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Research Paper

Out-of-Plane Vibration Analysis of Multiple-Stepped Circular Beam

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Abstract

Stepped circular beams are widely used in various engineering fields. Thus, studying the free vibration characteristics of those structures is an essential subject of research. The unified approach of the Complementary Functions Method (CFM) and the Laplace transform is employed in this paper to examine the out-of-plane free vibration analysis of the circular Timoshenko beams with multiple stepped cross-sections. The proposed procedure can be used to investigate the natural frequencies of circular rods consisting of an arbitrary number of steps through the curvature. The material of the beam is considered to be isotropic, homogeneous and elastic. By considering the effects of shear deformation and rotary inertia, the equations of motion of the circular beam are introduced. Obtained equations are transformed to the Laplace domain and solved numerically by the CFM. Comparisons of the results with those of ANSYS show that the suggested scheme is applicable and of high precision for out-of-plane free vibration analysis of multi-stepped circular beams.

Keywords: Out-of-plane free vibration; Multi-stepped circular beam; Laplace transform.

INTRODUCTION

Due to their wide practical applications in civil and mechanical engineering, curved beams have attracted the attention of many researchers. In-plane and out-of-plane free vibrations of curved beams with variable cross-sections was studied by Kawakami et al. [1] by applying the Green function. Howson and Jemah [2] presented the exact solution of out-of-plane natural frequencies of plannar structures made up of curved Timoshenko beams. Wu and Chiang [3] applied finite elements to study the natural frequencies of a horizontally circularly curved Timoshenko beam. Wu and Chen [4] determined the exact solution out-of-plane free vibration response of the curved beams. Doğruer [5] presented the exact solution of the dynamic response of an out-of-plane curved beam. Tüfekci and Doğruer [6] examined the out-of-plane free vibration of a circular arch by considering the effects of transverse shear and rotatory inertia due to both flexural and torsional vibrations. Eroğlu [7] studied the static and free vibration problems of curved rods with a finite element approximate and cross-section. Dynamic response of a straight beam was examined in the Laplace domain by Manolis and Beskos [8] and Beskos and Narayanan [9]. Aslan et al. [10] investigated the undamped forced vibration of out-of-plane loaded stepped circular rods in the Laplace domain with the CFM.

This paper is as special case of Noori [11].The present article investigates the out-of-plane free vibration response of the circular rods with multiple steps. The main objective of this study is to purpose an accurate scheme to obtain the out-of-plane natural frequencies of multi stepped curved beams. Complementary Functions Method was performed Yarımpabuç et al. [12] to solve the umerical model of FG pressure vessel for stresses and displacement.

The governing equations of the free vibration response of the considered structures are obtained in the time domain. Laplace transform, with respect to time, is then applied and the obtained canonical form of the first order ordinary differential equations (ODEs) has been solved by the CFM in the transformed domain. Computer programs are prepared with FORTRAN programming language to obtain the free vibration response. The natural frequencies obtained by proposed procedure are compared with the results of ANSYS [13]. Comparisons of the results demonstrated the accuracy and exactness of the present study.

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MATERIALS AND METHODS

The governing equations of the out-of-plane dynamic response of multi-stepped curved beam are obtained and given below.

$$\frac{\partial U_b}{\partial \phi}=r_0\Omega_n+r_0\frac{T_b\alpha_b}{GA(\phi)} \tag{1}$$

$$\frac{\partial \Omega}{\partial \phi}=\Omega_n+r_0\frac{M_t}{GI_t(\phi)} \tag{2}$$

$$\frac{\partial \Omega_n}{\partial \phi}=-\Omega_t+r_0\frac{M_n}{EI_n(\phi)} \tag{3}$$

$$\frac{\partial T_b}{\partial \phi}=r_0\rho A(\phi)\frac{\partial^2 U_b}{\partial t^2}-r_0q_b \tag{4}$$

$$\frac{\partial M_n}{\partial \phi}=r_0\rho I_t(\phi)\frac{\partial^2 U_n}{\partial t^2}+M_n-r_0m_t \tag{5}$$

$$\frac{\partial M_n}{\partial \phi}=r_0\rho I_n(\phi)\frac{\partial^2 \Omega_b}{\partial t^2}-M_n+r_0T_b-r_0m_n \tag{6}$$

Where T_b is vertical internal forces, M_t and M_n are the components of the torsional and internal bending moments U_b is the vertical displacement, Ω_t and Ω_n are the components related rotations, *E, ρ, h(ϕ),A(ϕ),* $I_t(\phi)$, $I_n(\phi)$, α_b, r_0, q_b, m_t and m_n indicate the modulus of elasticity, mass density, radius of the cross-section, area of cross-section sectional area, torsional moment of inertia, bending moment of inertia, shear correction factor, radius of curvature, distributed vertical load, distributed moment of torsion and bending, respectively.

The unknown column matrix, $\{\mathbf{Y}(\phi,t)\}$, for the forced vibration out-of-plane loaded rods is given as:

$$\{\mathbf{Y}(\phi,t)\}=\{U_b,\Omega_t,\Omega_n,T_b,M_t,M_n\}^T \tag{7}$$

Applying the Laplace transform to equations (1-6), converts these partial differential equations to variable-coefficient ODEs (8-13);

$$\frac{d\bar{U}_b}{d\phi}=-r_0\bar{\Omega}_n+r_0\frac{\bar{T}_b\alpha_b}{GA(\phi)} \tag{8}$$

$$\frac{d\bar{\Omega}_t}{d\phi}=\bar{\Omega}_n+r_0\frac{\bar{M}_t}{GI_t(\phi)} \tag{9}$$

$$\frac{d\bar{\Omega}_n}{d\phi}=-\bar{\Omega}_t+r_0\frac{\bar{M}_n}{EI_n(\phi)} \tag{10}$$

$$\frac{d\bar{T}_b}{d\phi}=r_0s^2\rho A(\phi)\bar{U}_b-r_0\bar{q}_b \tag{11}$$

$$\frac{d\bar{M}_t}{d\phi}=r_0s^2\rho A(\phi)\bar{\Omega}_t+\bar{M}_n-r_0\bar{m}_t \tag{12}$$

$$\frac{d\bar{M}_n}{d\phi}=r_0s^2\rho I_n(\phi)\bar{\Omega}_n-\bar{M}_t+r_0\bar{T}_b-r_0\bar{m}_n \tag{13}$$

Where the terms shown by $(\bar{\bullet})$ indicate the Laplace transform of the quantities.

The matrix notation of the ordinary differential equations ODEs (8-13) obtained in the Laplace domain is given as:

$$\frac{d\{\bar{\mathbf{Y}}(\phi,s)\}}{d\phi}=[\bar{\mathbf{A}}(\phi,s)]\{\bar{\mathbf{Y}}(\phi,s)\}+\{\bar{\mathbf{F}}(\phi,s)\} \tag{14}$$

Here, ϕ is independent variable and s is the Laplace transform parameter.

The general solution of the differential equation (14) which governs the out-of-plane free vibration response of the beam is given as follows:

$$\{\bar{\mathbf{Y}}(\phi,s)\}=\sum_{m=1}^{6}C_m\left[\bar{\mathbf{U}}^{(m)}(\phi,s)\right]+\{\bar{\mathbf{V}}(\phi,s)\} \tag{15}$$

In order to examine the free vibration of the considered structures the load vector are assumed to be zero and the Laplace parameter "*s*" is replaced with "*iω*". Also the inhomogeneous solution $\{\bar{\mathbf{V}}(\phi,s)\}$ is equal to zero. To determine the integration constants "C_m" of homogeneous solution from the boundary conditions, simultaneous equations are obtained and the matrix of the coefficients of those equations are performed. Since the mass and stiffness matrix of the system are not obtained separately by the presented procedure the eigenvalues and eigenvectors of the problem are not calculated thus the values of *ω* which make the determinant of coefficient's matrix zero are the natural frequencies of the structure.

NUMERICAL EXAMPLES AND DISCUSSION

In this paper the vibration of two, three and four stepped beams with circular and rectangular cross-sections is carried out. The results of three stepped circular beam with rectangular cross-sections are compared with those of ANSYS. The natural frequencies of fix-ended isotropic stepped circular beam, shown in Figure 1, are carried out. material properties are: mass density, ρ=7850x10^{-6} kgf/cm^3, Poisson's ratio, ν = 0.3, and modulus of elasticity, E=2.1x10^6 kgf/cm^2.

The radius of cross section (*h(ϕ))* of the two, three and four stepped beam is considered to be:

$$h(\phi)=\begin{cases}0.5\,, & -\frac{\pi}{6}\le\phi\le\frac{\pi}{6}\\ 1.0\,, & \frac{\pi}{6}\le\phi\le\frac{\pi}{4}\\ 1.0\,, & -\frac{\pi}{4}\le\phi\le-\frac{\pi}{6}\end{cases}\;;$$

$$h(\phi)=\begin{cases}0.5\,, & -\frac{\pi}{12}\le\phi\le\frac{\pi}{12}\\ 1.0\,, & \frac{\pi}{12}\le\phi\le\frac{\pi}{6}\\ 1.0\,, & -\frac{\pi}{12}\le\phi\le-\frac{\pi}{6}\\ 2.0\,, & \frac{\pi}{12}\le\phi\le\frac{\pi}{4}\\ 2.0\,, & -\frac{\pi}{12}\le\phi\le\frac{\pi}{4}\end{cases};$$

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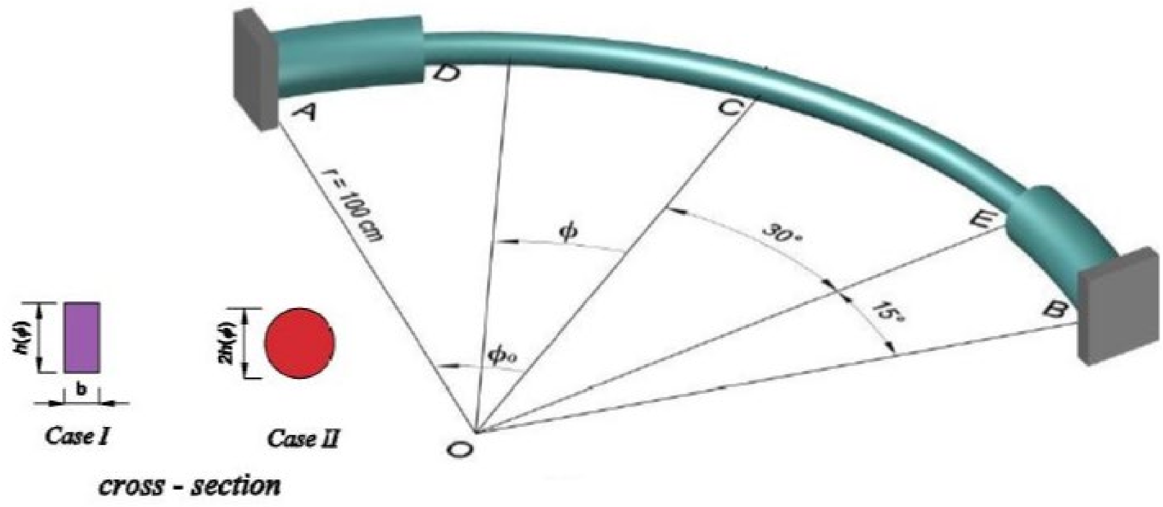

Figure 1: Fix–ended circular stepped rod

$$h(\phi)=\begin{cases}0.5\,, & -\dfrac{\pi}{16}\le\phi\le\dfrac{\pi}{16}\\ 1.0\,, & \dfrac{\pi}{16}\le\phi\le\dfrac{\pi}{8}\\ 1.0\,, & -\dfrac{\pi}{16}\le\phi\le-\dfrac{\pi}{8}\\ 2.0\,, & \dfrac{\pi}{8}\le\phi\le\dfrac{3\pi}{16}\\ 2.0\,, & -\dfrac{\pi}{8}\le\phi\le-\dfrac{3\pi}{16}\\ 4.0\,, & \dfrac{3\pi}{16}\le\phi\le\dfrac{\pi}{4}\\ 4.0\,, & -\dfrac{3\pi}{16}\le\phi\le-\dfrac{\pi}{4}\end{cases}$$

The flexural rigidity and cross section of rectangular and circular rod are given respectively, as;

$$GI_t(\phi)=G*0.141,\quad EI_n(\phi)=E\frac{bh^3(\phi)}{12},\quad A(\phi)=bh(\phi);$$

$$GI_t(\phi)=G*\frac{\pi h^4(\phi)}{2},\quad EI_n(\phi)=E\frac{\pi h^4(\phi)}{4},\quad A(\phi)=\pi h^2(\phi)$$

The geometric properties of the rods are tabulated in Table 1.

Table 1: Geometric properties of the rod.

Cross-section	$b(0)$ (cm)	$h(0)$ (cm)	r_0 (cm)	ϕ_0	α_n
Rectangular (Case I)	1	1	100	45	1.2
Circular (Case II)	-	0.5	100	45	1.11

At first the natural frequencies of a fix-ended, three steps beams with circular and rectangular cross-sections are carried out by the proposed approach. Obtained results compared with those of ANSYS. Comparison of the natural frequencies of the considered structure are listed in Table 2 for multiple stepped.

To demonstrate the accuracy of the proposed method, the natural frequencies obtained are compared with those of ANSYS. The parametric study on the free vibration of the stepped curved rods with circular and rectangular cross-section are studied and presented in Table 3.

As expected, the number of steps have a remarkable influence on the out-of-plane natural frequencies of the structures. As it can been seen in Table 3, the two-stepped beam with rectangular cross section has the lowest natural frequencies which corresponds to the highest period. Similarly the mentioned table shows that four-stepped beam with circular cross section has the highest natural frequencies and the lowest oscillating periods.

CONCLUSION

In this study, an efficient unified approach based on the combination of Laplace transform and the CFM is employed to examine out-of-plane free vibration response of the circular multi- stepped beams. The RK5 algorithm has been employed in the numerical solution process of initial value problems based on the CFM. Materials of the beam are considered to be isotropic, homogeneous and elastic.

Table 2: The natural frequencies of fix-ended rod with three stepped cross-section (Hz).

Mode	Present Study (Case I)	ANSYS (100 Elements)	Present Study (Case II)	ANSYS (100 Elements)
1	1.119	1.115	2.821	2.836
2	3.849	3.828	7.051	7.610
3	7.866	7.872	13.654	13.673
4	15.901	15.803	27.240	27.148
5	24.443	24.311	41.821	41.725

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Table 3: The natural frequencies of fix-ended rod with two stepped cross-section (Hz).

Mode	Two-stepped Beam		Three-stepped Beam		Four-stepped Beam	
	Case I	Case II	Case I	Case II	Case I	Case II
1	0.5771	1.0149	1.119	2.821	1.529	4.912
2	2.4498	4.6642	3.849	7.051	5.349	11.630
3	5.3832	8.9398	7.866	13.654	10.993	20.171
4	10.497	17.9403	15.901	27.240	21.247	40.246
5	16.494	30.1026	24.443	41.821	30.805	51.372

The canonical form of the first ODEs governing equations of the motion of the stepped circular beam has been solved by the CFM in the Laplace domain for a set of Laplace parameters. Several parametric results are presented. The proposed method can be apply to obtain the free vibration response of arbitrary stepped beams.

For the considered structures, computer program is coded in FORTRAN. The accuracy and exactness of the proposed procedure are demonstrated by comparing its results with the results of ANSYS. Good agreement is observed.

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