Çukurova Üniversitesi Eğitim Fakültesi Dergisi
Cilt: 45 Sayı:1 Sayfa: 123-142
www.cufej.com

An Analysis of Seventh-Grade Students' Mathematical Reasoning*

Emrullah ERDEM [a*], **Ramazan GÜRBÜZ**[a]

[a] Adıyaman Üniversitesi, Eğitim Fakültesi, Adıyaman/Türkiye

Article Info

DOI: 10.14812/cufej.2015.007

Article history:
Received 11 December 2014
Revised 10 February 2015
Accepted 25 March 2015

Keywords:
Mathematical reasoning,
Sevent-grade students,
Problems requiring reasoning.

Abstract

The aim of this study is to determine the seventh-graders' levels of mathematical reasoning and to reveal their performance. The present study was carried out with 167 seventh-grade students studying at randomly selected three middle schools that served low and middle socioeconomic areas in a city of Turkey. "Mathematical Reasoning Test (MRT)" was developed and used as data collection tool. In analyzing the data, participants' scores of the test was computed and which mathematical reasoning level they were in was determined. Sample responses of the some students regarding any question (Q7) in the test were presented directly and discussed. As a result of the analysis, it was found that about half of the students (45.5%) had medium and 27.5% of them had low level of mathematical reasoning. When the results are evaluated, it is probable to say that most of the students' mathematical reasoning is at medium or low level in general. On the other hand, it is remarkable that rather than the familiar classical problems, students need to be enabled to deal with the problems that they can do reasoning and thus their mathematical reasoning could be improved.

Yedinci Sınıf Öğrencilerinin Matematiksel Muhakemelerinin Bir Analizi

Makale Bilgisi

DOI: 10.14812/cufej.2015.007

Makale Geçmişi:
Geliş 11 Aralık 2014
Düzeltme 10 Şubat 2015
Kabul 25 Mart 2015

Anahtar Kelimeler:
Matematiksel muhakeme,
Yedinci sınıf öğrencileri,
Muhakeme gerektiren problemler.

Öz

Bu çalışmanın amacı, yedinci sınıf öğrencilerinin matematiksel muhakeme düzeylerini belirlemek ve bu yöndeki performanslarını ortaya koymaktır. Çalışma, Türkiye'nin bir ilindeki düşük ve orta sosyo-ekonomik düzeye sahip üç ortaokulunda öğrenim gören 167 yedinci sınıf öğrencisinin katılımıyla gerçekleştirilmiştir. Matematiksel Muhakeme Testi (MMT) geliştirilmiş ve veri toplama aracı olarak kullanılmıştır. Verilerin analizi için katılımcıların test puanları hesaplanmış ve hangi düzeyde oldukları belirlenmiştir. Bazı öğrencilerin testteki örnek bir soruya (Q7) ilişkin bazı cevapları doğrudan aktarılmış ve tartışılmıştır. Yapılan analiz sonucunda, katılımcıların yaklaşık yarısının (%45.5) matematiksel muhakemesinin orta, %27.5'inin ise düşük düzeyde olduğu tespit edilmiştir. Bu sonuçlar göz önüne alındığında, genel olarak öğrencilerin matematiksel muhakemelerinin orta ve düşük düzeyde olduğu söylenebilir. Matematiksel muhakemenin geliştirilebilmesi için öğrencilerin alışılmış klasik problemlerden ziyade muhakame yapmalarını gerektiren problemlerle uğraşmalarına imkân tanınmalıdır.

* This study is a part of master's thesis prepared by Res. Assist. Emrullah ERDEM under the advisory of Assoc. Prof. Dr. Ramazan GÜRBÜZ. The study was also presented at the 1th Turkish Computer and Mathematics Education Symposium.

* **Yazar:** eerdem@outlook.com

Introduction

Mathematical reasoning can be defined as higher level thinking process which is carried out through detailing a problem or phenomenon around the questions of "Why" and "How" and making it meaningful. Because maths knowledge is reached through reasoning rather than experiments or observations; without reasoning, mathematics cannot be fulfilled. Similarly, Curtis (2004) and Sparkes (1999) mentioned that reasoning is indispensable for doing mathematics. Toulmin, Rieke, and Janik (1984) stated that reasoning does not create new ideas and that the mission of reasoning is to make the best decision on a specific situation, subject or an event. Mathematical reasoning is structured via questioning maths knowledge. No matter at how advanced stage it is, unless an idea is based on knowledge and comprises logical approaches; it cannot be seen as reasoning (Umay, 2003). Thus, considering mathematics as an interpenetrating knowledge link is both the result of reasoning emphasis and forms a basis for an advance reasoning. (Umay & Kaf, 2005).

Lithner (2008) states that reasoning can possibly be evaluated as a process of thinking, the product of this process or both and he mentions the reasoning process in math language as in Figure 1.

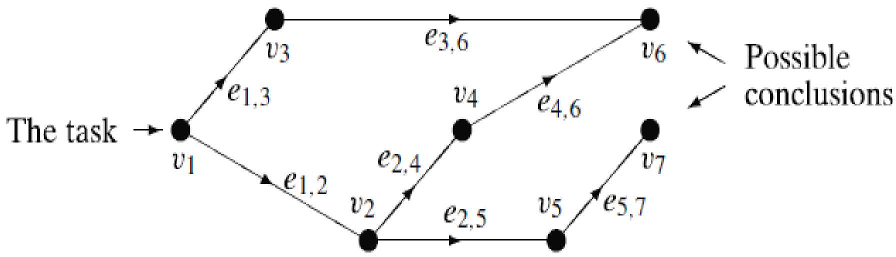


Figure 1. Reasoning process

In Figure 1, *a vertex v_n represents both a momentary state of knowledge and of the (sub)task. The reasoner makes a strategy choice among the edges leading from v_n. The strategy implementation is represented by a transition edge $e_{n,\ m}$. Here knowledge not already accessed in v_n is recalled or constructed and added up to form the new knowledge state in v_m, where the task is partially resolved and therefore a new task state is formulated. A reason is the motivation supporting transitions between vertices* (Lithner, 2008, p. 257).

According to Russell (1999), reasoning is a tool which provides students with comprehending the abstract statements that renders math a discipline. However, mathematical reasoning may also be defined as the process of reaching a decision by using critical, creative and logical thinking. Thus, it is crucially important that students fulfill mathematical reasoning in learning environments for effective learning. In national (MEB, 2009; 2013) and international studies which are carried out related to mathematical education (NCTM, 1989; 2000) and in a number of other relevant researches (Diezmann & English, 2001; English, 1998; Fischbein & Schnarch, 1997; Gürbüz & Erdem, 2014; Kramarski, Mevarech, & Lieberman, 2001; Lithner, 2000; Schoenfeld, 1985; Umay, 2003; White, Alexander, & Daugherty, 1998) it is mentioned that mathematical reasoning has a significant role on math learning. For instance, in their studies Diezmann & English (2001), Kramarski et al. (2001) and White et al. (1998) stated that there is a direct relationship between math learning and reasoning, and those who manage to do better reasoning produce more efficient solutions to the problems and fulfill much better associations.

The problems, most of which are stereotyped, well-structured and does not necessarily require reasoning and pushes to giving right answers that are presented to students in learning environments cause that students learn superficially. Frederiksen (1984) collected the problems within three categories: (1) *Well-structured problems* call for using a familiar method for problem solving and it can be easily solved through formula; (2) In addition to being identical to well-structured problems, *structured problems*, either the whole solution method or some aspects of it are produced by the problem solver. However, (3) *ill-structured problems* cannot be instantly formulated and a specific solution method is not available. Funke & Frensch (1995) stated that well-structured problems are similar to the problems within course books and includes only a few variables, while ill-structured problems manage to include a number of factors or variables that call for association through unpredictable ways. Francisco & Maher (2005) mentioned that the complicated applications which are presented to students will enable students to do more efficient reasoning in proportion to simple ones. Henningsen & Stein (1997) asserted that elimination of complexity of this application caused students tend to think at lower level. Hence, students need to be enabled to deal with the problems that requires higher level of reasoning.

One of the paramount aims of education is to enable individuals to produce efficient solutions to the problems that they encounter in their daily life. The path to achieve this goal in the best way goes through doing mathematics. Doing mathematics forces individuals to do reasoning while producing solution to the problem and thus helps them take all options into consideration and improves their decision making skills. Since people who do reasoning well manage to make up more correct and efficient decisions, they are more likely to be successful in their daily life. For this reason, in learning environments, it is essentially important to have students encounter with the problems which force them to do reasoning and to analyze their reasoning levels. In this sense, the aim of the current study is to determine seventh-graders’ levels of mathematical reasoning and to reveal their performance.

Method

Research Design

In this study, descriptive research method was used in order to find out the present level of mathematical reasoning of the seventh-graders. Many educational research methods are descriptive; that is, they set out to describe and to interpret what is (Cohen, Manion, & Morrison, 2000).

Participants

The participants were 167 seventh-grade students studying at randomly selected three elementary schools that served low and middle socioeconomic areas in a city of Turkey. These students were given code names such as “S1”, “S2”, “S3”, …

Data Collection

As data collection tool, by making use of literature, Mathematical Reasoning Test (MRT) (*some sample questions were presented in the appendix*) which was developed by the researchers and composed of 35 questions in different formats was used. The validity of the instrument was confirmed by two mathematics teachers and two mathematics educators. The pilot test was performed with 32 seventh-grade students who did not participate in the actual study. The pilot study revealed that questions in the test were understandable and clear for seventh-grade students. Nevertheless, 45 minutes given to the students in the pilot practice turned out to be insufficient and for this test, 60 minutes were allocated for the test. Also, the Cronbach's alpha reliability coefficient of the MRT was found to be .885.

Data Analysis

Students' answers were analyzed by using Statistical Package for Social Sciences (SPSS). Levels at Table 1 and score intervals are created according to the scoring scales on Table 2 and Table 3 because it is thought that this scoring will be more efficient and intelligable. The total score of each student is divided into the number of questions (35) and each student's level is determined. For example, the total score of a student who got 130 points out of Table 2 and Table 3, [mean:130/35=3.71; this score is between 3.00-3.99] (See Table 1) his mathematical reasoning is evaluated to be high. Moreover, responses related to one question (Q7) of one each of students at every level are given in detail.

Table 1.

Mathematical Reasoning Levels.

Level	**Score Interval**
Quite Low	0.00-0.99
Low	1.00-1.99
Medium	2.00-2.99
High	3.00-3.99
Quite High	4.00-5.00

Responses given to open ended questions were analyzed through scoring scale in Table 2 and responses given to two-phase questions *(1st Section-Multiple Choice,2nd Section-Open ended)* were analyzed through scoring scale in Table 3. In developing these scales, Gürbüz (2010) and Gürbüz & Birgin (2012) were taken advantage. According to Tablo 2 and Tablo 3, student responses were freely scored by a couple of experienced math trainers. Free evaluators who have come together for parellelism of scoring, agreed upon the consistency of scoring at the level of 85-90%.

Table 2.

The Scoring Scale Of Open Ended Questions.

Level	Score	Explanation	Sample Response
Completely Correct	5	Statements that are accepted to be completely true	Q_{25}: In the table, the addition of the numbers in the second row is 90. When we look at the order of the numbers, (10+20), (12+18), (14+16) each total is 30 and the whole total is 90.
Partly Correct-A	4	Missing statements according to complete true response	Q_{26}: The area of a quarter circle $=(\pi x20^2)/4=100\pi$ and the area of a circle piece which is 270^0 is $=3/4x(\pi x20^2)=300\pi$.The whole area is $100\pi+200\pi=300\pi$.
Partly Correct-B	3	Partly true statements that are fulfilled depending on the correct reason	Q_{31}: In the solution the problem a mistake was done. Because if the number of the master in duty is alot, the duration to complete the building decreases. Thus, the job that 5 masters finish in 10 days is completed by 10 masters in 5 days.
Partly Correct-C	2	Statements that is fulfilled by depending on a wrong cause or not depending on any kind of reason and accepted to be partly correct.	Q_{32}: The solution of the problem is correct. Because the speed of the vehicles that move from adverse directions is subtracted. In the same duration, the fast vehicle takes the lead more. *(The statement which is fulfilled by depending on the wrong reason and accepted to be partly correct).* Q_{31}: In the second situation, the construction of the house finishes sooner. *The statement which is fulfilled by not depending on any king of reason and accepted to be partly correct).*
Wrong	1	Statements of completely wrong or not completely related to the question.	Q_{26}: Since the rope that the sheep is tied to short, it cannot graze, it stays still. *(A completely wrong statement).* Q_{31}: The master may not have constructed the building strong enough while he was trying to finish it early. (*A statement that is not completely related to the question*).
Unanswered	0	Statements of no explanation is given or the question itself was given as responses	No explanation

Q_a: a. Question in MRT

Table 3.

Two-phased (1st section-multiple choice, 2nd section-open ended) questions scoring scale

Levels	Explanation	Assessment Criteria (1st-2nd Phase)	Score	Sample Response
Correct Justification	Answers that encompass all aspects of the valid justification	Correct Answer – Correct Justification	5	Q_7: The correct answer is B. Between the 1st and the 9th pages, 9 numbers are used. In order to find how many numbers are used from the 10th page to the 25th page, first we need to find how many numbers there are in this interval, there are (25-10+1)= 16 numbers and there are two numbers in each number. Thus, between the 10th and the 25th pages,16x2=32 numbers are used. Totally, 9+32=41 numbers have been used. Q_{10}: The correct answer is C. $\sqrt{36}(6^2) < \sqrt{39} < \sqrt{49}(7^2)$. So the edge length of a square garden becomes between 6m and 7 m.
		Incorrect Answer – Correct Justification	4	Q_{26}: The correct answer is A. Area of the 1st zone=$(\pi 20^2)$x3/4=300π, the area of the 2nd zone and 3rd are equal $(\pi 10^2)$x1/4= 25π. Q_{20}: The correct answer is D. (3/4)/(1/12)= (3/4)x(12)=9.
Partially Correct Justification	Answers that do not encompass all aspects of the valid justification	Correct Answer – Partially Correct Justification	3	Q_3: The correct answer is D. 1100-1095-1090-1085-1080-1075-1070-1065-1060-1055-… 700-715-730-745-760-775-790-805-820-835-850-865-890-… Q_{22}: The correct answer is D. Hour; from store A it is bought for 75 TRY, From store B for 80 TRY, From store C for 70 TRY, From store D for 70 TRY.
		Incorrect Answer Partially Correct Justification	2	Q_4: The correct answer is D. 17+12=29. Q_{11}: The correct answer is A. 235/10=23, because the least number of book is asked.
Wrong Justification	Answers that contain incorrect knowledge	Correct Answer – Wrong Justification	1	Q_{12}: The correct answer is A. 314-100=214; 314/2=157; 214-157=57. The correct answer is B. 25+10=35; 35+6=41
		Incorrect	0	Q_8: The correct answer is A . The

		Answer – Wrong Justification		result of the multiplication of 8 and 3 becomes the the highest. Q_9: The correct answer is A . (5/6)x(2/5)=1/3; 30x(1/3)=10.
No Justification	Correct, incorrect or blank answers with no justifications written	Correct Answer – No Justification	1	Q_9: The correct answer is B. No justification. Q_{12}: The correct answer is A. No justification
		Incorrect Answer – No Justification	0	Q_5: The correct answer is D. No justification. Q_{16}: The correct answer is A. No justification.
		No Answer – No Justification	0	No Answer No justification.

Q_a: a. Question in MRT

Results

The descriptive statistics about student scores are presented in Table 4 in percentage and frequency.

Table 4.

Descriptive Statistics Results

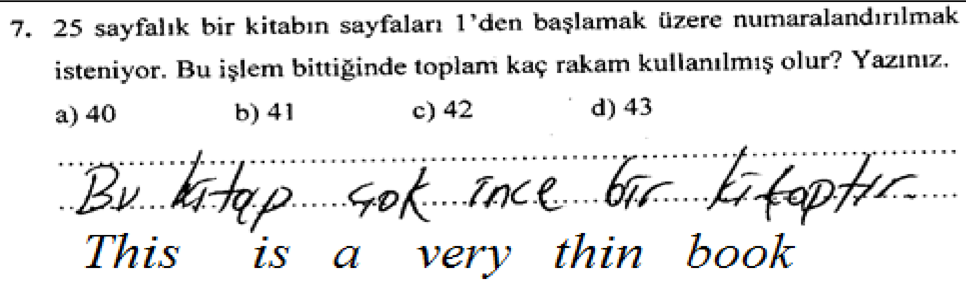
Level	Frequency (f)	Percent (%)
Quite Low	12	7.2
Low	46	27.5
Medium	76	**45.5**
High	28	16.8
Quite High	5	3.0
Total	**167**	**100.0**

As obviously seen on Table 4, mathematical reasoning level of the 7.2% of the students is quite low, of 27.5% is low, of 45.5 % is medium and of 16.8 % is high and of 3% is quite high. When we look at these results, it is possible to say that about half of the students (45.5%) have medium level of mathematical reasoning. The total percentage of the students at low and quite low level is 34.7% and the total percentage of the students at high and quite high level is calculated to be 19.8%. When these percentages are compared, it may be said that the students whose mathematical reasoning are low are more than the ones whose mathematical reasoning are high. Here it is possibly inferred that in general terms, the mathematical reasoning level of the students is medium and low.

This section covers answers and comments on the same question (Q7) related to each level of mathematical reasoning.

S4-MRT-Q7

Note: S4-MRT-Q7= Response by S4 to Q7 in MRT


7. 25 sayfalık bir kitabın sayfaları 1'den başlamak üzere numaralandırılmak isteniyor. Bu işlem bittiğinde toplam kaç rakam kullanılmış olur? Yazınız.

a) 40 b) 41 c) 42 d) 43

Bu kitap çok ince bir kitaptır

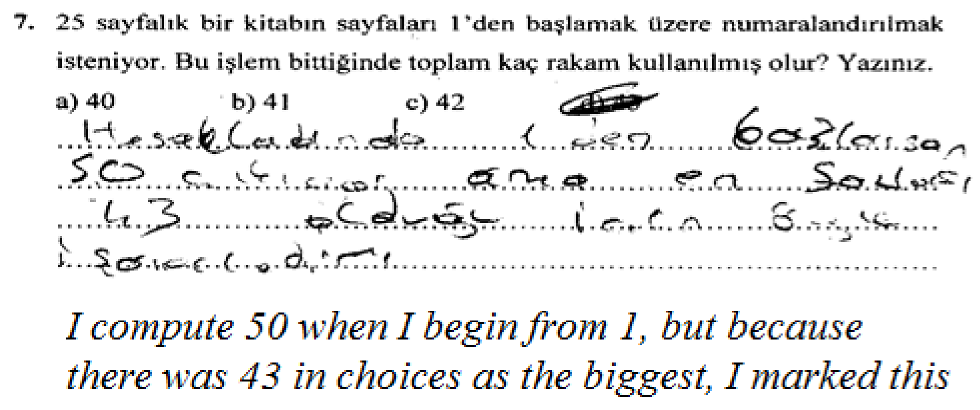
This is a very thin book

Figure 2. Response by S4 to Q7 in MRT

In Figure 2, when the response by the S4 is analyzed, it is seen that this student did not use any kind of mathematical statement and did not do any mathematical operation, either. The reason why the student could not mark any choice could be that he could not understand the question. The student only gave an explanation of *"This is a very thin book"*. It may be concluded from this explanation that the student commented by considering the physical dimension of the book. That the student could not understand the question, focusing on only the physical aspect of the book and the answer he gave did not include maths may possibly be interpreted in the way that the mathematical reasoning of the student is quite low. However, the average score that S4 got out of MRT is calculated to be 0.25. This average equals to *"quite low"* level of the interval (0.00-0.99).

S48-MRT-Q7

Note: S48-MRT-Q7= Response by S48 to Q7 in MRT


7. 25 sayfalık bir kitabın sayfaları 1'den başlamak üzere numaralandırılmak isteniyor. Bu işlem bittiğinde toplam kaç rakam kullanılmış olur? Yazınız.

a) 40 b) 41 c) 42

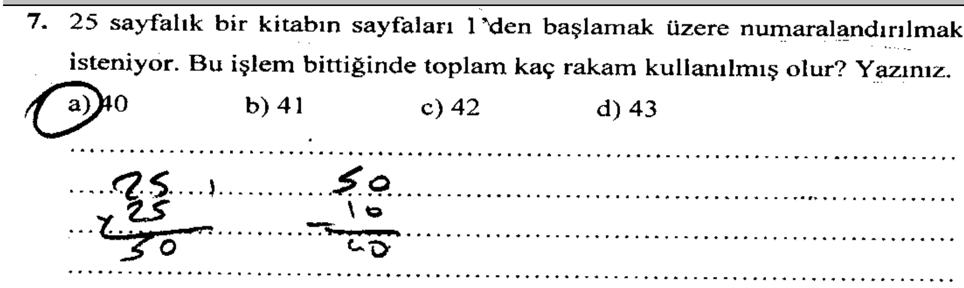
I compute 50 when I begin from 1, but because there was 43 in choices as the biggest, I marked this

Figure 3. Response by S48 to Q7 in MRT

As it is seen in the response by S48 in Figure 3, this student initially thought that on each leaf of the book there are two pages are used so he reached the conclusion of 2x25=50 through a rough calculation. However, as he could not see the result he got in the answer choices he responded 43 by explaining that *"I compute 50 when I begin from 1, but because there was 43 in choices as the biggest, I marked this".* That the student could not understand the question well enough and chose the closest response to the result instead of supporting the one he found himself may be interpreted in the way that mathematical reasoning of the student is at low level. However, the average score that S48 got from MRT is calculated to be 1.74. This average equals to *"low"* interval (1.00-1.99).

S93-MRT-Q7

Note: S93-MRT-Q7= Response by S93 to Q7 in MRT


7. 25 sayfalık bir kitabın sayfaları 1'den başlamak üzere numaralandırılmak isteniyor. Bu işlem bittiğinde toplam kaç rakam kullanılmış olur? Yazınız.

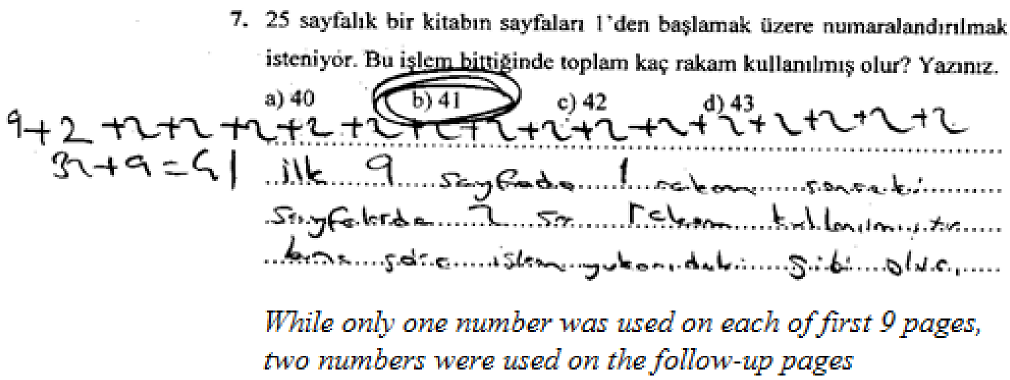
a) 40 b) 41 c) 42 d) 43

Figure 4. Response by S93 to Q7 in MRT

As it is seen in the response by S93 in Figure 4, this student initially thought that on each page two numbers are used so he did a calculation such 25+25=50 and then considering that on the first 10 pages one number is used; he reached the result of 50-10=40. That the student thought that one number is used on the 10th page as well (he knows that on the first 9 pages one number is used); and not being able to write a justification to the result he got may be interpreted in the way that mathematical reasoning of the student is at medium level. However, the average score that S93 got from MRT is calculated to be 2.46. This average equals to *"medium"* interval (2.00-2.99) .

S123-MRT-Q7

Note: S123-MRT-Q7= Response by S123 to Q7 in MRT


7. 25 sayfalık bir kitabın sayfaları 1'den başlamak üzere numaralandırılmak isteniyor. Bu işlem bittiğinde toplam kaç rakam kullanılmış olur? Yazınız.

a) 40 b) 41 c) 42 d) 43

While only one number was used on each of first 9 pages, two numbers were used on the follow-up pages

Figure 5. Response by S123 to Q7 in MRT

As it is seen in the response by S123 in Figure 5, this student reached the correct answer through realizing that on the first 9 pages one by one number; on the other pages two by two numbers were used. That student managed to express their opinions and justify like that "*While only one number was used on the each of first 9 pages, two numbers were used on the follow-up pages*" but fell short of doing the generalization which is necessary for the basic logic of mathematics may be interpreted in the way that mathematical reasoning of the student is at high level rather than quite high. However, the average score that S123 got from MRT is calculated to be 3.91. This average equals to *"high"* interval (3.00-3.99).

S149-MRT-Q7
Note: S149-MRT-Q7= Response by S149 to Q7 in MRT

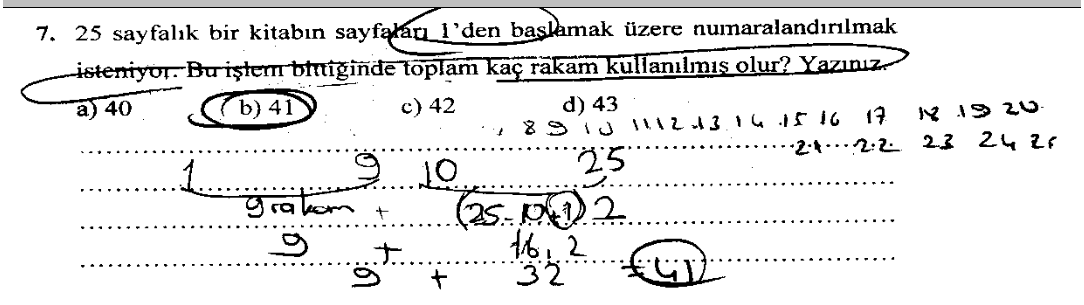


Figure 6. Response by S149 to Q7 in MRT

As it is seen in the response by S149 in Figure 6, this student reached the correct answer through realizing that on the each of first 9 pages one number; on the other pages two numbers were used. That student managed to express their opinions and justifications and do generalization may be interpreted in the way that mathematical reasoning of the student is at quite high level. However, the average score that S149 got from MRT is calculated to be 4.74. This average equals to *"quite high"* interval (4.00-5.00) .

Discussion, Conclusion & Implementation

As a result of analysis, it was determined that the mathematical reasoning of 7.2 % of the students is quite low; of 27.5 %, it is low; of 45.5% it is medium; of 16.8% it is high and of 3% it is quite high. When these results are evaluated, it is probable to say that that most of the students' mathematical reasoning level is at medium or low in general.

When the solutions of the students at *quite low level* are analyzed*,* three phenomenons are seen: The first one is no intervention for response, the second is marking any option focusing only the options and giving no explanation about this option; and the third one is marking a wrong answer randomly and writing statements that are not related to maths or do not match the maths' logic. For example, it was seen that related to the Q7, students at this level used statements like *"this is a very thin book", "it depends on the person who counts the numbers"*, which have nothing to do with the solution. Here, it is possible to say that the mathematical reasoning level of these students is poor. In literature, similar student responses which are non-mathematical are seen (Amir & Williams, 1999; Batanero & Serrano, 1999; Çimen, 2008; Fischbein, Nello, & Marino, 1991; Gürbüz & Erdem, 2014; Lecoutre, 1992; Mandacı-Şahin, 2007; Nilsson, 2007; 2009; Pilten, 2008; Pratt, 1998).

When the solutions of the students at *low level* are analyzed; three phenomenons are seen as well. In the first one, students focused solely upon multiple choices and marked one. In the second one, they reached the wrong answer through wrong justifications. For example, related to the Q11, one student went to the wrong answer through a justification of *"225:15=15. Because 10+5=15".* In the third situation, they reached the right answer through presenting wrong justifications. For example, related to the Q5 in MRT, one student went to the the right answer through the wrong justification of *"our teacher said that we could calculate the circumference of earth via the formula of 2πr. No matter how longer the radius is, we need to extend the rope up to 2π''*. Here it is likely to say that students at this level are short of adequate mathematical reasoning which stems from comprehending what they read and express it in math language. In this context, literature also stated about the effect of mathematical language upon learning (Çalıkoğlu-Bali, 2003; Gibbs & Orton, 1994; NCTM, 1989; Orton & Frobisher, 1996; Lansdell, 1999; Moore, 1994; Raiker, 2002; Schroeder, 1993).

When the solutions of the students at *medium* level are analyzed; it is probable to say that they produced wrong solutions that stems from insufficient mathematical reasoning. For example, from the statements of a couple of students about the Q33 as *"there is a relationship between the number of created pieces and cutting sequence number in the multiples, but..."* and *"two pieces out of one cut,4 pieces out of 2 cutting,8 pieces out of 3 cutting in going on like that. However, we cannot know how many pieces will finally come up"*, it is possible to mention that they did reasoning insufficiently. As it is seen from students' answers, students at this level could understand what they read but they could not reach the correct answer that stems from insufficient reasoning. Similarly, in their studies Diezmann & English (2001), Kramarski et al. (2001), White et al. (1998) and Gürbüz & Erdem (2014) supported this inference through parallel statements.

Students at *high level* fulfilled correct mathematical reasoning and could submit correct solutions. Nevertheless, they could not give the correct answers to some of the questions which stem from incorrect mathematical generalizations. For example, one student at this level tried to create a solution as "*if 1/2+x=1; x=1/2 and -1/2+x=-1/2+1/2=0; hmmm"* in his first trial related to the Q19. However, when he could not reach the correct answer, in his second trial *" if 1/2.x=1; x=2 and -1/2.2=-1, -1.2=-2 gosh, gosh… haa. 1/2.2=1 hmmm"* and in his third trial *"1/2+1/2=1, 1+(-1/2)=1/2, 1/2+(-1)=-1/2 ole*y, *I dare I found it!. If -1/2+?=-1; ?=-1/2*", out of these trials, it is possible to say that students at this level are short of fulfilling mathematical generalization. Because some students gave up creating solutions after they could not find it in one or two trials. The path to generalization in maths goes through conceptual learning. In parallel, literature stresses the importance of conceptual learning in maths (Baki, 1998; Baker & Czarnocha, 2002; Camacho, 2002; Gürbüz, 2010; İşleyen & Işık 2003; Rittle-Johnson, & Koedinger, 2002; Soylu & Soylu, 2006).

As students at *quite high level* manage to present correct justifications to their correct solutions, it is likely to infer that their mathematical reasoning is adequate, because these students could create fully correct solutions to the questions. For example, The statement of a student at this stage to the Q34, as*"as a triangle is created by three edges, the number of edges is more than the number of triangles. Even, the number of triangles is 2 less than the number of edges. Then if the number of edges is n; the number of triangle is n-2"* confirms this argument. It is possible to say that students at this level are able to to comprehend what they read; use mathematical language; learn mathematics in conceptual dimension and do mathematical reasoning.

For further research, studies about designing the learning environments that improve the mathematical reasoning of students may be included. Through studying with less number of students, a more detailed picture can be obtained related to the mathematical reasoning of students. Moreover, in learning environments, rather than the familiar classical problems, students need to be enabled to deal with the problems that they can do reasoning and thus their mathematical reasoning could be improved.

Geniş Özet

Giriş

Matematiksel muhakeme, bir problem ya da durumu "Neden" ve "Nasıl" soruları etrafında detaylandırıp anlamlandırarak yapılan bir üst düzey düşünme süreci olarak tanımlanabilir. Matematikte gerçeklere deney ya da gözlemle değil, muhakemede bulunularak ulaşıldığı için muhakeme olmaksızın matematik yapılamaz. Benzer şekilde Curtis (2004) ve Sparkes (1999), muhakemenin matematik yapmak için olmazsa olmazlardan olduğunu ifade etmişlerdir. Toulmin, Rieke, & Janik (1984), muhakemenin yeni fikirler oluşturmadığını ve muhakemenin görevinin, belli bir durum, konu ya da olay hakkında en iyi kararı vermek olduğunu belirtmişlerdir. İleri düzeylerde de olsa bir düşünce bilgi temeline dayanmıyorsa, gerekçelendirilemiyorsa, mantıklı yaklaşımlar içermiyorsa muhakeme olarak kabul edilemez (Umay, 2003). Bu nedenle, matematiği iç içe geçmiş bir bilgiler ağı olarak görme hem muhakeme vurgusunun bir sonucu olmakta, hem de daha ileri bir muhakeme için bir temel oluşturmaktadır (Umay & Kaf, 2005).

Russell'e (1999) göre muhakeme, öğrencilerin matematiği bir disiplin yapan soyut ifadeleri anlamayı sağlayan bir araçtır. Matematiksel muhakeme ise; kritik düşünme, yaratıcı düşünme ve mantıksal düşünme gibi çeşitli düşünme tarzlarını işe koşup bir karara varma süreci olarak ifade edilebilir. Bu nedenle öğrenme ortamlarında öğrencilerin matematiksel muhakemede bulunmaları etkili öğrenmelerin gerçekleşmesi için önem arz etmektedir. Nitekim matematik eğitimiyle ilgili yapılan ulusal (MEB, 2009; 2013) öğretim programlarında ve uluslararası reform çalışmalarında (NCTM, 1989; 2000) ve diğer birçok araştırmada (Diezmann & English, 2001; English, 1998; Fischbein & Schnarch, 1997; Kramarski, Mevarech, & Lieberman, 2001; Lithner, 2000; Schoenfeld, 1985; Umay, 2003; White, Alexander, & Daugherty, 1998) matematiksel muhakemenin matematik öğrenme üzerinde önemli rolü olduğundan bahsedilmektedir. Örneğin, Diezmann & English (2001), Kramarski vd. (2001) ve White vd. (1998) yaptıkları çalışmalarda, matematik öğrenmeyle muhakeme arasında doğru bir ilişkinin olduğunu, daha iyi muhakemede bulunanların problemler karşısında daha etkili çözümler üretebildiklerini ve daha iyi ilişkilendirmelerde bulunduklarını belirtmişlerdir.

Öğrenme ortamlarında sunulan çoğu alışılagelmiş, iyi yapılandırılmış, muhakemede bulunmayı pek gerektirmeyen ve doğru cevaplamaya yönlendiren problemler, öğrencilerin yüzeysel öğrenmelerine neden olmaktadır. Francisco & Maher (2005) öğrencilere sunulan kompleks uygulamaların basit olanlara oranla onların daha etkili bir şekilde muhakemede bulunmalarını sağlayacağını ifade etmişlerdir. Aynı paralelde Henningsen & Stein (1997) bir uygulamanın kompleksliğini azaltmanın öğrencilerin düşük düzeyde düşünmelerine yol açtığını belirtmişlerdir. Bu bağlamda matematiksel muhakemeyi değerlendirmede farklı türden sorular kullanılabilmesine rağmen çözümüne hemen ulaşılamayan açık uçlu problemlerin daha etkili olacağı düşünülmektedir.

Eğitimin en önemli hedeflerinden birisi de bireylerin gündelik yaşamda karşılaştıkları problemlere etkili çözümler üretebilmelerini sağlamaktır. Eğitimin bu hedefini en iyi şekilde gerçekleştirebilmenin yolu, matematik yapmaktan geçmektedir. Çünkü matematik yapmak, bireyleri bir probleme çözüm üretirken muhakeme yapmaya zorlayarak olası bütün seçenekleri göz önüne almalarını sağlamakta ve karar verme yeteneklerini geliştirmektedir. İyi muhakemede bulunan insanlar, daha doğru ve etkili kararlar verebildikleri için günlük yaşamlarında daha başarılı olabilmektedirler. Bu sebeple öğrenme ortamlarında öğrencileri muhakeme yapmaya zorlayan problemlerle buluşturmak ve matematiksel muhakeme düzeylerini incelemek önemlidir. Bu çalışmanın amacı, yedinci sınıf öğrencilerinin matematiksel muhakeme düzeylerini belirlemek ve bu yöndeki performanslarını ortaya koymaktır.

Yöntem

Araştırma Deseni

Bu çalışmada, katılımcıların matematiksel muhakemelerinin mevcut durumu incelendiğinden betimsel araştırma yöntemi kullanılmıştır. Nitekim birçok eğitim araştırmasının betimsel nitelikte olduğu ve bu araştırmalarda ilgili durumun betimlenerek yorumlandığı belirtilmektedir (Cohen, Manion, & Morrison, 2000).

Katılımcılar

Çalışma, Türkiye'nin bir ilindeki düşük ve orta sosyo-ekonomik düzeye sahip üç ortaokulunda öğrenim gören 167 yedinci sınıf öğrencisinin katılımıyla gerçekleştirilmiştir. Araştırma etiği gereği katılımcılara Ö1, Ö2, Ö3, … şeklinde kodlar verilmiştir.

Verilerin Toplanması

Veri toplama aracı olarak literatürden faydalanılarak geliştirilen ve 35 sorudan oluşan Matematiksel Muhakeme Testi (MMT) kullanılmıştır. Testi geliştirme sürecinde kapsam geçerliği için uzman görüşlerine başvurulmuştur. Testin Cronbach Alfa katsayısı ".885" olarak hesaplanmıştır. Gerçek uygulamaya katılmayan 32 yedinci sınıf öğrencisinin katılımıyla gerçekleştirilen pilot uygulama sonucunda, testteki soruların anlaşılır ve açık olduğu tespit edilmiştir. Ayrıca pilot uygulamada bu test için verilen 45 dakikalık sürenin gerçek uygulamada 60 dakikaya çıkarılmasına karar verilmiştir.

Verilerin Analizi

Öğrenci cevapları uygun istatistiksel programlar kullanılarak analiz edilmiştir. Tablo 1'deki düzeyler ve puan aralığı Tablo 2 ve Tablo 3'teki puanlama ölçeklerine göre oluşturulmuştur. Her öğrencinin aldığı toplam puan, soru sayısına (35) bölünerek öğrencinin düzeyi belirlenmiştir. Örneğin Tablo 2 ve Tablo 3'ten 130 puan alan bir öğrencinin [130/35=3.71 puanı 3.00-3.99 aralığındadır] (Bakınız Tablo 1) matematiksel muhakemesi yüksek olarak değerlendirilmiştir. Bunun yanı sıra araştırmaya katılan öğrencilerin matematiksel muhakemelerinin ne düzeyde olduğunu daha net görebilmek amacıyla bazı öğrencilerin (her bir düzeydeki öğrenciler) testte yer alan bir soruya (Q7) ilişkin cevapları doğrudan aktarılarak detaylı bir şekilde ele alınmıştır.

Tablo 1.

Matematiksel Muhakeme Düzeyleri

Düzey	Puan Ortalaması ($\bar{X}$)
Oldukça Düşük	0.00-0.99
Düşük	1.00-1.99
Orta	2.00-2.99
Yüksek	3.00-3.99
Oldukça Yüksek	4.00-5.00

Öğrenci cevapları, tecrübeli iki matematik eğitimcisi tarafından bağımsız bir şekilde puanlanmıştır. Yapılan puanlamanın paralelliği için bir araya gelen bağımsız değerlendirmeciler, puanlamanın %85-90 düzeyinde tutarlılığı konusunda hemfikir olmuşlardır.

Bulgular ve Yorum

Yapılan analiz sonucunda, öğrencilerin %7.2'sinin matematiksel muhakemesinin *oldukça düşük*; %27.5'inin *düşük*; %45.5'inin *orta*; %16.8'inin *yüksek* ve %3'ünün *oldukça yüksek* düzeyde olduğu ortaya çıkmıştır. Bu değerlere bakıldığında öğrencilerin yaklaşık yarısının (%45.5) orta düzeyde matematiksel muhakemeye sahip olduğu söylenebilir. Oldukça düşük ve düşük düzeydeki öğrencilerin toplam yüzdesi %34.7 ve oldukça yüksek ve yüksek düzeydeki öğrencilerin toplam yüzdesi %19.8 olarak hesaplanmıştır. Bu değerler karşılaştırıldığında matematiksel muhakemesi düşük düzeyde olan öğrencilerin matematiksel muhakemesi yüksek düzeyde olan öğrencilerden daha fazla olduğu söylenebilir. Bu sonuçlardan hareketle, genel olarak öğrencilerin matematiksel muhakemelerinin orta ve düşük düzeyde olduğu söylenebilir.

Sonuç ve Tartışma

Bu araştırma sonucunda, yedinci sınıf öğrencilerinin matematiksel muhakemelerinin genel olarak orta ve düşük düzeyde olduğu tespit edilmiştir. Katılımcıların matematiksel muhakeme düzeylerinin daha net bir resmine ulaşmak için aşağıda her bir düzey literatürle ilişkilendirilerek tartışılmıştır.

*Oldukça düşük düzey*deki öğrencilerin çözümleri incelendiğinde üç durumla karşılaşılmaktadır. Birincisi, cevap için herhangi bir müdahalede bulunulmaması, ikincisi, sadece seçeneklere odaklanarak bir seçeneğin işaretlenmesi ve bu seçeneğe ilişkin herhangi bir açıklamanın yapılmaması ve üçüncüsü ise, yine rastgele yanlış bir seçeneğin işaretlenerek matematikle ilgisi olmayan ya da matematik mantığına uymayan ifadelerin yazılması şeklinde sıralanabilir. Örneğin, bu düzeydeki öğrencilerin MMT'deki 7. soruya ilişkin *"bu kitap çok ince bir kitaptır", "rakamları sayan kişiye bağlıdır"* gibi çözümle ilgisi olmayan ifadeler kullandıkları görülmüştür. Buradan bu düzeydeki öğrencilerin matematiksel muhakemelerinin yetersiz olduğu söylenebilir. Literatürde matematiksel olmayan benzer öğrenci cevaplarına rastlanmaktadır (Fischbein, Nello, & Marino, 1991; Lecoutre, 1992; Pratt, 1998; Amir & Williams, 1999; Batanero & Serrano, 1999; Nilsson, 2007; 2009; Mandacı-Şahin, 2007; Çimen, 2008; Pilten, 2008).

*Düşük düzey*deki öğrencilerin çözümleri incelendiğinde de üç durumla karşılaşılmaktadır. Birinci durumda öğrenciler sadece cevap seçeneklerine odaklanarak birini işaretlemişlerdir. İkinci durumda yanlış gerekçelerle yanlış cevaba ulaşmışlardır. Örneğin, MMT'deki 11. soruya ilişkin bir öğrenci *"225:15=15. Çünkü 10+5=15'tir"* şeklindeki yanlış bir gerekçeden yanlış cevaba gitmiştir. Üçüncü durumda ise yanlış gerekçeler sunarak doğru cevaba ulaşmışlardır. Örneğin, MMT'deki 5. soruya ilişkin bir öğrenci*"öğretmenimiz dünyanın çevresini 2πr formülüyle hesaplayacağımızı söylemişti. Yarıçap ne kadar uzun olursa olsun, ipi 2π kadar uzatmamız gerekir"* şeklindeki yanlış bir gerekçeden doğru cevaba gitmiştir. Buradan bu düzeydeki öğrencilerin okuduklarını anlamalarından ve anladıklarını matematiksel dile aktarmalarından kaynaklı yeterli matematiksel muhakemede bulunamadıkları söylenebilir. Literatürde de matematiksel dilin öğrenme üzerindeki etkisinden bahsedilmektedir (NCTM, 1989; Schroeder, 1993; Gibbs & Orton, 1994; Moore, 1994; Orton & Frobisher, 1996; Lansdell, 1999; Raiker, 2002; Çalıkoğlu-Bali, 2003).

*Orta düzey*deki öğrencilerin çözümleri incelendiğinde eksik matematiksel muhakemeden kaynaklı hatalı çözümler ürettikleri söylenebilir. Örneğin, MMT'deki 33. soruya ilişkin iki öğrenci *"oluşan parça sayısı ile kesme sıra numarası arasında 2'nin katlarında bir ilişki var ancak…" ve "bir kesmeden iki parça, 2 kesmeden 4 parça, 3 kesmeden 8 parça böyle devam edip gidiyor. Ama biz bilemeyiz ki en sonda kaç parça oluşacak"* şeklindeki ifadelerinden eksik muhakemede bulundukları ifade edilebilir. Bu öğrenci cevaplarından da görülebileceği gibi bu düzeydeki öğrenciler okuduklarını anlayabilmişlerdir ancak eksik muhakemeden kaynaklı tam doğru cevaba gidememişlerdir. Nitekim Diezmann & English (2001), Kramarski vd. (2001) ve White vd. (1998) yaptıkları çalışmalarda aynı paralelde ifadelerle bu çıkarsamayı desteklemişlerdir.

*Yüksek düzey*deki öğrenciler doğru matematiksel muhakemede bulunup, doğru çözümler sunabilmişlerdir. Ancak eksik matematiksel genelleme yapmaktan kaynaklı bazı sorulara tam doğru cevap verememişlerdir. Örneğin, bu düzeydeki bir öğrencinin MMT'deki 19. soruya ilişkin ilk denemesinde*"1/2+x=1 ise x=1/2 ve -1/2+x=-1/2+1/2=0 hımmm"* şeklinde bir çözüm üretmeye çalışmıştır. Ancak bu yaklaşımla doğru cevaba ulaşamayınca ikinci denemesinde *"1/2.x=1 ise x=2 ve -1/2.2=-1, -1.2=-2 alla, alla… haa. 1/2.2=1 hımmm"* ve üçüncü denemesinde *"1/2+1/2=1, 1+(-1/2)=1/2, 1/2+(-1)=-1/2 oley galiba buldum. -1/2+?=-1 ise ?=-1/2"* şeklindeki denemelerinden bu düzeydeki öğrencilerin matematiksel genelleme yapmada az da olsa eksik oldukları söylenebilir. Çünkü bazı öğrenciler bir ya da iki denemede bulamadıktan sonra çözüm üretmekten vazgeçmişlerdir. Matematikte genelleme yapabilmenin yolu kavramsal boyutta öğrenmeden geçmektedir. Nitekim literatürde de matematikte kavramsal öğrenmenin önemine vurgu yapılmaktadır (Baki, 1998; Rittle-Johnson & Koedinger, 2002; Baker & Czarnocha, 2002; Camacho, 2002; İşleyen & Işık 2003; Soylu & Soylu, 2006; Gürbüz, 2010).

Oldukça yüksek düzeydeki öğrenciler doğru çözümlerine doğru gerekçeler sunabildikleri için matematiksel muhakemelerinin yeterli olduğu söylenebilir. Çünkü bu öğrenciler sorulara tam doğru çözümler üretebilmişlerdir. Örneğin, bu düzeydeki bir öğrencinin MMT'deki 34. soruya ilişkin *"üç kenar bir üçgen oluşturduğuna göre ve tabloda da kenar sayısı üçgen sayısından fazladır. Hatta üçgen sayısı kenar sayısının 2 eksiği çıkıyor. O halde kenar sayısı n ise üçgen sayısı n-2 olur"* şeklindeki ifadesi bu yargıyı doğrulamaktadır. Bu düzeydeki öğrencilerin okuduğunu anlamada, matematiksel dili kullanmada, matematiği kavramsal boyutta öğrenmede ve matematiksel muhakemede bulunmada yeterli oldukları söylenebilir.

İleride yapılacak araştırmalarda, öğrencilerin matematiksel muhakemelerini geliştirecek öğrenme ortamları tasarlanıp değerlendirilebilir. Ayrıca az sayıda öğrenciyle çalışılarak, öğrencilerin matematiksel muhakemelerine ilişkin daha detaylı bir resme ulaşılabilir. Öte yandan, matematiksel muhakemenin geliştirilebilmesi için öğrencilerin alışılmış klasik problemlerden ziyade muhakeme yapmalarını gerektiren problemlerle uğraşmalarına imkân tanınmalıdır.

References

Amir, G. & Williams, J. (1999). Cultural influences on children's probabilistic thinking. *Journal of Mathematical Behavior, 18*, 85-107.

Baker, W. & Czarnocha, B. (2002). *Written meta-cognition and procedural knowledge*. Proceedings of the 2nd International Conference on the Teaching of Mathematics. University of Crete, Hersonissos Crete, Greece, 1-6 July 2002.

Baki, A. (1998). *Matematik öğretiminde işlemsel ve kavramsal bilginin dengelenmesi*. Atatürk Üniversitesi 40. Kuruluş Yıldönümü Matematik Sempozyumu, Erzurum.

Batanero, C. & Serrano, L. (1999). The meaning of randomness for secondary school students. *Journal for Research in Mathematics Education, 30*(5), 558-567.

Camacho, J. E. D. (2002*). Comparing declarative and procedural learning strategies under a problem based learning approach*. Unpublished doctoral dissertation, United States International University, San Diego.

Cohen, L., Manion, L., & Morrison, K. (2000). *Research methods in education*. London: Routledge Falmer.

Curtis, J. (2004). *A comparative analysis of walled lake consolidated schools' mathematics assessment program and the state of Michigan's educational assessment program*. Unpublished master's thesis, Wayne State University.

Çalıkoğlu-Bali, G. (2003). Matematik öğretmen adaylarının matematik öğretiminde dile ilişkin görüşleri. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi*, *25*, 19-25.

Çimen, E. E. (2008). *Matematik öğretiminde, bireye "Matematiksel Güç" kazandırmaya yönelik ortam tasarımı ve buna uygun öğretmen etkinlikleri geliştirilmesi*. Unpublished doctoral dissertation, Dokuz Eylül Üniversitesi, Eğitim Bilimleri Enstitüsü, İzmir.

Diezmann, C. & English, L. D. (2001). Developing young children's mathematical power. *Roeper Review*, *24*(1), 11-13.

English, L. D. (1998). Reasoning by analogy in solving comparison problems, *Mathematical Cognition, 4*(2), 125-146.

Fischbein, E., Nello, M. S., & Marino, M. S. (1991). Factors affecting probabilistic judgments in children and adolescents. *Educational Studies in Mathematics, 22*, 523-549.

Fischbein, E. & Schnarch, D. (1997). The evolution with age of probabilistic, intuitively based misconceptions. *Journal of Research in Science Teaching*, *28*(1), 96-105.

Francisco, J. M. & Maher, C. A. (2005). Conditions for promoting reasoning in problem solving: Insights from a longitudinal study. *Journal of Mathematical Behavior, 24*, 361–372.

Frederiksen, N. (1984). Implications of cognitive theory for instruction in problem solving. *Review of Educational Research*, *54*, 363-407.

Gibbs, W. & Orton, J. (1994). Language and mathematics. In A. Orton & G. Wain (Eds.), *Issues in teaching mathematics* (pp. 95-116). London: Cassell.

Gürbüz, R. (2010). The effect of activity based instruction on conceptual development of seventh grade students in probability. *International Journal of Mathematical Education in Science and Technology*, *41*(6), 743-767.

Gürbüz, R. & Birgin, O. (2012). The effect of computer-assisted teaching on remedying misconceptions: The case of the subject "probability". *Computers and Education, 58*(3), 931-941.

Gürbüz, R. & Erdem, E. (2014). Matematiksel ve olasılıksal muhakeme arasındaki ilişkinin incelenmesi: 7. sınıf örneği. *Adıyaman Üniversitesi Sosyal Bilimler Enstitüsü Dergisi,* 7(16), 205-230.

Henningsen, M. & Stein, M. K. (1997). Mathematical tasks and student cognition: classroom based factors that support and ınhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education, 28*(5), 524-549.

İşleyen, T. & Işık, A. (2003). Conceptual and procedural learning inmathematics. *Journal of The Korea Society of Mathematical Education SeriesD: Research in Mathematical Education*, *7(*2), 91–99.

Kramarski, B. A., Mevarech, Z. R., & Lieberman A. (2001). Effects of multilevel versus unilevel metacognitive training on mathematical reasoning. *Journal of Educational Research*, *94*(5), 292-300.

Lansdell, J. M. (1999). Introducing young children to mathematical concepts: Problems with new terminology. *Educational Studies*, *25*(3), 327-333.

Lithner, J. (2000). Mathematical reasoning in task solving. *Educational Studies in Mathematics*, *41*, 165-190.

Lecoutre, M. P. (1992). Cognitive models and problem spaces in "purely random" situations. *Educational Studies in Mathematics, 23*, 557-568.

Lithner, J. (2008). A research framework for creative and imitative reasoning. *Educational Studies in Mathematics, 67*, 255-276.

Mandacı-Şahin, S. (2007). 8. *Sınıf öğrencilerinin matematik gücünün belirlenmesi*. Unpublished doctoral dissertation Karadeniz Teknik Üniversitesi, Fen Bilimleri Enstitüsü, Trabzon.

MEB (2009). *İlköğretim matematik dersi 1-5. sınıflar öğretim programı.* T.C. Milli Eğitim Bakanlığı. Talim ve Terbiye Kurulu Başkanlığı, Ankara.

MEB (2013). *Ortaokul matematik dersi (5, 6, 7 ve 8. Sınıflar) öğretim programı.* T.C. Milli Eğitim Bakanlığı. Talim ve Terbiye Kurulu Başkanlığı, Ankara.

Moore, R. C. (1994). Making the transition to formal proof. *Educational Studies in Mathematics, 27*, 249-266.

National Council of Teachers of Mathematics [NCTM] (1989). *Curriculum and evaluation standards for school mathematics.* Reston: Virginia.

National Council of Teachers of Mathematics [NCTM] (2000). *Principles and standards for school mathematics*. Reston, VA.

Nilsson, P. (2007). Different ways in which students handle chance encounters in the explorative setting of a dice game. *Educational Studies in Mathematics*, *66*, 293-315.

Nilsson. P. (2009). Conceptual variation and coordination in probability reasoning. *Journal of Mathematical Behavior*, *28*, 247-261.

Orton, A. & Frobisher, L. (1996). *Insights into teaching mathematics*. London: Cassell.

Pilten, P. (2008). *Üstbiliş stratejileri öğretiminin ilköğretim beşinci sınıf öğrencilerinin matematiksel muhakeme becerilerine etkisi*. Unpublished doctoral dissertation Gazi Üniversitesi, Eğitim Bilimleri Enstitüsü, Ankara.

Pratt, D. (1998). *The construction of meanings in and for a stochastic domain of abstraction*. Unpublished doctoral dissertation, Institue of Education, University of London.

Raiker, A. (2002). Spoken language and mathematics. *Cambridge Journal of Education*, *32*(1),45- 60.

Rittle-Johnson, B. & Koedinger, K. R. (2002). Comparing instructional strategies for integrating conceptual and procedural knowledge. In D. S. Mewborn, P. Sztajin, D. Y. White, H. G. Wiegel, R. L. Bryant, & K. Nooney (Eds.), *Proceedings of the 24th Annual Meeting of the North American Chapters of the International Group for the Psychology of Mathematics Education* (pp. 969-978). Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education.

Russell, S. J. (1999). Mathematical reasoning in the middle grades. In L. V. Stiff and F. R. Curcio (Eds.), *Developing mathematical reasoning in grades* K-12 (pp. 1–12). Reston, VA: National Council of Teachers of Mathematics.

Schoenfeld, A. H. (1985). *Mathematical problem solving*. Orlando: Academic Press.

Schroeder, T. L. (1993). Mathematical connections: two cases from an evaluation of students' mathematical problem solving, Annual Meeting of NCTM, Seattle, Mart.

Soylu, Y. & Soylu, C. (2006). Matematik derslerinde başarıya giden yolda problem çözmenin rolü. *İnönü Üniversitesi Eğitim Fakültesi Dergisi, 7*(11), 97-111.

Sparkes, J. J. (1999). *NCTM's vision of mathematics assessment in the secondary school: Issues and challenges.* Unpublished Master's Thesis. Memorial University of Newfoundland.

Toulmin, S., Rieke, R., & Janik, A. (1984). *An introduction to reasoning* (Second Edition). Macmillan Publishing Co., Inc. New York.

Umay, A. (2003). Matematiksel muhakeme yeteneği. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi, 24*, 234-243.

Umay, A. & Kaf, Y. (2005). Matematikte kusurlu akıl yürütme üzerine bir çalışma. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi, 28,* 188-195.

White, C. S., Alexander, P. A., & Daugherty, M. (1998). The relationship between young children's analogical reasoning and mathematical learning. *Mathematical Cognition, 4*(2), 103-123.

Appendix. Some Questions in Mathematical Reasoning Test (MRT)

Q_3	**Q_4**
In a school with 1100 students, 5 students lessen each year. In an other school with 700 students, 15 students increase each year. How many years later will the number of the students in both schools become equal? Please write. a) 12 b) 15 c) 18 d) 20	Erdem calculates that in a bread queue, he is the 17th from the beginning and the 12th from the end. According to this how many persons are there in the queue totally? Please write. a) 26 b) 27 c) 28 d) 29
Q_5	**Q_7**
Imagine that there is a rope that tightly envelops the earth on the equator. If the radius of the earth were 1 meter longer, how many meters would we need to extend the rope to wrap the earth tightly? Please write. a) π b)2π c) 3π d) can not be known	Book of 25 pages are numbered from number 1. How many numbers have been used in this numbering? Please write. a) 40 b) 41 c) 42 d) 43
Q_8	**Q_9**
A dolphin jumped up 8 meters while swimming 3 meters of dept under water. How many meters did this dolphin jumped above the water level? Please write. a) 11 m b) 5 m c) 24 m d) 10 m	1/6 of the eggs within a basket has been broken. 2/5 out of the rest of them is sold. As 30 eggs left within the basket, how many eggs have been sold? Please write. a) 10 b) 20 c) 30 d) 40
Q_{10}	**Q_{11}**
Which interval is the length of an edge located of a garden which is square shaped whose area is 39 m^2? Why? a) between 4 m and 5 m b) between 5 m and 6 m c) between 6 m and 7 m d) between 7 m and 8 m	Ahmet paid 235 TRYs for all the books that he bought for 5 and 10 TRYs each. According to this, how many books did Ahmet buy al least? Please write. a) 23 b) 24 c) 45 d) 46
Q_{12}	**Q_{19}**
In a farm where there are sheep and hens, the number of feet is 34 and the number of head is 100. According to this information, what is the number of the sheep in this farm? Please write. a) 57 b) 60 c) 63 d) 66	1/2, 1, 1/2, -1/2, -1, ? a) 1 b) -1/2 c) 1/2 d) -1

Q $_{20}$

The shape above which is created through combinig three sticks 1/4 in size, how many sticks 1/12 in size is necessary to create it? Please explain.

a) 3 b) 6 c) 9 d) 12

Q $_{22}$

The ticket price in various stores of a clock whose all features are all the same is given below. In which store is this clock bought cheapest after the discounts made? Please explain.

a) Store A/100 TL - 25 % discount

b) Store B /90 TL - 10 % discount

c) Store C /90 TL - 20 % discount

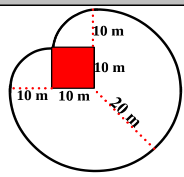
d) Store D/100 TL - 30% discount

Q $_{25}$

Row	Numbers	Total
1. row	2,3,4,5,6,7,8 ,9,10	
2. row	10,12,14,16,18,20	
3. row	5, 7, 9, 11, 13, 15	
4. row	3,6,9,12,15,18,21	

Develop a strategy which indicates that the total sequential numbers within each row above is 90.

Q $_{26}$



As it is seen above, to the edge of a garden whose bottom is quadrate (10m×10m), a sheep is tied with a rope of 20 meters. When the rope is tight, what is the maximum square meter area that the sheep can graze? Please explain.

Evaluate the solutions of the 31th and the 32th questions and write your own comment on each step.

Q $_{31}$

As 5 masters finish building a house of 100 m^2 in 10 days; in how many days 10 masters with the same qualifications finish building a house of 150 m^2?

Solution Way

1st step: If 5 masters finish a house of 100 m^2 in 10 days; 10 masters finish it in 5 days.

2nd step: If 10 masters finish a house of 100 m^2 in 20 days; they finish a house 150 m^2 in (150x20)/100=30 days.

Q $_{32}$

Two reciprocal vehicles from two cities whose distance is 240 km set off at the same time. As the speed of on per hour is 50 km an the other one's 70 km; how many hours later these vehicles meet after their depurture?

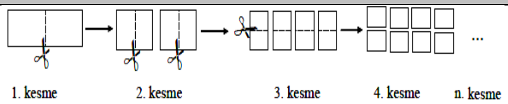
Solution Way:

1st step: The distance between two vehicles is 240 kms.

2nd step: It is essential to calculate the speed difference of both in order to find how many hours later they will meet. 70-50=20

3rd step: 240/20=12 hours later they will meet.

Q_{33}

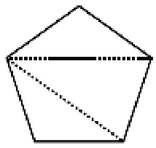


His teacher wants Ali cut the paper strip into two equal pieces with scissors as above and recut each of these pieces and go on this work.

Cutting Row Number	The Number of The Piece Created	The Display of Numbers in Exponential Notation
1	2	2^1
2	4	2^2
3	8	2^3
. . .	. . .	. . .
n		?

Ali does not manage to know what to write in the place of "?" where the generalization is done. In your opinion, what should Ali write in the place of "?" Please explain.

Q_{34}

	Edge Number	Triangle Number
	3	1
	4	2
	5	3
	. . .	. . .
	n	?

According to the table above, what should be placed instead of "?" please explain.