literature [5, 22]. The solutions for the values of $\alpha = 0.25$, $\alpha = 0.5$, $\alpha = 0.75$ and $\alpha = 1$ have been shown with the figures. According to the figures of the solutions, we argue that the option prices increase when $\alpha$ values decrease. Furthermore, results of this study show that the ADM is quite effective technique which gives an accurate and exact solution for FBSE and GFBSE with the initial conditions. We have also declared that the results have verified the validity and effectiveness of the suggested method in Caputo fractional derivative sense. Meanwhile, we can see that the option has the lowest price in exercise time of the option $(\tau = T)$ when $\alpha = 1$. As $\alpha$ decreases, the payoff of the option increases. When $\alpha = 0.25$, we observe that the option is overpriced.

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