Araştırma Makalesi **BAUN Fen Bil. Enst. Dergisi, 20(1), 430-439, (2018)**

DOI: 10.25092/baunfbed.418471 **J. BAUN Inst. Sci. Technol., 20(1), 430-439, (2018)**

Approximate solution of time-fractional KdV equations by residual power series method

Mehmet ŞENOL*, Ayşe ATA

Nevşehir Hacı Bektaş Veli University Faculty of Science and Literature, Department of Mathematics, Nevşehir.

Geliş Tarihi (Recived Date): 09.11.2017
Kabul Tarihi (Accepted Date): 14.03.2018

Abstract

In this article, approximate solutions of the time-fractional Korteveg de Vries (KdV) and modified version of it is obtained by using the residual power series method (RPSM). Numerical results are given and then they are compared with the exact solutions both numerically and graphically. The results show that the present method is very successful, effective and reliable.

Keywords: *Residual power series method, KdV equation, Caputo fractional derivative, Fractional partial differential equation.*

Zaman kesirli KdV denklemlerinin residual kuvvet serisi yöntemi ile yaklaşık çözümü

Özet

Bu çalışmada zaman-kesirli Korteveg de Vries (KdV) denkleminin ve modifiye edilmiş halinin rezidual kuvvet serisi metodu (RPSM) ile yaklaşık çözümü elde edilmiştir. Nümerik sonuçlar verilmiş ve bu sonuçlar tam çözümle nümerik ve grafiksel olarak karşılaştırılmıştır. Bulunan sonuçlar kullanılan yöntemin gayet başarılı, etkili ve güvenilir olduğunu ortaya koymaktadır.

Anahtar kelimeler: *Residual kuvvet serisi yöntemi, KdV denklemi, Caputo kesirli türevi, Kesirli kısmi diferansiyel denklem.*

* Mehmet ŞENOL, msenol@nevsehir.edu.tr, http://orcid.org/0000-0001-8110-7739
Ayşe ATA, ayseataoz@gmail.com, http://orcid.org/0000-0002-0911-6830

1. Introduction

In recent years, fractional calculus has found countless applications in different branches of engineering and science such as fractional differential equations (FDE), fluid flow, electrical network, mathematical physics, biology, image and signal processing, viscoelasticity and control.

There are some common methods that are used to obtain approximate or analytical solutions of nonlinear fractional partial differential equations in literature. Adomian decomposition method (ADM) [1], Laplace analysis method (LAM) [2], homotopy analysis method (HAM) [3], homotopy perturbation method (HPM) [4], differential transformation method (DTM) [5] and perturbation-iteration algoritm (PIA) [6] are among them.

In this article, a new technique, namely, Residual power series method (RPSM) [7-14], is used to obtain approximate solution of time- fractional KdV equation. In this method, the coefficients of the power series are calculated by means of the concept of residual error with the help of one or more variable algebraic equation chains, and finally, in practice, a so-called truncated series solution is obtained [7].

The main advantage of this method over other methods is that it can be applied directly to the problem without linearization, perturbation or discretization and without any transformation by selecting appropriate initial conditions [8].

There are a few definition of fractional derivative of order $\alpha > 0$. The most widely used are the Riemann-Liouville and Caputo fractional derivatives.

1.1. ***Definition*** The Riemann –Liouville fractional derivative operator $\mathrm{D}^{\alpha} f(\mathrm{x})$ for $\alpha > 0$ and $q-1 < \alpha < q$ defined as [15]:

$$D^{\alpha} f(x) = \frac{d^q}{dx^q}\left[\frac{1}{\Gamma(q-\alpha)}\int_a^x \frac{f(t)}{(x-t)^{\alpha+1-q}}\,dt\right]. \tag{1}$$

1.2. ***Definition*** The Caputo fractional derivative of order $\alpha > 0$ for $n\epsilon\mathbb{N}$, $n-1 < \alpha < n$, D_*^{α}, defined as [15]:

$$D_*^{\alpha} \mathrm{f}(x) = \mathrm{J}^{n-\alpha} D^n f(x) = \frac{1}{\Gamma(n-\alpha)}\int_a^x (x-t)^{n-\alpha-1}\left(\frac{d}{dt}\right)^n f(t)dt. \tag{2}$$

The relation between Riemann-Liouville and Caputo fractional derivatives is expressed in the following theorem.

1.3. ***Theorem*** Let > 0, $n-1 < \alpha < n$, for $n \in \mathbb{N}$. Then [11]:

$$D_*^{\alpha} f(x) = D^{\alpha} f(x) - \sum_{k=0}^{n-1} \frac{x^{k-\alpha}}{\Gamma(k-\alpha+1)} D^k f(0). \tag{3}$$

1.4. ***Definition*** The Power series expansion of the form [8]:

$$\sum_{m=0}^{\infty} c_m (t-t_0)^{m\alpha} = c_0 + c_1(t-t_0)^{\alpha} + c_2(t-t_0)^{2\alpha} + \cdots, \ 0 \le n-1 < \alpha \le n, \quad t \ge t_0 \tag{4}$$

is called fractional power series about $t = t_0$.

1.5. ***Theorem*** For the fractional power series $\sum_{m=0}^{\infty} c_m \, t^{m\alpha}$ for $t \ge 0$ there are only three cases [10].
1) The series converges only when $t = 0$,
2) The series converges for each $t \ge 0$,
3) There is a positive real number R such that the series converges whenever $0 \le t < R$ and diverges whenever $t \ge R$.

1.6. ***Theorem*** Suppose that f has a FPS representation at of the form [10]:

$$f(t) = \sum_{n=0}^{\infty} c_n (t-t_0)^{n\alpha}, \ \ 0 < m-1 < \alpha \le m, \qquad t_0 \le t < t_0 + R \tag{5}$$

If $f(t) \in C[t_0, t_0+R]$ and $D_{t_0}^{n\alpha} f(t) \in C(t_0, t_0+R)$ for $n = 0,1,2,...$
then the coefficients c_n will take the form:

$$c_n = \frac{D_{t_0}^{n\alpha} f(t_0)}{\Gamma(n\alpha+1)}, \qquad D_{t_0}^{n\alpha} = D_{t_0}^{\alpha}.D_{t_0}^{\alpha}.D_{t_0}^{\alpha} \dots D_{t_0}^{\alpha} \ (n - times) \tag{6}$$

2. Residual power series algorithm

To illustrate the basic idea of RPSM, let's take a nonlinear fractional differential equation of the form:

$$D_t^{n\alpha} u(x,t) + R[x]u(x,t) + N[x]u(x,t) = g(x,t), \quad t > 0, x \in R, n-1 < n\alpha \le n \tag{7}$$

expressed by initial condition

$$f_0(x) = u(x,0) = f(x) \tag{8}$$

which is $R[x]$ is a linear operator and $N[x]$ is a non-linear operator and $g(x,t)$ are continuous functions.
The RPSM method consists of expressing the solution of the equation given below as the fractional power series expansion around $t = 0$.

$$f_{n-1}(x) = D_t^{(n-1)\alpha} u(x,0) = h(x) \tag{9}$$

The expansion form of the solution is given by:

$$u(x,t) = \sum_{n=0}^{\infty} f_n(x) \frac{t^{n\alpha}}{\Gamma(1+n\alpha)}, 0 < \alpha \le 1, \quad x \in I, \quad 0 \le t < R \tag{10}$$

In the next step, the k. truncted series of $u(x,t)$, that is $u_k(x,t)$ can be written as:

$$u_k(x,t) = \sum_{n=0}^{k} f_n(x) \frac{t^{n\alpha}}{\Gamma(1+n\alpha)}, \qquad 0 < \alpha \le 1, \qquad x \in I, \qquad 0 \le t < R, \qquad k = 1,2,3,\ldots \tag{11}$$

If the 1. RPS approximate solution $u_1(x,t)$ is written as:

$$u_1(x,t) = f(x) + f_1(x) \frac{t^{\alpha}}{\Gamma(1+\alpha)} \tag{12}$$

then $u_k(x,t)$ could be reformulated as:

$$u_k(x,t) = f(x) + f_1(x) \frac{t^{\alpha}}{\Gamma(1+\alpha)} + \sum_{n=2}^{k} f_n(x) \frac{t^{n\alpha}}{\Gamma(1+n\alpha)}, \qquad 0 < \alpha \le 1, \qquad x \in I, \qquad 0 \le t < R, \qquad k = 2,3,4,\ldots \tag{13}$$

First we define the residual function as:

$$Res(x,t) = D_t^{n\alpha} u(x,t) + R[x]u(x,t) + N[x]u(x,t) - g(x,t) \tag{14}$$

and the k. residual function as:

$$Res_k(x,t) = D_t^{n\alpha} u_k(x,t) + R[x]u_k(x,t) + N[x]u_k(x,t) - g(x,t), k = 1,2,3,\ldots \tag{15}$$

It is clear that $Res(x,t) = 0$ and $\lim_{k\to\infty} Res_k(x,t) = Res(x,t)$ for each $x \in I$ and $t \ge 0$. In fact this lead to $D_t^{(n-1)\alpha} Res_k(x,t_0)$ for $n = 1,2,3,\ldots,k$ because the fractional derivative of a constant is zero in the Caputo sense.

Solving the equation $D_t^{(n-1)\alpha} Res_k(x,0) = 0$ gives us the desired $f_n(x)$ coefficients. Thus the $u_n(x,t)$ approximate solutions can be obtained respectively.

3. Application of the RPSM for fractional KdV equations

3.1. ***Example*** Consider the time-fractional Korteveg de Vries (KdV) equation [15]:

$$D_t^{a} u + 6uu_x + u_{xxx} = 0, 0 < a \le 1 \tag{16}$$

with the initial condition:

$$u(x,0) = \frac{1}{2} sech^2 \left(\frac{1}{2} x\right) \tag{17}$$

The known exact solution of the problem for $\alpha = 1$ is:

$$u(x,t) = \frac{1}{2} sech^2 \left(\frac{1}{2}(x-t)\right) \tag{18}$$

For residual power series

$$u(x,t) = \sum_{n=0}^{\infty} f_n(x) \frac{t^{n\alpha}}{\Gamma(1+a)} \tag{19}$$

and k. truncated series

$$u_k(x,t) = \sum_{n=0}^{k} f_n(x) \frac{t^{n\alpha}}{\Gamma(1+a)} \tag{20}$$

the k_1 th residual function for the KdV equation is:

$$Resu_k(x,t) = D_t^{\alpha} u(x,t) + 6u(x,t)u_x(x,t) + u_{xxx}(x,t), k = 1,2,3,... \tag{21}$$

So the fractional power series expansion of $u(x,t)$ about $t = 0$ is

$$u_k(x,t) = f(x) + f_1(x) \frac{t^{\alpha}}{\Gamma(1+a)} + \sum_{n=2}^{k} f_n(x) \frac{t^{n\alpha}}{\Gamma(1+a)}, \qquad k = 2,3,4,... \tag{22}$$

To determine the first unknown coefficient $f_1(x)$, in the expansion of (2.7) we substitute the 1th truncated series $u_1(x,t)$ into the 1th residual function $Resu_1(x,t)$ to get

$$Resu_1(x,t) = D_t^{\alpha} u_1(x,t) + 6u_1(x,t)(u_1)_x(x,t) + (u_1)_{xxx}(x,t) \tag{23}$$

Since $u_1(x,t) = f(x) + f_1(x)\frac{t^{\alpha}}{\Gamma(1+\alpha)}$, the Eq. (23) leads to:

$$Resu_1(x,t) = f_1(x) + 6\left(f(x) + \frac{t^{\alpha} f_1(x)}{\Gamma(1+\alpha)}\right)\left(f'(x) + \frac{t^{\alpha} f_1{}'(x)}{\Gamma(1+\alpha)}\right) + f^{(3)}(x) + \frac{t^{\alpha} f_1{}^{(3)}(x)}{\Gamma(1+\alpha)} \tag{24}$$

Now for the substitution of $t = 0$ through Eq.(24) we obtain:

$$\mathrm{Res}_1(x,0) = f_1(x) + 6f(x)f'(x) + f^{(3)}(x) \tag{25}$$

Thus for $\mathrm{Res}_1(x,0) = 0$

$$f_1(x) = 4\text{csch}^3(x)\sinh^4\left(\frac{x}{2}\right) \tag{26}$$

Hence, the 1st RPS approximate solution of Eq. (16) can expressed as:

$$u_1(x,t) = \frac{\Gamma(1+\alpha) + t^{\alpha}\tanh\left(\frac{x}{2}\right)}{(1+\text{Cosh}(x))\Gamma(1+\alpha)} \tag{27}$$

In the same manner, to obtain second unknown coefficient $f_2(x)$, we substitute the 2nd truncated series $u_2(x,t) = f(x) + f_1(x)\frac{t^{\alpha}}{\Gamma(1+\alpha)} + f_2(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}$, into the 2nd residual function $Resu_2(x,t) = D_t^{\alpha}u_2(x,t) + 6u_2(x,t)(u_2)_x(x,t) + (u_2)_{xxx}(x,t)$ and we get:

$$\begin{aligned} Resu_2(x,t) = {} & f_2(x) + 6f(x)f'(x) + \frac{6t^{\alpha}f_1(x)f'(x)}{\Gamma(1+\alpha)} + \frac{6t^{2\alpha}f_2(x)f'(x)}{\Gamma(1+2\alpha)} \\ & + \frac{6t^{\alpha}f(x){f_1}'(x)}{\Gamma(1+\alpha)} + \frac{6t^{2\alpha}f_1(x){f_1}'(x)}{\Gamma(1+\alpha)^2} + \frac{6t^{3\alpha}f_2(x){f_1}'(x)}{\Gamma(1+\alpha)\Gamma(1+2\alpha)} \\ & + \frac{6t^{2\alpha}f(x){f_2}'(x)}{\Gamma(1+2\alpha)} + \frac{6t^{3\alpha}f_1(x){f_2}'(x)}{\Gamma(1+\alpha)\Gamma(1+2\alpha)} + \frac{6t^{4\alpha}f_2(x){f_2}'(x)}{\Gamma(1+2\alpha)^2} + f^{(3)}(x) \\ & + \frac{t^{\alpha}{f_1}^{(3)}(x)}{\Gamma(1+\alpha)} + \frac{t^{2\alpha}{f_2}^{(3)}(x)}{\Gamma(1+2\alpha)} \end{aligned} \tag{28}$$

Now applying D_t^{α} on both sides of Eq. (28) and equating it to 0 for $t = 0$ gives:

$$f_2(x) = \frac{1}{4}\big(-2+\cosh(x)\big)\text{sech}^4\left(\frac{x}{2}\right) \tag{29}$$

Therefore, the 2nd RPS approximate solution of Eq. (16) is obtained as:

$$u_2(x,t) = \frac{1}{4}\text{sech}^2\left(\frac{x}{2}\right)(2 + \frac{t^{2\alpha}(-2+\cosh(x))\text{sech}^2\left(\frac{x}{2}\right)}{\Gamma(1+2\alpha)} + \frac{2t^{\alpha}\tanh\left(\frac{x}{2}\right)}{\Gamma(1+\alpha)}) \tag{30}$$

Similarly, by applying the same procedure for $n = 3,4,5$ we obtain the following results respectively,

$$\begin{aligned} f_3(x) = {} & \frac{((39 - 32\cosh(x) + \cosh(2x))\Gamma(1+\alpha)^2}{16\Gamma(1+\alpha)^2} \\ & + \frac{12(-2+\cosh(x))\Gamma[1+2\alpha])\text{sech}^6\left(\frac{x}{2}\right)\tanh\left(\frac{x}{2}\right)}{16\Gamma(1+\alpha)^2} \end{aligned} \tag{31}$$

$$\begin{aligned} u_3(x,t) = {} & \frac{1}{2}\text{sech}^2\left(\frac{x}{2}\right) + \frac{t^{2\alpha}\big(-2+\cosh(x)\big)\text{sech}^4\left(\frac{x}{2}\right)}{4\Gamma(1+2\alpha)} + \frac{4t^{\alpha}\text{csch}^3(x)\sinh^4\left(\frac{x}{2}\right)}{\Gamma(1+\alpha)} \\ & + \frac{t^{3\alpha}((39 - 32\cosh(x) + \cosh(2x))\Gamma(1+\alpha)^2}{16\Gamma(1+\alpha)^2\Gamma(1+3\alpha)} \\ & + \frac{12(-2+\cosh(x))\Gamma(1+2\alpha))\text{sech}^6\left(\frac{x}{2}\right)\tanh\left(\frac{x}{2}\right)}{16\Gamma(1+\alpha)^2\Gamma(1+3\alpha)} \end{aligned} \tag{32}$$

and so on. Other iterations are not given for brevity but fifth iteration solution $u_5(x,t)$ is calculated in this manner. Using a symbolic computation program like Mathematica, the other iterations can be calculated. In Table 1, the fifth order approximate RPSM results and the exact solutions are compared. Also in Figure 1 they are compared graphically.

3.2. ***Example*** Consider the time-fractional modified Korteveg de Vries (mKdV) equation [16]:

$$D_t^{\alpha}u + 6u^2u_x + u_{xxx} = 0,\ 0 < \alpha \leq 1 \tag{33}$$

with the initial condition:

$$u(x,0) = \sqrt{c}\mathrm{sech}(k + \sqrt{c}x) \tag{34}$$

The known exact solution of the problem for $\alpha = 1$ is:

$$u(x,t) = \sqrt{c}\mathrm{sech}(k + \sqrt{c}(x - ct)) \tag{35}$$

Following the same manner as in the previous example, we obtain the successive iterations as:

$$f_1(x) = c^2\mathrm{sech}\left(k + \sqrt{c}x\right)\tanh\left(k + \sqrt{c}x\right) \tag{36}$$

$$u_1(x,t) = \frac{\sqrt{c}\mathrm{sech}\left(k + \sqrt{c}x\right)(\Gamma[1+\alpha] + c^{3/2}t^{\alpha}\tanh\left(k + \sqrt{c}x\right))}{\Gamma(1+\alpha)} \tag{37}$$

$$f_2(x) = \frac{1}{2}c^{7/2}(-3 + \cosh\,(2\left(k + \sqrt{c}x\right))\mathrm{sech}^3\left(k + \sqrt{c}x\right) \tag{38}$$

$$\begin{aligned} u_2(x,t) = \sqrt{c}\mathrm{sech}\left(k + \sqrt{c}x\right)(1 &- \frac{c^3t^{2\alpha}\left(-1 + 2\mathrm{sech}^2\left(k + \sqrt{c}x\right)\right)}{\Gamma(1+2\alpha)} \\ &+ \frac{c^{3/2}t^{\alpha}\tanh\left(k + \sqrt{c}x\right)}{\Gamma(1+\alpha)} \end{aligned} \tag{39}$$

$$\begin{aligned} f_3(x) = \frac{1}{8\Gamma(1+\alpha)^2\Gamma(1+2\alpha)}&c^5((315 - 164\cosh[2(k + \sqrt{c}x)] + \cosh[4(k \\ &+ \sqrt{c}x)])\Gamma(1+\alpha)^2 + 24(-7 + 3\cosh[2(k \\ &+ \sqrt{c}x)])\Gamma(1+2\alpha)^2)\mathrm{sech}^5\left(k + \sqrt{c}x\right)\tanh\left(k + \sqrt{c}x\right) \end{aligned} \tag{40}$$

$$\begin{aligned} u_3(x,t) = \frac{1}{(8\Gamma(1+\alpha)^2\Gamma(1+2\alpha)\Gamma(1+\alpha))}&(\sqrt{c}\mathrm{sech}\,(k + \sqrt{c}x)(8\Gamma(1+\alpha)^2\Gamma(1 \\ &+ 2\alpha)\Gamma(1+3\alpha) + 4c^3t^{2\alpha}(-3 + \cosh\,[2(k + \sqrt{c}x)])\Gamma(1+\alpha)^2\Gamma(1 \\ &+ 3\alpha)\mathrm{sech}^2\left(k + \sqrt{c}x\right) + 8c^{3/2}t^{\alpha}\Gamma(1+\alpha)\Gamma(1+2\alpha)\Gamma(1 \\ &+ 3\alpha)\tanh\,(k + \sqrt{c}x) + c^{9/2}t^{3\alpha}((315 - 164\cosh[2(k + \sqrt{c}x)] \\ &+ \cosh[4(k + \sqrt{c}x)])\Gamma(1+\alpha)^2 + 24(-7 + 3\cosh[2(k \\ &+ \sqrt{c}x)])\Gamma(1+2\alpha)^2)\,\mathrm{sech}(k + \sqrt{c}x)^4\tanh(k + \sqrt{c}x))) \end{aligned} \tag{41}$$

Following this manner the other iteration results could be calculated. In Table 2, the third order approximate RPSM results are compared with exact solutions numerically and absolute errors of RPSM solutions are computed by different values of α and for different time points. In fact, the results show competitive solutions of RPSM. Figure 2 is extracted based on illustrating approximate solutions of RPSM in the similar manner and by choosing equal parameters and also as seen, they have similar patterns with exact solutions. In addition, they prove that both PIA and RPSM give remarkably approximate results.

Table 1. Comparison of numerical values of $u_5(x,t)$ in 3.1. Example for $x = 10$.

	$\alpha = 0.25$	$\alpha = 0.50$	$\alpha = 0.75$	$\alpha = 1.00$		
t	$u_5(x,t)$	$u_5(x,t)$	$u_5(x,t)$	$u_5(x,t)$	Exact solution	Absolute error
0	9.07616E-5	9.07616E-5	9.07616E-5	9.07916E-5	9.07916E-5	0
0.1	2.10313E-4	1.34953E-4	1.10732E-4	1.00339E-4	1.00339E-4	1.84812E-13
0.2	2.61221E-4	1.63130E-4	1.27525E-4	1.10890E-4	1.10890E-4	1.00999E-11
0.3	3.04835E-4	1.90686E-4	1.44887E-4	1.22551E-4	1.22551E-4	1.09388E-10
0.4	3.44949E-4	2.19003E-4	1.63411E-4	1.35438E-4	1.35439E-4	6.02626E-10
0.5	3.82956E-4	2.48619E-4	1.83425E-4	1.49678E-4	1.49681E-4	2.283610E-9
0.6	4.19542E-4	2.79821E-4	2.05178E-4	1.65413E-4	1.65420E-4	6.819710E-9
0.7	4.55102E-4	3.12787E-4	2.28887E-4	1.82797E-4	1.82815E-4	1.726760E-8
0.8	4.89888E-4	3.47646E-4	2.54761E-4	2.01999E-4	2.02037E-4	3.873280E-8
0.9	5.24070E-4	3.84494E-4	2.83004E-4	2.23202E-4	2.23281E-4	7.918610E-8
1.0	5.57768E-4	4.23410E-4	3.13820E-4	2.46608E-4	2.46758E-4	1.504510E-7

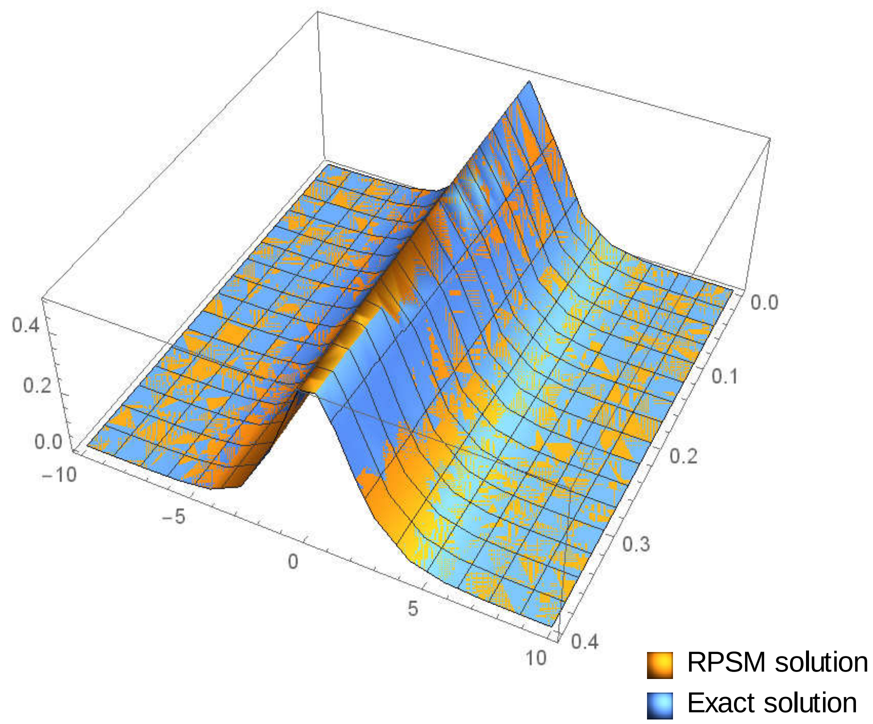

Figure 1. The surface graph of $u_5(x,t)$ and exact solution of 3.1. Example.

Table 2. Comparison of numerical values of $u_3(x,t)$ in 3.2. Example for $x = 10$.

	$\alpha = 0.25$	$\alpha = 0.50$	$\alpha = 0.75$	$\alpha = 1.00$		
t	$u_3(x,t)$	$u_3(x,t)$	$u_3(x,t)$	$u_3(x,t)$	Exact solution	Absolute error
0	9.07999E-5	9.07999E-5	9.07999E-5	9.07999E-5	9.07999E-5	0
0.1	1.99357E-4	1.34439E-4	110679E-4	1.00341E-4	1.00349E-4	7.95268E-9
0.2	2.36952E-4	1.60889E-4	127173E-4	1.10836E-4	1.10903E-4	6.68370E-8
0.3	2.66246E-4	1.85381E-4	143856E-4	1.22330E-4	1.22567E-4	2.36879E-7
0.4	2.91337E-4	2.09199E-4	161181E-4	1.34868E-4	1.35457E-4	5.89415E-7
0.5	3.13772E-4	2.32797E-4	179326E-4	1.48496E-4	1.49704E-4	1.20806E-6
0.6	3.34328E-4	2.56387E-4	198387E-4	1.63258E-4	1.65448E-4	2.18998E-6
0.7	3.53465E-4	2.80084E-4	218419E-4	1.79201E-4	1.82848E-4	3.64737E-6
0.8	3.71481E-4	3.03954E-4	239464E-4	1.96370E-4	2.02079E-4	5.70897E-6
0.9	3.88580E-4	3.28038E-4	261548E-4	2.14810E-4	2.23332E-4	8.52185E-6
1.0	4.04912E-4	3.52361E-4	284694E-4	2.34566E-4	2.46820E-4	1.22533E-5

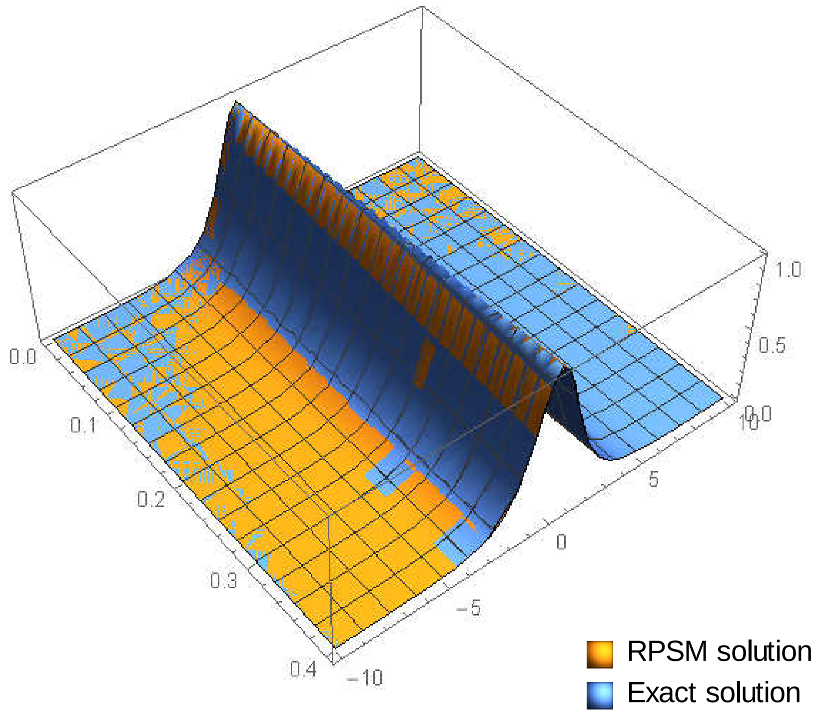

Figure 2. The surface graph of $u_5(x,t)$ and exact solution of 3.2. Example.

4. Conclusion

In this study, residual power series method was introduced to obtain approximate solutions for time-fractional Korteveg de Vries (KdV) and modified KdV partial differential equations. Numerical results and comparison with the exact solutions show that the present method is very powerful and reliable technique and producing highly approximate results. Comparing to other tecniques the method is very simple to apply without linearization, perturbation or discretization or any transformations. Also it is a good tool to use to calculate the approximate solutions of a wide range of fractional partial differential equations.

References

[1] Ray, S.S., and Bera, R.K., An approximate solution of a nonlinear fractional differential equation by Adomian decomposition method, **Applied Mathematics and Computation**, 167(1), 561-571, (2005).

[2] Kexue, L., and Jigen, P., Laplace transform and fractional differential equations, **Applied Mathematics Letters**, 24(12), 2019-2023, (2011).

[3] Zurigat, M., Momani, S., Odibat, Z., and Alawneh, A., The homotopy analysis method for handling systems of fractional differential equations, **Applied Mathematical Modelling**, 34(1), 24-35, (2010).

[4] Yildirim, A., An algorithm for solving the fractional nonlinear Schrödinger equation by means of the homotopy perturbation method, **International Journal of Nonlinear Sciences and Numerical Simulation**, 10(4), 445-450, (2009).

[5] Arikoglu, A., and Ozkol, I., Solution of fractional differential equations by using differential transform method, **Chaos, Solitons & Fractals**, 34(5), 1473-1481, (2007).

[6] Şenol, M., and Dolapci, I.T., On the Perturbation–Iteration Algorithm for fractional differential equations, **Journal of King Saud University-Science**, 28(1), 69-74, (2016).

[7] Kumar, A., Kumar, S., and Singh, M., Residual power series method for fractional Sharma-Tasso-Olever equation, **Communications in Numerical Analysis**, 1-10, (2016).

[8] Jaradat, H.M., Al-Shara, S., Khan, Q.J., Alquran, M., and Al-Khaled, K., Analytical solution of time-fractional Drinfeld-Sokolov-Wilson system using residual power series method, **IAENG International Journal of Applied Mathematics**, 46(1), 64-70, (2016).

[9] Alquran, M., Analytical solutions of fractional foam drainage equation by residual power series method, **Mathematical sciences**, 8(4), 153-160, (2015).

[10] El-Ajou, A., Arqub, O.A., Zhour, Z.A., and Momani, S., New results on fractional power series: theories and applications, **Entropy**, 15(12), 5305-5323, (2013).

[11] Ahmad, R.S., An analytical solution of the fractional Navier-Stokes equation by residual power series method, Zarqa University, Doctoral dissertation, 10-90, (2015).

[12] Al Qurashi M.M., Korpinar, Z., Baleanu, D., and Inc, M., A new iterative algorithm on the time-fractional Fisher equation: Residual power series method, **Advances in Mechanical Engineering**, 9(9) 1–8, (2017).

[13] Tchier, F., Inc, M., Korpinar, Z., and Baleanu, D., Solutions of the time fractional reaction–diffusion equations with residual power series method, **Advances in Mechanical Engineering**, 8(10) 1–10, (2016).

[14] Inc, M., Korpinar, Z., Al Qurashi M.M., and Baleanu, D., A new method for approximate solutions of some nonlinear equations: Residual power series method, **Advances in Mechanical Engineering**, 8(4) 1–7, (2016).

[15] Momani, S., Odibat, Z., and Alawneh, A., Variational iteration method for solving the space□and time□fractional KdV equation", **Numerical Methods for Partial Differential Equations**, 24(1), 262-271, (2008).

[16] Abdulaziz, O., Hashim, I., and Ismail, E.S., Approximate analytical solution to fractional modified KdV equations, **Mathematical and Computer Modelling**, 49(1-2), 136-145, (2009).