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Process capability: A New Criterion for Loss Function–Based Quality Improvement

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Abstract: Response surface methodology (RSM) – the method most preferred by quality engineers – is a natural and effective tool to achieve the desired process quality. Most of the current literature on process quality does not focus on information relating to how much better or worse a process is and also the degree of the process performance. On the other hand, although the process performance criteria are able to predict process capability, they cannot provide significant information relating to the process quality in terms of rate of rejects and losses. Therefore, this paper takes into account these two concepts and defines a criterion based on the process capability indices for the upside-down normal loss function (UDNLF). The proposed approach determines the optimal settings of a given process by minimizing the expected UDNLF which is defined in terms of C_p and C_{pm} indices. The proposed procedure and its merits are illustrated on the basis of an example.

Süreç Yeterliliği: Kayıp Fonksiyonuna Dayalı Kalite Geliştirme için Yeni Bir Kriter

Anahtar Kelimeler
Dayanıklı tasarım,
Yanıt yüzey metodolojisi,
Kayıp fonksiyonu,
Süreç yeterlilik indeksleri

Özet: Yanıt yüzey metodolojisi (RSM), kalite mühendisleri tarafından oldukça tercih edilen bir yöntem olarak, arzu edilen süreç kalitesine ulaşmak için doğal ve etkin bir araçtır. Süreç kalitesi üzerine birçok çalışma, bir sürecin ne kadar iyi ya da kötü olduğu ve aynı zamanda süreç performans derecesi hakkında bilgi vermeye odaklanmamaktadır. Diğer taraftan, süreç performans kriterleri bir sürecin yeterliliğini tahmin etme özelliğine sahip olsa da, red ve kayıp oranları ile süreç kalitesi arasında önemli miktarda bilgi sağlayamamaktadır. Bu çalışma, iki kavramı göz önüne alır ve ters çevrilmiş normal kayıp fonksiyonu (UDNLF) için süreç yeterliliği indekslerine dayalı bir kriter tanımlar. Önerilen yaklaşım, C_p ve C_{pm} indekslerine ile tanımlanmış beklenen UDNLF' yi minimize ederek, bir sürecin optimal çözümünü belirler. Önerilen yöntem ve avantajları bir örnek üzerinde gösterilmiştir.

1. Introduction

Traditionally, loss functions have played an irreplaceable role in the development of robust design. In a statistical sense, loss functions speak in monetary terms and determine the loss resulting from the deviations of a quality characteristic from its desired value. In this context, one is dealing with financial and social losses caused by poorly designed, poorly constructed or poorly operated products. According to Taguchi's philosophy, this result should be assessed in two respects: the company view (for example, costs relating to returned products, rework, scrap and repair) and the customer view (for example, customer dissatisfaction related to unsatisfactory product performance). The quality loss concept is therefore known as 'loss to society'.

In terms of the old traditional definition of loss, if the quality characteristics are within the specification limits, then there are no losses. However, this definition was found inadequate and unreal by [1]. Moreover, Taguchi claimed that it is possible for the loss to be incurred even if quality characteristics are within the predefined specifications, and he proposed the quadratic loss function, which is continuous and differentiable everywhere. However, Taguchi's quadratic loss function has some shortcomings in practice – i.e. unbounded, assessing same loss when the process is too far from the target. Consequently, a new loss function based on inverted normal density function, inverted normal loss function (INLF), is

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proposed by [2] – see also, [3]. The UDNLF, proposed by [4], is one minus a scaled normal density function and is a special form of INLF. Besides being zero at the target and asymptotically approaching to one, it also has a finite maximum loss. Therefore, the UDNLF offers effective results in modelling losses accurately and leads to optimal decisions in the field of manufacturing. Even though, these mentioned loss functions are the inversions of probability density functions, they do not give sufficient information about the process optimization for the purposes of obtaining the best operating conditions. Design and relating approaches such as RSM and loss function-based methods are sound robust design strategies to optimize the systems.

Taguchi's robust parameter design (RPD) can be defined as an experimental procedure used in determining the quality of a process and, which tries to reduce the process variability by selecting the best operating condition. Taguchi's RPD approach has been regarded with interest by influential scientists; however, his methodologies and analysis techniques have been criticized by the statistical communities. Consequently, new methodologies have been proposed in harmony with advanced statistical procedures. The RSM, first developed by [5], was revisited in the 1990s and was subsequently popularized. The RSM is a natural and effective tool to assign the optimal control factor setting of a process, which optimizes the response over a region of interest. [6] discussed a constrained optimization technique called the dual response surface (DRS). Their approach is based on modeling the primary and secondary response surfaces, and optimizing the primary response subject to a constraint on the secondary response. A nonlinear programming based on inequality constraints for the DRS problems is proposed by [7]. An alternative approach, by using the mean squared error (MSE) criterion, is proposed by [8] to minimize the estimated MSE by taking into account the distance from the target along with the variability in response. [9] proposed UDNLF as the objective function where the process mean and standard deviation responses are fitted by the response surface models. Their approach is based on minimizing the estimator of the expected UDNLF under the 'target is best' case. Following these articles, the current literature involves several other methods for the DRS; see, for example, [10-19].

The process should be regarded as a whole made up of all materials, methods, equipment, and persons that produce a measurable response. In line with the nature of the process, all the processes involve unwanted fluctuations that affect the system performance. Current literature about quality improvement focuses on reducing this inherent variability to improve the process quality. Since less variation indicates better quality, process variability is a measure of uniformity of the responses. Process capability is a fundamental concept to quantify this uniformity of a process in quality control. Since the 1970s, process capability analysis – which includes techniques as histograms, probability plots, design of experiments, control charts, and process capability indices – has received a great deal of attention by quality engineers and statisticians. In fact, process capability indices, as the quantitative indicators, are used in the purposes of characterizing the process quality. Therefore, with the assistance of these indices, one can obtain information relating to how good the process performance is with respect to specifications. In practice, the widely used statistics, which relate the natural tolerance limits of a given process to the specification limits to measure the process capability, are C_p, C_{pk} and C_{pm}.

Many quality improvement techniques focus on reducing process variation in line with the "loss to society" concept. The widespread use of loss functions in industrial applications has increased their popularity. However, most of current literature does not focus on how much better or worse a process is and also the degree of process performance. Although the process performance criteria are able to predicting process capability, they cannot provide significant information relating to process quality in terms of the rate of rejects and losses. In order to bridge this gap, this paper takes into account these two concepts and presents a new criterion based on process capability indices for the UDNLF. In addition, this proposed approach demonstrates the relationship between process capability indices which are estimated by response surfaces. The proposed approach finds the optimal settings of a given process with the minimum loss, and with additional information such as the process capability. The ability to consider the spread of the specifications and the possible effects of the process distribution between the specification limits on the quality loss makes this proposed optimization technique feasible with regard to the 'loss to society' concept.

The rest of this paperis divided into four sections. Section 2 provides a brief overview of the UDNLF and the process capability indices. The proposed optimization technique is introduced in Section 3. All our findings are illustrated by a numerical example at the end. Finally, the paper ends with conclusion.

2. Material and Method

2.1. UDNLF overview

The UDNLF, proposed by [4], is a weighted loss function which evaluates the losses with a more reasonable risk, and is defined by the following formula,

$$L_{UDN}(y|\tau) = 1 - exp\left(-\frac{(y-\tau)^2}{2\lambda^2}\right) \quad (1)$$

where y represents the process measurements, τ is the target, and λ is a scale parameter. The UDNLF is zero at the target and asymptotically approaches to one. A pragmatic choice is setting λ to 42.5% of the specification ranges. Thus, the loss is approximately 50% when the quality characteristic is at its specification limits. The scale parameter λ adjusts the penalty associated with any deviation from the target. Figure 1 presents a graphical illustration of the UDNLF for various values of λ when $\tau = 0$. In Figure 1, a larger value of λ signifies that the relatively large deviations from the target can be tolerated, so the loss reaches its maximum more slowly.

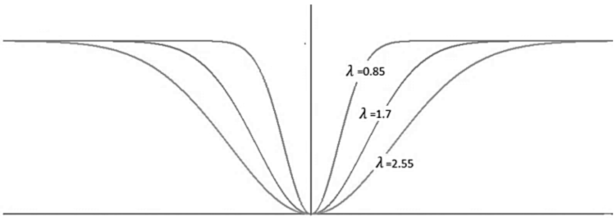

Figure 1. The UDNLF with $\lambda = 2.55, 1.7$ and 0.85.

A simple analytical formula is presented by [4] for the expected loss as follows:

$$EL_{UDN} = 1 - \frac{\lambda}{\sqrt{\sigma^2+\lambda^2}} exp\left(\frac{-(\mu-\tau)^2}{2(\sigma^2+\lambda^2)}\right) \quad (2)$$

when the process quality characteristic has a normal density with the process mean μ and process variance σ^2. This formula quantifies the economic impacts of process changes by combining the company view – i.e. the information about the systems – and the customer feedback, i.e. the unsatisfactory product performance caused by a deviation from the target. [9] proposed a reasonable estimator of Equation (2) as follows:

$$\hat{E}L_{UDN} = 1 - \frac{\lambda}{\sqrt{\hat{\sigma}^2(x)+\lambda^2}} exp\left(\frac{-(\hat{\mu}(x)-\tau)^2}{2(\hat{\sigma}^2(x)+\lambda^2)}\right) \quad (3)$$

where the fitted mean and variance response surfaces are denoted by $\hat{\mu}(x)$ and $\hat{\sigma}^2(x)$, respectively. They used this estimator as an objective function to obtain the best operating conditions of a system.

2.2. Process capability indices

The process capability indices are statistics widely used by quality engineers and statisticians as a quantitative measure of the capability of producing items within the customer specification limits. Beginning with C_p index, several process capability indices have been proposed in the field of quality improvement and control.

The C_p index is the measurement of the potential capability of the process and is defined as follows:

$$C_p = \frac{(USL - LSL)}{6\sigma} \quad (4)$$

where USL and LSL are upper and lower specification limits, respectively, and σ is known process variance. The value of C_p is 1, which means that the process is just capable. C_p index considers the spread of the specifications due to the six sigma; in other words, it does not focus on the location of the process mean, μ.

The other well-known index C_{pk}, which is a process capability ratio for off-center processes, is defined as follows:

$$C_{pk} = \frac{\min(USL-\mu, \mu - LSL)}{3\sigma} \quad (5)$$

Unlike C_p, C_{pk} takes into account the process variance besides the departures from μ. When the value C_{pk} is greater than 2, then process capability exceeds the 6σ level. In fact, this explains why it is called a 6σ robust design.

Chan et al. introduce C_{pm} index which considers the proximity to the target value, τ, besides the process variation when assessing process performance [20]. When μ and σ are known, then C_{pm} is defined as follows:

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\tau-\mu)^2}} = \frac{C_p}{\sqrt{1+\frac{(\tau-\mu)^2}{\sigma^2}}} \quad (6)$$

C_{pm} possess the necessary properties required for assessing process capability. C_{pm} reflects the process drifts from its desired value, τ, so it is a convenient indicator where τ is not the midpoint of the USL and LSL. On the other hand, since C_{pm} incorporates quadratic loss, it is also referred to as the Taguchi index. The loss function appears in the denominator, the term $6\sqrt{\sigma^2 + (\tau-\mu)^2}$ gives the average loss per piece for a given system. Additionally, when the process attains its target, C_{pm} is similar to C_p.

In practice, the mean and variance of the process are generally unknown process parameters. The reasonable estimators of Equations (4)-(6) can be obtained by using the sample standard deviation, s, and the sample mean, $\bar{x}$, instead of process parameters, σ and μ, respectively. On the other hand, in the current literature, there are certain other indices such as, C_{pmk}, C_{pw}, and C_{rp}. The reader is referred to the studies of [21-23].

The process capability indices are widely used by optimization techniques in quality improvement. [24]

used C_{pm} and goal programming as an optimization technique for the purposes of quality improvement of the multi-responses systems. [25] exploited the target costing technique considering C_{pk} , and different loss function and quality control charts. [26] proposed the use of C_{pm} as an alternative optimization criterion to MSE for multi-responses optimization. [27] considered the maximization of the process capability as a criterion in the process design to obtain the best operating condition. Using the ideas set out in these articles as a point of departure, the current literature includes several other approaches – see, [28-31].

2.3. Central composite design

Cental composite design (CCD), first introduced by [5], is the most popular class of second-order design. In fact, CCD is a sequential experimentation technique. It involves the use of a two-level factorial or fraction combined with the $2k$ axial or star points. As a result, the CCD involves F factorial points, $2k$ axial points, and n_c_center runs. Therefore, while the factorial points represent a variance-optimal design, center runs provide information about the curvature in the system. A general illustration of CCD is presented in Table 1 (see, [5]).

Table 1. An illustration of the CCD

x_1	x_2	…	x_k
-α	0	…	0
α	0	…	0
0	-α	…	0
0	α	…	0
…	…	…	…
0	0	…	-α
0	0	…	α

The flexibility in the use of CCD is caused by the selection of α, the axial distance, and n_c, the number of center runs. While the choice of α depends to a great extent on the region of operability and region of interest, the choice of n_c often has an influence on the distribution of the variability in the region of interest. It is important for a second-order design to possess a reasonably stable distribution of the scaled prediction variance throughout the experimental design region. A reliability technique to control this situation is the notion of design relatability. In the case of CCD, rotatability is achieved by making a proper choice of α, the axial distance. The first condition is that the factorial portion must be a full 2^k design. The second condition is that $\alpha = \sqrt[4]{F}$. Table 2, which is borrowed from [32] (p. 549), gives value of α, for a rotatable design for various numbers of design variables, where N is the total experimental run.

Table 2. Values of α for a rotatable design for various numbers of design variables

k	F	N	α
2	4	8+n_c	1.414
3	8	14+n_c	1.682
4	16	24+n_c	2.000
5	32	42+n_c	2.378
6	64	76+n_c	2.828
7	128	78+n_c	3.364

3. Results

3.1. The proposed procedure

The proposed approach is related minimizing the estimator of expected UDNLF in Equation (3) with a new criteria based on C_{pm} and C_p. In this context, first the relations between λ and, C_p and C_{pm} are defined. [4] set the scale parameter λ to 42.5% of the specification ranges for UDNLF. Thus, considering $\lambda = 0.425(USL - LSL)$, the following equations are obtained:

$$\lambda = 2.55\, \sigma\, C_p \tag{7}$$

and

$$(\tau - \mu)^2 = \frac{\sigma^2}{{C_{pm}}^2}\left({C_p}^2 - {C_{pm}}^2 \right) \tag{8}$$

Taking into account Equations (7)-(8), and the expected loss in Equation (2) can be rewritten in the following form in terms of C_{pm} and C_p,

$$\begin{aligned} &{EL_{UDN}}^* \\ &= 1 \\ &- \frac{2.55\, C_p}{1 + 2.55^2 {C_p}^2} exp\left(\frac{-\left({C_p}^2 - {C_{pm}}^2\right)}{2{C_{pm}}^2\left(1 + 2.55^2 {C_p}^2\right)} \right) \end{aligned} \tag{9}$$

This new formula quantifies the economic impacts of process changes by combining the company view and the customer feedback, besides speaking in terms of process capability. A reasonable estimator of Equation (9) is proposed as follows:

$$\begin{aligned} &{\hat{E}L_{UDN}}^* \\ &= 1 \\ &- \frac{2.55\, \hat{C}_p}{1 + 2.55^2 {\hat{C}_p}^2} exp\left(\frac{-\left({\hat{C}_p}^2 - {\hat{C}_{pm}}^2\right)}{2{\hat{C}_{pm}}^2\left(1 + 2.55^2 {\hat{C}_p}^2\right)} \right) \end{aligned} \tag{10}$$

where $\hat{C}_p$ and $\hat{C}_{pm}$ are the estimators of C_p and C_{pm} and are defined in the following forms:

$$\hat{C}_p = \frac{USL - LSL}{6\hat{\sigma}(x)} \quad (11)$$

and

$$\hat{C}_{pm} = \frac{USL - LSL}{6\sqrt{\hat{\sigma}^2(x) + (\tau - \hat{\mu}(x))^2}} \quad (12)$$

Here, $\hat{\mu}(x)$ and $\hat{\sigma}(x)$ are the fitted second-order response surfaces for the process mean and standard deviation, respectively.

Finally, the optimal factor settings of the process are obtained by minimizing the $\hat{E}L_{UDN}{}^*$ given in Equation (10) under the region of interest, $\mathbf{x} \in R$, as follows:

$$\begin{gathered}\text{Min } \hat{E}L_{UDN}{}^* \\ \textit{Subject to } \mathbf{x} \in R\end{gathered} \quad (13)$$

Generally, two regions of interest are considered: spherical and cubodial. For cubodial designs, the constraint is of the form $-1 \le x_i \le 1, i = 1, \dots k$, and for spherical desings the constraint is defined by $\mathrm{x'x} \le \rho^2$, where ρ is the design radius.

3.2. Example

A Roman-style catapult experiment was introduced by [33] and has been revisited by many researchers, such as [13] and [9]. This experiment was designed by the Texas Instrumens' Education and Development Center. As noted by [33], the catapult, which possesses many of the elements of a real-life problem, is ideally suited to the classroom environment. This experiment focuses on determining the effects of three components – arm length (x_1), stop angle (x_2) and pivot height (x_3) – on the catapult performance to predict the distance (y) to the point where a projectile landed from the base of the catapult. A CCD with three replicates is conducted. According to the Table 2, for $k = 3, \alpha = 1.682$ and with six center points, $n_c = 6$, the total number of experiments run was obtained as twenty. In Table 3, $\bar{y}$ and s are the point estimates of the process mean and standard deviation, respectively. Minitab 15 statistical software package was used for the analysis.

The fitted response surfaces for the process mean and standard deviation were obtained by [13] as follows:

$$\begin{aligned}\hat{\mu}(x) = 84.88 &+ 15.29x_1 + 0.24x_2 + 18.80x_3 \\ &- 0.52x_1^2 - 11.80x_2^2 \\ &+ 0.39x_3^2 + 0.22x_1x_2 \\ &+ 3.60x_1x_3 - 4.42x_2x_3\end{aligned} \quad (14)$$

and

$$\begin{aligned}\hat{\sigma}(x) = 4.53 &+ 1.84x_1 + 4.28x_2 + 3.73x_3 \\ &+ 1.16x_1^2 + 4.40x_2^2 \\ &+ 0.94x_3^2 + 1.20x_1x_2 \\ &+ 0.73x_1x_3 + 3.49x_2x_3\end{aligned} \quad (15)$$

Table 3. The catapult study data

u	x_1	x_2	x_3	$\bar{y}$	s
1	-1	-1	-1	38.3	4.0
2	-1	-1	1	80.7	10.0
3	-1	1	-1	47.0	4.4
4	-1	1	1	75.0	19.5
5	1	-1	-1	60.3	7.5
6	1	-1	1	114.7	11.6
7	1	1	-1	69.0	7.8
8	1	1	1	94.7	30.7
9	-1.682	0	0	56.7	4.9
10	1.682	0	0	111.3	8.1
11	0	-1.682	0	50.0	7.0
12	0	1.682	0	60.0	2.4
13	0	0	-1.682	54.7	5.0
14	0	0	1.682	116.7	6.8
15	0	0	0	84.7	5.9
16	0	0	0	83.3	3.8
17	0	0	0	85.3	3.8
18	0	0	0	86.0	3.6
19	0	0	0	84.3	4.7
20	0	0	0	85.7	5.9

The target value for the mean response is 80 with specification limits [60,100]. Additionally, the desired value for the standard deviation should not exceed 3.5. The optimization problem requires the "target is best" case. Figure 2 presents the capability six-pack results for the catapult data; see also [9].

Twenty subgroups of size three were used in the analysis; see Figure 2(a). Figures 2(b) and 2(c) indicate that while the X-bar chart exhibits many out-of-control points, the R chart is almost in control. This indicates that the operator is having no difficulty in making consistent measurements. From Figure 2(d), p-value for the Anderson-Darling test is greater than 0.05, so this value indicates that the process has approximately a normal density function. On the other hand, $C_p = 0.58$ is less than one, so the process spread is greater than specification limits – see, the capability histogram in Figure 2(e). In addition, $C_{pm} = 0.28$, given in Figure 2(f), indicates anoff-centered process and lower capability – i.e., poor process.

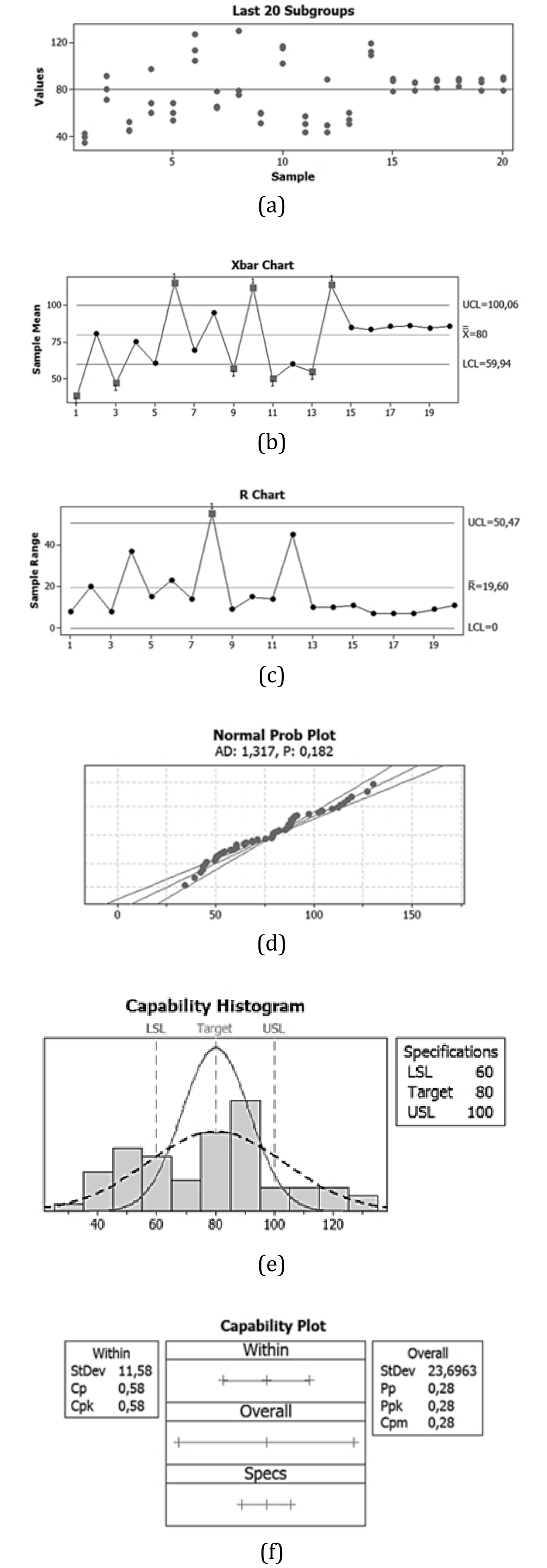

Figure 2. MINITAB's capability six-pack results for the catapult data: (a) subgroups, (b) X-bar chart, (c) R chart, (d) Normal probability plot, (e) capability histogram, (f) capability plot.

In line with the mentioned information about the catapult experiment, the optimal factor settings are obtained by minimizing the following optimization problem:

$$\begin{gathered} Min \ \ \hat{E}L_{UDN}^{\ \ *} \\ Subject\ to\ -1 \le x_i \le 1\ , i = 1,2,3 \end{gathered} \tag{16}$$

where $\hat{E}L_{UDN}^{\ \ *}$ is defined by Equation (10),

$$\hat{C}_p = \frac{6.67}{\hat{\sigma}(x)} \text{ and } \hat{C}_{pm} = \frac{6.67}{\sqrt{\hat{\sigma}^2(x)+(\tau-\hat{\mu}(x))^2}}. \tag{17}$$

The important point here is that since the problem requires the "target is best" case, one expects that the estimation of the process mean hits the target with minimum variance. If this expectation is met, $\tau = \hat{\mu}(x)$ is obtained. This means that $\hat{C}_p = \hat{C}_{pm}$. Therefore, for the purposes of the optimization problem, a new criterion for the constraints as $\hat{C}_p = \hat{C}_{pm}$ is added. This new constraint force estimates the process mean at the target. The optimization results of the proposed approach and a comparative summary are illustrated in Table 4.

From Table 4, the optimal factor setting for the proposed approach is $x^* = (0.12661, -028594, -0{,}28247)$ where $\hat{\mu}(x) = 80$, $\hat{\sigma}(x) = 3.1510$ with the minimum expected loss $\hat{E}L_{UDN} = 0.01670$ under $\hat{C}_p = 2.1157$ and $\hat{C}_{pm} = 2.115$. These results demonstrate that the estimated process mean hits the target and with an acceptable variability besides the minimum expected loss. Table 4 also contains solutions to the catapult problem from different approaches. [6]'s method hits the target and [8] estimate the process mean with a little bias with an acceptable variability. Additionally, [9] estimate the process mean with a little bias, and their approach provides additional information – for example, that the process mean and standard deviation are obtained with the minimum expected loss. The MSE of [8] is a measure of risk and cannot provide any information about how much better the process is.

However, as mentioned by [9], the results from different approaches cannot be compared in a straightforward manner since the different methods have different optimization criteria. They can be compared on the basis of the additional information they provide. The proposed approach gives an additional information about the process such as $\hat{C}_{pm}$ and $\hat{C}_p$. The obtained $\hat{C}_{pm}$ indicates that the process is centered at the midpoint of the specifications and the process capability around the target is estimated as equal to value of $\hat{C}_p$. In line with the obtained results, it is obvious that the process mean is at the midpoint of specifications and is spread between the specifications. It can therefore be said that the process is highly capable of producing. The proposed

Table 4. Comparisons of the best operating conditions for the catapult data.

	$x*$	$\hat{\mu}(x)$	$\hat{\sigma}(x)$	$\widehat{EL}_{UDN}$	$\hat{C}_p$	$\hat{C}_{pm}$
The proposed method	(0.12661, −028594, −0,28247)	80	3.1510	0.01670	2.1157	2.1157
Vining and Myers (1990)	(0.12913, −0.28511, −0.28461)	80	3.1511	Unknown	Unknown	Unknown
Lin and Tu (1995)	(0.12621, −027890, −0,30238)	79.6496	3.1116	Unknown	Unknown	Unknown
Köksoy and Fan (2012)	(0.12622, −0.27891, −0.30238)	79.6498	3.1116	0.01654	Unknown	Unknown

approach combines the information related to the process and the voice of customer to determine the damage caused by a deviation from the target; it also characterizes this damage on the basis of process capability indices. In fact, the obtained optimal factor settings for the proposed approach have three important features: *i.* estimate process mean at the target value with minimum variability, *ii.* estimate the process mean and variance with minimum loss, *iii.* estimate process capability indices with minimum loss. As a result, the proposed model and optimization approach provides much more information about the process.

4. Discussion and Conclusion

Various technologies in the current literature focus on obtaining the best operating settings for a DRS problem. This is typically done by either minimizing the loss or minimizing/maximizing a particular response surface of the quality characteristic under some constraints/no constraint for a given system. Many researchers have proposed novel improvements on robust design and have sought to understand the possible relationships between the quality loss and the behavior of the spread of the data. This paper introduces a new perspective on this relationship and presents a model which characterizes the loss with process capability indices, $\hat{C}_{pm}$ and $\hat{C}_p$. In fact, this is the most important feature that distinguishes this proposed approach from the others. As previously discussed, although many of existing methods – for example, the studies of [6], [7], [12], [14] and [34] – are effective and sound approaches, they do not give any information about how much better or worse a process is, since they do not construct based on the process loss minimization based-optimization. On the other hand, quality experts commonly utilize loss functions to develop new methodologies for quality improvement. Therefore, loss function based methods have an important effect on risk assessment in respect to quality. The studies of [8], [9], [35], and [36] provide valuable strategies based on several loss/risk functions. The basic idea behind these studies is to obtain the best operating condition by minimizing the penalty caused by being off-target. Following this fundamental philosophy, integrating a new criterion – i.e. process capability indices – is proposed for the expected UDNLF in this paper. The proposed approach is constructed by minimizing the expected UDNLF where the process capability indexes, $\hat{C}_{pm}$ and $\hat{C}_p$ are fitted by quadratic the response surfaces. Since the expected loss of UDNLF is used as an objective function, the scale parameter of loss function provides a reasonable way to combine the voice of the customer and information relating to the process capability. The proposed approach is constructed under the “target is best” case. Moreover, it yields more realistic and informative results in terms of defining the general characteristics of the process, besides establishing how much better the process is. The proposed approach and its merits are illustrated by a well-known design of experiments study in the literature. As a future study, an approach based on estimating the UDNLF with a simulation study and an appropriate optimization criterion can be studied.

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