Sakarya University Journal of Science, 22 (6), 1787-1792, 2018.

SAKARYA UNIVERSITY
JOURNAL OF SCIENCE

e-ISSN: 2147-835X
http://www.saujs.sakarya.edu.tr

Received
02-03-2018
Accepted
14-06-2018

Doi
10.16984/saufenbilder.400272

SAKARYA UNIVERSITY

Geometric Kinematic Approach to Rigid Objects with Point Contact Based on A Local-Surface Frame

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Abstract

Controlling the relative motion between two contacting rigid bodies is necessary when manipulation and locomotion tasks are encountered in robotics. In this paper, we express the kinematics of spin-rolling motion of rigid objects with point contact based on the local-surface frame method. We give the relationship between the Darboux frame and local-surface frame, and also between the Frenet-Serret frame and local-surface frame. Additionally, we obtain velocity of the moving object according to the local-surface frame curvatures of the respective contact curve and geometric invariants. We can have proper information about trajectory planning if we take the formulations of moving object into consideration according to the local-surface frame.

Keywords: Frenet-Serret frame, Darboux frame, Local-Surface frame, kinematics, point contact.

1. INTRODUCTION

During the robot manipulation tasks, the robot arm needs to be able to control the contact motion, when it interacts with an object. If the robot grasps an object it may be necessary to roll or to slide the object in a specified manner [1]. The point contact motion can be defined as the combination of sliding, spin, and rolling motion.

Kinematics of point contact has been examined widely by researchers. Some of them studied on contact between planar rigid bodies (for example, [2, 3, 4, 5, 6]). Recently, some researchers have studied on the kinematics of 3D contact between two rigid bodies. Pars describes the configuration space as five-dimensional associated with the relative motion between two rigid bodies in point contact [7]. Cai and Roth study the kinematics of two contacting bodies in point contact according to spatial motion [8, 9]. They examined the relative motion between two rigid bodies with point contact by using local geometric properties of each rigid body [9]. However, they studied the spatial motion of the rigid bodies with line contact [10]. Montana derives the equations of contact from a geometric perspective [11]. Cui and Dai applies Darboux frame method to developing geometric kinematics of rigid objects with point contact [12].

This paper presents the local-surface frame method to express the kinematics of rigid bodies with point contact. Also, we show some relationships between the Darboux frame and local-surface frame, and the Frenet-Serret frame and local-surface frame. Moreover, we give the velocity equations with respect to local-surface frame curvatures of the respective contact curve and geometric invariants.

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2. PRELIMINARIES

Let $\boldsymbol{\alpha}(s)$ be a unit speed curve that satisfies $\|\boldsymbol{\alpha}'(s)\| = 1$ in E^3 and the Frenet-Serret frame $\{\boldsymbol{\alpha};\ \boldsymbol{t}, \boldsymbol{N}, \boldsymbol{B}\}$ of this curve parametrized by arc length parameter s, is:

$$\boldsymbol{\alpha}'(s) = \boldsymbol{t}$$

$$\frac{\boldsymbol{t}'(s)}{\|\boldsymbol{t}'(s)\|} = \boldsymbol{N}(s) \qquad (1)$$

$$\boldsymbol{t}(\boldsymbol{s}) \times \boldsymbol{N}(s) = \boldsymbol{B}(s)$$

where the vectors $\boldsymbol{t}$, $\boldsymbol{N}$, and $\boldsymbol{B}$ are the unit tangent vector, unit principal normal vector, and unit binormal normal vector, and $\kappa = \kappa(s) = ||\boldsymbol{t}'(s)||$ and $\tau = \tau(s) = ||\boldsymbol{B}'(s)||$ are the curvature and torsion of the curve $\boldsymbol{\alpha}(s)$ at s_0, respectively. The Frenet vector and the unit Frenet vector of this curve can be defined as follows:

$$\boldsymbol{f} = \tau\boldsymbol{t} + \kappa\boldsymbol{B} \qquad (2)$$

and

$$\boldsymbol{w} = \frac{\boldsymbol{f}}{\|\boldsymbol{f}\|} = \frac{\tau\boldsymbol{t} + \kappa\boldsymbol{B}}{\sqrt{\tau^2 + \kappa^2}}.$$

Let $\boldsymbol{X}(u, v)$ be a regular surface and the curve $\boldsymbol{\alpha}(s)$, parametrized by arc length parameter s, be on the surface $\boldsymbol{X}(u, v)$. Since the curve $\boldsymbol{\alpha}(s)$ is also a space curve, the curve $\boldsymbol{\alpha}(s)$ has the Frenet-Serret frame as mentioned above. The curve $\boldsymbol{\alpha}(s)$ lies on the surface $\boldsymbol{X}(u, v)$, and there exists another frame called as a Darboux frame $\{\boldsymbol{t}, \boldsymbol{g}, \boldsymbol{n}\}$ of the curve $\boldsymbol{\alpha}(s)$ at s_0, where $\boldsymbol{t}$ is the unit tangent vector of the curve $\boldsymbol{\alpha}(s)$, $\boldsymbol{n}$ is the unit normal vector of the surface $\boldsymbol{X}(u, v)$ and $\boldsymbol{g}$ is a unit vector given by $\boldsymbol{g} = \boldsymbol{n} \times \boldsymbol{t}$. The derivative formulas of Darboux frame are given as follows:

$$\begin{bmatrix} \boldsymbol{t}' \\ \boldsymbol{g}' \\ \boldsymbol{n}' \end{bmatrix} = \begin{bmatrix} 0 & \kappa_g & \kappa_n \\ -\kappa_g & 0 & \tau_g \\ -\kappa_n & -\tau_g & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{t} \\ \boldsymbol{g} \\ \boldsymbol{n} \end{bmatrix}, \qquad (3)$$

where, κ_g is the geodesic curvature, κ_n is the normal curvature, and τ_g is the geodesic torsion of the curve $\boldsymbol{\alpha}(s)$. The Darboux vector and the unit Darboux vector of this curve are [13]:

$$\boldsymbol{d} = \tau_g\boldsymbol{t} + \kappa_n\boldsymbol{g} + \kappa_g\boldsymbol{n} \qquad (4)$$

and

$$\boldsymbol{d_0} = \frac{\boldsymbol{d}}{\|\boldsymbol{d}\|} = \frac{\tau_g\boldsymbol{t} + \kappa_n\boldsymbol{g} + \kappa_g\boldsymbol{n}}{\|\tau_g\boldsymbol{t} + \kappa_n\boldsymbol{g} + \kappa_g\boldsymbol{n}\|}.$$

Let the right-handed system, which we call as a Local-Surface frame, of the surface $\boldsymbol{X}(u, v)$ be defined as $(\boldsymbol{t_1}, \boldsymbol{t_2}, \boldsymbol{n})$ which can then be given as

$$\boldsymbol{t_1} = \frac{\boldsymbol{X_u}(\boldsymbol{u}, \boldsymbol{v})}{\|\boldsymbol{X_u}(\boldsymbol{u}, \boldsymbol{v})\|}$$

$$\boldsymbol{t_2} = \frac{\boldsymbol{X_v}(\boldsymbol{u}, \boldsymbol{v})}{\|\boldsymbol{X_v}(\boldsymbol{u}, \boldsymbol{v})\|} \qquad (5)$$

$$\boldsymbol{n} = \frac{\boldsymbol{X_u}(\boldsymbol{u}, \boldsymbol{v}) \times \boldsymbol{X_v}(\boldsymbol{u}, \boldsymbol{v})}{\|\boldsymbol{X_u}(\boldsymbol{u}, \boldsymbol{v}) \times \boldsymbol{X_v}(\boldsymbol{u}, \boldsymbol{v})\|}$$

where $\boldsymbol{X_u}$ and $\boldsymbol{X_v}$ are the partial derivatives of $\boldsymbol{X}(u, v)$ with regard to parameters u and v [14]. In this paper, we suppose that Local-Surface frame $(\boldsymbol{t_1}, \boldsymbol{t_2}, \boldsymbol{n})$ of surface $\boldsymbol{X}(u, v)$ is an orthonormal coordinate system.

3. EXAMINATION OF THE RELATIONSHIPS BETWEEN THREE FRAMES

Initially, we can express the relationships between the Frenet-Serret frame and Darboux frame. Let the angle between the unit vectors $\boldsymbol{n}$ and $\boldsymbol{N}$ be θ as shown in Figure (1). We can write the following equations:

$$\boldsymbol{n} = \boldsymbol{N}\cos\theta + \boldsymbol{B}\sin\theta$$

$$\boldsymbol{g} = \boldsymbol{N}\sin\theta - \boldsymbol{B}\cos\theta \qquad (6)$$

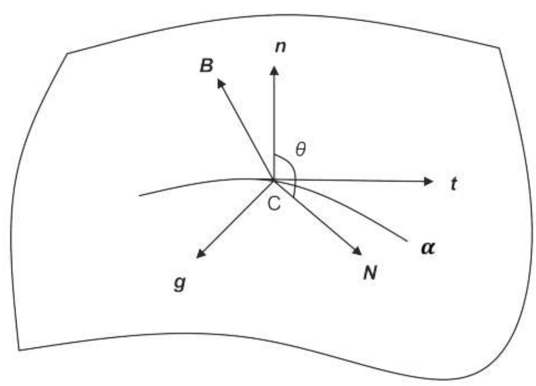

Figure 1: The relationship between the Darboux and Serret-Frenet frames of the curve $\boldsymbol{\alpha}$

On the other hand, let the parameter curves $\boldsymbol{X_u}(\boldsymbol{u},\boldsymbol{v})$ and $\boldsymbol{X_v}(\boldsymbol{u},\boldsymbol{v})$ be lines of curvature and the angle between the unit vectors $\boldsymbol{t}$ and $\boldsymbol{t_1}$ be φ as shown in Figure (2).

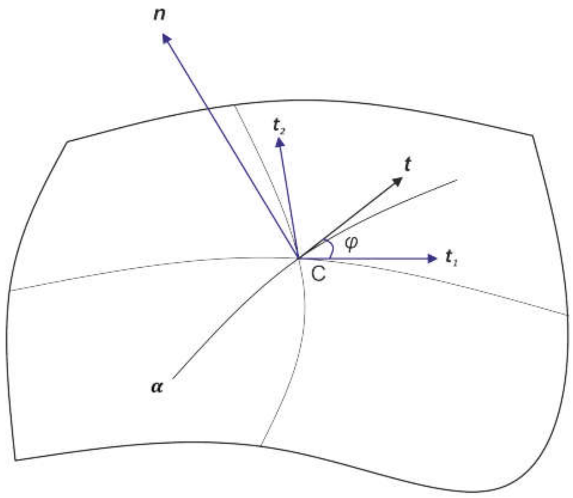

Figure 2: The relationship between the Darboux and Local-Surface frames of the curve $\boldsymbol{\alpha}$

Then, we can express the relationships below:

$$\boldsymbol{t} = \boldsymbol{t_1}\cos\varphi + \boldsymbol{t_2}\sin\varphi$$

$$\boldsymbol{g} = \boldsymbol{t_1}\sin\varphi - \boldsymbol{t_2}\cos\varphi \qquad (7)$$

$$\boldsymbol{n} = \boldsymbol{t_1}\times\boldsymbol{t_2}$$

Using the equations (3) and (7), we can define the derivative formulas of Local-Surface frame as:

$$\begin{bmatrix}\boldsymbol{t_1}'\\ \boldsymbol{t_2}'\\ \boldsymbol{n}'\end{bmatrix} = \begin{bmatrix}0 & 0 & \rho_1\\ 0 & 0 & \rho_2\\ -\rho_1 & -\rho_2 & 0\end{bmatrix}\begin{bmatrix}\boldsymbol{t_1}\\ \boldsymbol{t_2}\\ \boldsymbol{n}\end{bmatrix}, \qquad (8)$$

where $\rho_1 = \kappa_n\cos\varphi - \tau_g\sin\varphi$ and $\rho_2 = \kappa_n\sin\varphi + \tau_g\cos\varphi$. As known that we can write the Local-Surface frame vector with

$$\boldsymbol{l} = \boldsymbol{l_{t1}} + \boldsymbol{l_{t2}} + \boldsymbol{l_n}. \qquad (9)$$

Therefore, we can write the Local-Surface frame vector using the equations (8) and (9) as follows:

$$\boldsymbol{l} = \rho_2\,\boldsymbol{t_1} - \rho_1\,\boldsymbol{t_2}. \qquad (10)$$

Remark: We can write the following relations using the relationships $\rho_1 = \kappa_n\cos\varphi - \tau_g\sin\varphi$ and $\rho_2 = \kappa_n\sin\varphi + \tau_g\cos\varphi$:

i. If $\kappa_n = 0$, then the curve $\boldsymbol{\alpha}$ is asymptotic curve, $\rho_1 = -\tau_g\sin\varphi$ and $\rho_2 = \tau_g\cos\varphi$.
   - $\varphi = 0 \Rightarrow \rho_1 = 0$ and $\rho_2 = \tau_g$,
   - $\varphi = \pi/2 \Rightarrow \rho_1 = -\tau_g$ and $\rho_2 = 0$.

ii. If $\tau_g = 0$, then the curve $\boldsymbol{\alpha}$ is line of curvature, $\rho_1 = \kappa_n\cos\varphi$ and $\rho_2 = \kappa_n\sin\varphi$.
   - $\varphi = 0 \Rightarrow \rho_1 = \kappa_n$ and $\rho_2 = 0$,
   - $\varphi = \pi/2 \Rightarrow \rho_1 = 0$ and $\rho_2 = \kappa_n$.

iii. $\rho_1{}^2 + \rho_2{}^2 = \kappa_n{}^2 + \tau_g{}^2$.
   - If $\tau_g = 0$, then the curve $\boldsymbol{\alpha}$ is line of curvature, and $\kappa_n = \sqrt{\rho_1{}^2 + \rho_2{}^2}$,
   - If $\kappa_n = 0$, then the curve $\boldsymbol{\alpha}$ is asymptotic curve, and $\tau_g = \sqrt{\rho_1{}^2 + \rho_2{}^2}$.

iv. $\kappa_n = \rho_1\cos\varphi + \rho_2\sin\varphi$,
   $\tau_g = -\rho_1\sin\varphi + \rho_2\cos\varphi$, and $\kappa_g = -1$.

Finally, we can examine the relationships between the Frenet-Serret frame and Local-Surface frame as follows:

$$\begin{bmatrix}\boldsymbol{t}\\ \boldsymbol{N}\\ \boldsymbol{B}\end{bmatrix} = \begin{bmatrix}\cos\varphi & \sin\varphi & 0\\ \frac{-\sin\varphi}{D} & \frac{\cos\varphi}{D} & \frac{\rho_1\cos\varphi+\rho_2\sin\varphi}{D}\\ \frac{\rho_1\sin\varphi\cos\varphi+\rho_2\sin^2\varphi}{D} & \frac{-(\rho_1\cos^2\varphi+\rho_2\sin\varphi\cos\varphi)}{D} & \frac{1}{D}\end{bmatrix}\begin{bmatrix}\boldsymbol{t_1}\\ \boldsymbol{t_2}\\ \mathbf{n}\end{bmatrix}, \qquad (11)$$

where, $D = \sqrt{(\rho_1\cos\varphi + \rho_2\sin\varphi)^2 + 1}$.

4. GEOMETRIC KINEMATIC EXAMINATIONS OF SPIN-ROLLING MOTION

Let the objects E and $\bar{E}$, and the surfaces S and $\bar{S}$ which belong to these objects be given, and we suppose that they make spin-rolling motion without slipping at every moment. The curves $\boldsymbol{\alpha}$ and $\bar{\boldsymbol{\alpha}}$ are the contact trajectory curves on the surfaces S and $\bar{S}$ of the objects E and $\bar{E}$, respectively. We suppose that the object E is a fixed and object E has spin-rolling motion relative to the object $\bar{E}$. Hence, the Local-Surface frames $\{E\}$ and $\{\bar{E}\}$ belong to the objects E and $\bar{E}$ are fixed and moving Local-Surface frames, respectively. Let the contact point of the curves $\boldsymbol{\alpha}$ and $\bar{\boldsymbol{\alpha}}$ be C, and the moving and fixed Local-Surface frames attached to the contact point C be $(\boldsymbol{t_1},\boldsymbol{t_2},\boldsymbol{n})$ and $(\bar{\boldsymbol{t_1}},\bar{\boldsymbol{t_2}},\bar{\boldsymbol{n}})$, respectively. Since, the unit tangent vectors of the curves $\boldsymbol{\alpha}$ and $\bar{\boldsymbol{\alpha}}$ are same, i.e since

$$\frac{d\bar{c}}{d\bar{s}} = \boldsymbol{t}_{\mathbf{C}},$$

we can write the equation below.

$$\boldsymbol{t}_{\mathbf{C}} = \boldsymbol{t}_{\mathbf{1}} \cos\varphi + \boldsymbol{t}_{\mathbf{2}} \sin\varphi \qquad \boldsymbol{t}_{\mathbf{C}} = \overline{\boldsymbol{t}_{\mathbf{1}}} \cos\overline{\varphi} + \overline{\boldsymbol{t}_{\mathbf{2}}} \cos\overline{\varphi} \tag{12}$$

Then, we can express the Local-Surface frame $(\overline{\boldsymbol{t}_1}, \overline{\boldsymbol{t}_2}, \overline{\boldsymbol{n}})$ as follows:

$$\overline{\boldsymbol{t}_{\mathbf{1}}} = \boldsymbol{t}_{\mathbf{1}} \cos\phi + \boldsymbol{t}_{\mathbf{2}} \sin\phi \qquad \overline{\boldsymbol{t}_{\mathbf{2}}} = \boldsymbol{t}_{\mathbf{1}} \sin\phi - \boldsymbol{t}_{\mathbf{2}} \cos\phi \qquad \overline{\boldsymbol{n}} = \boldsymbol{n}, \tag{13}$$

where ϕ is the angle between the vectors $\boldsymbol{t}_1$ and $\overline{\boldsymbol{t}_1}$. If we have $\phi = 0$, then it will be true that $\boldsymbol{t}_1 = \overline{\boldsymbol{t}_1}$, $\boldsymbol{t}_2 = \overline{\boldsymbol{t}_2}$, and $\boldsymbol{n} = \overline{\boldsymbol{n}}$. Also, these frames will always coincide because of the rolling constraints. Additionally, let s and $\bar{s}$ be arc lengths of the curves $\boldsymbol{\alpha}$ and $\overline{\boldsymbol{\alpha}}$, and also let us take an arbitrary point $\bar{P}$ be on the object E. We can write the position vector of the point $\bar{P}$ of the object E according to the frame $\{\bar{E}\}$ as

$$\overline{\boldsymbol{p}} = \overline{\boldsymbol{c}} + \overline{\mu_1}\overline{\boldsymbol{t}_{\mathbf{1}}} + \overline{\mu_2}\overline{\boldsymbol{t}_{\mathbf{2}}} + \overline{\mu_3}\overline{\boldsymbol{n}}. \tag{14}$$

Then if we take derivative of the equation (14) according to $\bar{s}$, we have:

$$\begin{aligned} \frac{d\overline{\boldsymbol{p}}}{d\bar{s}} &= \left(\cos\overline{\varphi} + \frac{d\overline{\mu_1}}{d\bar{s}} - \overline{\mu_3}\overline{\rho_1}\right)\overline{\boldsymbol{t}_{\mathbf{1}}} \\ &+ \left(\sin\overline{\varphi} + \frac{d\overline{\mu_2}}{d\bar{s}} - \overline{\mu_3}\overline{\rho_2}\right)\overline{\boldsymbol{t}_{\mathbf{2}}} \\ &+ \left(\frac{d\overline{\mu_3}}{d\bar{s}} + \overline{\mu_1}\overline{\rho_1} + \overline{\mu_2}\overline{\rho_2}\right)\overline{\boldsymbol{n}}, \end{aligned} \tag{15}$$

where $\overline{\rho_1} = \overline{\kappa_n}\cos\overline{\varphi} - \overline{\tau_g}\sin\overline{\varphi}$ and $\overline{\rho_2} = \overline{\kappa_n}\sin\overline{\varphi} + \overline{\tau_g}\cos\overline{\varphi}$. Since the point $\bar{P}$ of the object $\bar{E}$ is a fixed point, we can write the following relation.

$$\frac{d\overline{\boldsymbol{p}}}{d\bar{s}} = 0. \tag{16}$$

Using the equations (15) and (16) we get the equation below:

$$\begin{aligned} \frac{d\overline{\mu_1}}{d\bar{s}} &= \overline{\mu_3}\overline{\rho_1} - \cos\overline{\varphi} \\ \frac{d\overline{\mu_2}}{d\bar{s}} &= \overline{\mu_3}\overline{\rho_2} - \sin\overline{\varphi} \\ \frac{d\overline{\mu_3}}{d\bar{s}} &= -\overline{\mu_1}\overline{\rho_1} - \overline{\mu_2}\overline{\rho_2}. \end{aligned} \tag{17}$$

On the other hand, the point $\bar{P}$ can also be written according to frame $\{\bar{E}\}$ as

$$\boldsymbol{p} = \boldsymbol{c} + \mu_1\boldsymbol{t}_{\mathbf{1}} + \mu_2\boldsymbol{t}_{\mathbf{2}} + \mu_3\boldsymbol{n}. \tag{18}$$

Similarly, if we take derivative of the equation (18) according to s, it yields

$$\begin{aligned} \frac{d\boldsymbol{p}}{ds} &= \left(\cos\varphi + \frac{d\mu_1}{d\bar{s}} - \mu_3\rho_1\right)\boldsymbol{t}_{\mathbf{1}} \\ &+ \left(\sin\varphi + \frac{d\mu_2}{d\bar{s}} - \mu_3\rho_2\right)\boldsymbol{t}_{\mathbf{2}} \\ &+ \left(\frac{d\overline{\mu_3}}{d\bar{s}} + \mu_1\rho_1 + \mu_2\rho_2\right)\boldsymbol{n}. \end{aligned} \tag{19}$$

During rolling motion, the velocities of the contact curves $\boldsymbol{\alpha}$ and $\overline{\boldsymbol{\alpha}}$ will be equal because of the rolling constraints. Hence, it will be true that $s = \bar{s}$. On the other hand, if the equation (13) is substituted into (15), then we have the following equation.

$$\begin{aligned} \frac{d\overline{\boldsymbol{p}}}{d\bar{s}} = &\left[\left(\cos\overline{\varphi} + \frac{d\overline{\mu_1}}{d\bar{s}} - \overline{\mu_3}\overline{\rho_1}\right)\cos\phi \right. \\ &\left. - \left(\sin\overline{\varphi} + \frac{d\overline{\mu_2}}{d\bar{s}} - \overline{\mu_3}\overline{\rho_2}\right)\sin\phi\right]\boldsymbol{t}_{\mathbf{1}} \\ &+ \left[\left(\cos\overline{\varphi} + \frac{d\overline{\mu_1}}{d\bar{s}} - \overline{\mu_3}\overline{\rho_1}\right)\sin\phi \right. \\ &\left. + \left(\sin\overline{\varphi} + \frac{d\overline{\mu_2}}{d\bar{s}} - \overline{\mu_3}\overline{\rho_2}\right)\cos\phi\right]\boldsymbol{t}_{\mathbf{2}} \\ &+ \left(\frac{d\overline{\mu_3}}{d\bar{s}} + \overline{\mu_1}\overline{\rho_1} + \overline{\mu_2}\overline{\rho_2}\right)\boldsymbol{n} \end{aligned} \tag{20}$$

Using the relation in (16), we can obtain the same relationships in (17). Due to rolling constraints, the Local-Surface frames $\{E\}$ and $\{\bar{E}\}$ will coincide at any moment. Hence, it will be true that:

$$\mu_1 = \overline{\mu_1}, \quad \mu_2 = \overline{\mu_2}, \quad and \quad \mu_3 = \overline{\mu_3},. \tag{21}$$

and

$$\frac{d\mu_1}{ds}=\frac{d\overline{\mu_1}}{d\bar{s}},\quad \frac{d\mu_2}{ds}=\frac{d\overline{\mu_2}}{d\bar{s}}\quad and\quad \frac{d\mu_3}{ds}=\frac{d\overline{\mu_3}}{d\bar{s}}. \tag{22}$$

From now on, we can use s instead of s and $\bar{s}$, and we can use μ_i instead of μ_i and $\overline{\mu_i}$ because of contact situation and coincidence. Therefore, substituting the equations (15) and (22) into (19) we have the following equation:

$$\begin{aligned}\frac{d\boldsymbol{p}}{ds} &= \left(\cos\varphi - \cos\bar{\varphi} + \mu_3\rho_1^*\right)\boldsymbol{t_1}\\ &+\left(\sin\varphi - \sin\bar{\varphi} + \mu_3\rho_2^*\right)\boldsymbol{t_2}\\ &+\left(-\mu_1\rho_1^* - \mu_2\rho_2^*\right)\boldsymbol{n}.\end{aligned} \tag{23}$$

where

$$\rho_1^* = \overline{\rho_1} - \rho_1 \quad and \quad \rho_2^* = \overline{\rho_2} - \rho_2. \tag{24}$$

5. LOCAL-SURFACE FRAME BASED VELOCITY OF THE SPIN-ROLLING MOTION

In this section, we find the Local-Surface frame based translational velocity of the object $\bar{E}$ according to the fixed local-surface frame $\{E\}$ by using the equation (23) as

$$\begin{aligned}\boldsymbol{v_p} &= \frac{d\boldsymbol{p}}{ds}\frac{ds}{dt} = \vartheta\left(\cos\varphi - \cos\bar{\varphi} + \mu_3\rho_1^*\right)\boldsymbol{t_1}\\ &+\vartheta\left(\sin\varphi - \sin\bar{\varphi} + \mu_3\rho_2^*\right)\boldsymbol{t_2}\\ &+\vartheta\left(-\mu_1\rho_1^* - \mu_2\rho_2^*\right)\boldsymbol{n},\end{aligned} \tag{25}$$

where, $\vartheta = ds/dt$ is the magnitude of the rolling velocity. Additionally, using the angular velocity

$$\boldsymbol{\omega} = \omega_1\boldsymbol{t_1} + \omega_2\boldsymbol{t_2} + \omega_3\boldsymbol{n}, \tag{26}$$

we can determine the velocity of the point P according to the fixed local-surface frame $\{E\}$ with

$$\begin{aligned}\boldsymbol{v_p} = \boldsymbol{\omega}\times\boldsymbol{\Gamma}_{CP} &= \left(-\mu_2\omega_3 + \mu_3\omega_2\right)\boldsymbol{t_1}\\ &+\left(\mu_1\omega_3 - \mu_3\omega_1\right)\boldsymbol{t_2}\\ &+\left(-\mu_1\omega_2 + \mu_2\omega_1\right)\boldsymbol{n},\end{aligned} \tag{27}$$

where $\boldsymbol{\Gamma}_{CP} = \mu_1\boldsymbol{t_1} + \mu_2\boldsymbol{t_2} + \mu_3\boldsymbol{n}$. Due to coincidence we can write $\boldsymbol{t_1} = \overline{\boldsymbol{t_1}}$ and $\boldsymbol{t_2} = \overline{\boldsymbol{t_2}}$, and also $\varphi = \bar{\varphi}$. Hence, it is true that $\cos\varphi - \cos\bar{\varphi} = 0$ and $\sin\varphi - \sin\bar{\varphi} = 0$. Therefore, using the equations (25) and (27) we can express the equation below:

$$\omega_1 = -\vartheta\rho_2^*,\quad \omega_2 = -\vartheta\rho_1^*,\quad and \quad \omega_3 = 0. \tag{28}$$

Therefore, the angular velocity of object $\bar{E}$ according to fixed local-surface frame $\{E\}$ can be written with the following equation:

$$\boldsymbol{\omega} = \vartheta\left(-\rho_2^*\boldsymbol{t_1} + \rho_1^*\boldsymbol{t_2}\right). \tag{29}$$

6. CONCLUSION

In this paper, we gave the kinematics of spin-rolling motion of rigid objects with point contact with respect to the Local-Surface frame method. We studied on the relationships between the Darboux frame and Local-Surface frame, and also between the Frenet-Serret frame and Local-Surface frame. On the other hand, we found the velocity formulations of a moving object based on the Local-Surface frame curvatures of the contact curve and geometric invariants.

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