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ON A TYPE OF α-COSYMPLECTIC MANIFOLDS

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Abstract. The object of this paper is to study α-cosymplectic manifolds admitting a W_2-curvature tensor.

1. Introduction

A $(2m+1)$-dimensional differentiable manifold M of class C^∞ is said to have an almost contact structure if the structural group of its tangent bundle reduces to $U(m)\times 1$ ([3], [14]), equivalently an almost contact structure is given by a triple (φ,ξ,η) satisfying certain conditions. Many different types of almost contact structures are defined in the literature. In [12], Pokhariyal and Mishara have introduced new tensor fields which is called W_2 and E-tensor fields in a Riemmanian manifold and studied their properties. Then, Pokhariyal [13] has studied some properties of this tensor fields in Sasakian manifold. Recently, Matsumoto et al. [9] have studied P-Sasakian manifolds admitting W_2 and E-tensor fields and De and Sarkar [5] have studied Sasakian manifolds admitting tensor field. The curvature tensor W_2 is defined by

$$W_2(X,Y,U,V)=R(X,Y,U,V)+\frac{1}{n-1}[g(X,U)S(Y,V)-g(Y,U)S(X,V)], \quad (1)$$

where S is a Ricci tensor of type $(0,2)$ [12]. In [16], Yildiz and De have studied geometric and relativistic properties of Kenmotsu manifolds admitting W_2-curvature tensor.

In the present paper, we have studied the some curvature conditions on α-cosymplectic manifolds. We also have classified α-cosymplectic manifolds which satisfy the conditions $P.W_2=0$, $\tilde{Z}.W_2=0$, $C.W_2=0$ and $\tilde{C}.W_2=0$ where P is the projective curvature tensor, $\tilde{Z}$ is the concircular curvature tensor, $\tilde{C}$ is the quasi-conformal curvature tensor and C is the conformal curvature tensor.

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2. Preliminaries

Let $(M^n, \varphi, \xi, \eta, g)$ be an n-dimensional (where $n = 2m+1$) almost contact metric manifold, where φ is a $(1,1)$-tensor field, ξ is the structure vector field, η is a 1-form and g is the Riemannian metric. It is well know that the (φ, ξ, η, g) structure satisfies the conditions [4].

$$\varphi\xi = 0,\ \eta(\varphi\xi) = 0,\ \eta(\xi) = 1, \tag{2}$$

$$\varphi^2 X = -X + \eta(X)\xi,\ g(X,\xi) = \eta(\xi) = 1, \tag{3}$$

$$g(\varphi X, \varphi Y) = g(X,Y) - \eta(X)\eta(Y), \tag{4}$$

for any vector fields X and Y on M^n.

If moreover

$$\nabla_X \xi = -\alpha\varphi^2 X, \tag{5}$$

$$(\nabla_X \eta)Y = \alpha[g(X,Y) - \eta(X)\eta(Y)], \tag{6}$$

where ∇ denotes the Riemannian connection of hold and α is a real number, then $(M^n, \varphi, \xi, \eta, g)$ is called a α-cosymplectic manifold [8]. (See also: [1])

In this case, it is well know that [10]

$$R(X,Y)\xi = \alpha^2[\eta(X)Y - \eta(Y)X], \tag{7}$$

$$S(X,\xi) = -\alpha^2(n-1)\eta(X), \tag{8}$$

where S denotes the Ricci tensor. From (7), it easily follows that

$$R(X,\xi)Y = \alpha^2[g(X,Y)\xi - \eta(Y)X] \tag{9}$$

$$R(X,\xi)\xi = \alpha^2[\eta(X)\xi - X]. \tag{10}$$

The notion of the quasi-conformal curvature tensor was given by Yano and Sawaki [15].

According to them a quasi-conformal curvature tensor $\tilde{C}$ is defined by

$$\tilde{C}(X,Y)Z = aR(X,Y)Z + b[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY] \tag{11}$$

$$-\frac{r}{n}[\frac{a}{n-1} + 2b][g(Y,Z)X - g(X,Z)Y],$$

where a and b are constants and R, S, Q and η are the Riemannian curvature tensor type of $(1,3)$, the Ricci tensor of type $(0,2)$, the Ricci operator defined by $g(QX,Y) = S(X,Y)$ and scalar curvature of the manifold respectively. If $a = 1$ and $b = -\frac{1}{n-2}$ then takes the form

$$\tilde{C}(X,Y)Z = R(X,Y)Z - \frac{1}{n-2}[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY] \tag{12}$$

$$+\frac{r}{(n-1)(n-2)}[g(Y,Z)X - g(X,Z)Y] = C(X,Y)Z,$$

where C is the conformal curvature tensor[7].

We next define endomorphisms $R(X,Y)$ and $X \wedge_A Y$ of $\chi(M)$ by

$$R(X,Y)W = \nabla_X \nabla_Y W - \nabla_Y \nabla_X W - \nabla_{[X,Y]} W,$$
$$(X \wedge_A Y)W = A(Y,W)X - A(X,W)Y,$$

respectively, where $X, Y, W \in \chi(M)$ and A is the symmetric $(0,2)$-tensor. On the other hand, the projective curvature tensor P and the concircular curvature tensor $\tilde{Z}$ in a Riemannian manifold (M^n, g) are defined by

$$P(X,Y)W = R(X,Y)W - \frac{1}{n-1}(X \wedge_S Y)W, \tag{13}$$

$$\tilde{Z}(X,Y)W = R(X,Y)W - \frac{r}{n(n-1)}(X \wedge_g Y)W, \tag{14}$$

respectively [16].

An α-cosymplectic manifold is said to be an η-Einstein manifold if Ricci tensor S satisfies condition

$$S(X,Y) = \lambda_1 g(X,Y) + \lambda_2 \eta(X)\eta(Y), \tag{15}$$

where λ_1, λ_2 are certain scalars.

A Riemannian or a semi-Riemannian manifold is said to semi-symmetric if $R(X,Y).R = 0$, where $R(X,Y)$ is considered as a derivation of the tensor algebra at each point of the manifold for the tangent vectors X and Y [16].

In a α-cosymplectic manifold, using (8) and (9), equations (11), (12), (13) and (14) reduce to

$$P(\xi,X)Y = -\alpha^2 g(X,Y)\xi - \frac{1}{n-1}S(X,Y)\xi \tag{16}$$

$$\tilde{Z}(\xi,X)Y = (\alpha^2 + \frac{r}{n(n-1)})[-g(X,Y)\xi + \eta(Y)X] \tag{17}$$

$$\begin{aligned} C(\xi,Y)W = & \frac{\alpha^2(n-1)+r}{(n-1)(n-2)}[g(Y,W)\xi - \eta(W)Y] \\ & - \frac{1}{n-2}[S(Y,W)\xi - \eta(W)QY], \end{aligned} \tag{18}$$

$$\tilde{C}(\xi,Y)W = K[\eta(W)Y - g(Y,W)\xi] - b[S(Y,W)\xi - \eta(W)QY], \tag{19}$$

respectively, where $K = a\alpha^2 + b\alpha^2(n-1) + \frac{r}{n}(\frac{a}{n-1} + 2b)$.

A α-cosymplectic manifold M^n is said to be Einstein if its Ricci tensor S is of the form

$$S(X,Y) = \lambda_1 g(X,Y), \tag{20}$$

for any vector fields X, Y and λ_1 is a certain scalar.

Theorem 1. *A cosymplectic manifold is locally the Riemannian product of an almost Kaehler manifold with the real line*[11].

3. α-COSYMPLECTIC MANIFOLDS SATISFYING $W_2 = 0$

In this section we consider a α-cosymplectic manifold satisfying $W_2 = 0$.

Theorem 2. *Let M be an n-dimensional $(n > 3)$ α-cosymplectic manifold satisfying $W_2 = 0$. Then M is an Einstein manifold and M is locally isometric to the hyperbolic space $H^n(-\alpha^2)$.*

Proof. If M be an n-dimensional α-cosymplectic manifold satisfying $W_2 = 0$,then we have from (1)

$$R(X,Y,U,V) = \frac{1}{n-1}[g(Y,U)S(X,V) - g(X,U)S(Y,V)]. \tag{21}$$

Using $X = U = \xi$ in (21), we get

$$R(\xi,Y,\xi,V) = \frac{1}{n-1}[g(Y,\xi)S(\xi,V) - g(\xi,\xi)S(Y,V)].$$

From (2), (8) and (10), we get

$$S(Y,V) = -\alpha^2(n-1)g(Y,V). \tag{22}$$

Thus M is an Einstein manifold. Now using (22) in (21), we get

$$R(X,Y,U,V) = -\alpha^2 g(Y,U)g(X,V) + \alpha^2 g(X,U)g(Y,V).$$

Hence M is of constant curvature $-\alpha^2$. Then M is locally isometric to the hyperbolic space $H^n(-\alpha^2)$. □

4. W_2-SEMISYMMETRIC α-COSYMPLECTIC MANIFOLDS

Definition 3. *An n-dimensional α-cosymplectic manifolds is called W_2-semisymmetric if it satisfies*

$$R(X,Y).W_2 = 0, \tag{23}$$

where $R(X,Y)$ is to be considered as a derivation of the tensor algebra at each point of the manifold for tangent vectors X, Y.

Proposition 4. *Let M be an n-dimensional α-cosymplectic manifold. Then the W_2-curvature tensor on M satisfies the condition*

$$W_2(X,Y,U,\xi) = 0. \tag{24}$$

Proof. The proof is clear from (1) and (7). □

Theorem 5. *A W_2-semisymmetric α-cosymplectic manifold is a locally the Riemannian product of an almost Kaehler manifold with the real line or a locally isometric to the hyperbolic space $H^n(-\alpha^2)$.*

Proof. From (23) we have

$$(R(X,Y).W_2)(Z,U)V = R(X,Y)W_2(Z,U)V - W_2(R(X,Y)Z,U)V \\ - W_2(Z,R(X,Y)U)V - W_2(Z,U)R(X,Y)V = 0. \tag{25}$$

If we multiply this equation by ξ, we have

$$g(R(X,Y)W_2(Z,U)V,\xi) - g(W_2(R(X,Y)Z,U)V,\xi) \\ - g(W_2(Z,R(X,Y)U)V,\xi) - g(W_2(Z,U)R(X,Y)V,\xi) = 0. \tag{26}$$

Putting $X = \xi$ in (26) we obtain

$$g(R(\xi,Y)W_2(Z,U)V,\xi) - g(W_2(R(\xi,Y)Z,U)V,\xi) \\ - g(W_2(Z,R(\xi,Y)U)V,\xi) - g(W_2(Z,U)R(\xi,Y)V,\xi) = 0. \tag{27}$$

Using (7), (9) and (10) in (27), we get

$$- \alpha^2 g(Y,W_2(Z,U)V) + \alpha^2\eta(W_2(Z,U)V)\eta(Y) \\ + \alpha^2 g(Y,Z)g(W_2(\xi,U)V,\xi) - \alpha^2\eta(Z)g(W_2(Y,U)V,\xi) \\ + \alpha^2 g(Y,U)g(W_2(Z,\xi)V,\xi) - \alpha^2\eta(U)g(W_2(Z,Y)V,\xi) \\ + \alpha^2 g(Y,V)g(W_2(Z,U)\xi,\xi) - \alpha^2\eta(V)g(W_2(Z,U)Y,\xi) = 0. \tag{28}$$

Using (24) in (28), we obtain

$$\alpha^2 W_2(Z,U,V,Y) = 0.$$

Then $\alpha = 0$ or $W_2 = 0$.The proof is completed from Theorem 1 and Theorem 2. □

5. α-COSYMPLECTIC MANIFOLDS SATISFYING $P(X,Y).W_2 = 0$

In this section we consider a α-cosymplectic manifold M^n satisfying the condition

$$P(X,Y).W_2 = 0.$$

This equation implies

$$P(X,Y)W_2(Z,U)V - W_2(P(X,Y)Z,U)V \\ - W_2(Z,P(X,Y)U)V - W_2(Z,U)P(X,Y)V = 0. \tag{29}$$

Taking the inner product with ξ and putting $X = \xi$

$$g(P(\xi,Y)W_2(Z,U)V,\xi) - g(W_2(P(\xi,Y)Z,U)V,\xi) \\ - g(W_2(Z,P(\xi,Y)U)V,\xi) - g(W_2(Z,U)P(\xi,Y)V,\xi) = 0. \tag{30}$$

Using (16) in (30), we have

$$\begin{aligned}&-\alpha^2 g(Y,W_2(Z,U)V)-\frac{1}{n-1}S(Y,W_2(Z,U)V)\\&+\alpha^2 g(Y,Z)g(W_2(\xi,U)V,\xi)+\frac{1}{n-1}S(Y,Z)g(W_2(\xi,U)V,\xi)\\&+\alpha^2 g(Y,U)g(W_2(Z,\xi)V,\xi)+\frac{1}{n-1}S(Y,U)g(W_2(Z,\xi)V,\xi)\\&+\alpha^2 g(Y,U)g(W_2(Z,U)\xi,\xi)+\frac{1}{n-1}S(Y,U)g(W_2(Z,U)\xi,\xi)=0.\end{aligned} \tag{31}$$

Using (24) in (31) we get

$$S(Y,W_2(Z,U)V)=\alpha^2(1-n)g(Y,W_2(Z,U)V). \tag{32}$$

So, M^n is an Einstein manifold. Now using (1) in (32) we get

$$\begin{aligned}&\alpha^2 R(Z,U,V,Y)+\frac{\alpha^2}{n-1}[g(Z,V)S(U,Y)-g(U,V)S(Z,Y)]\\&+\frac{1}{n-1}R(Z,U,V,QY)+\frac{1}{(n-1)^2}[g(Z,V)S(U,QY)-g(U,V)S(Z,QY)]=0.\end{aligned} \tag{33}$$

Again using $Z=V=\xi$ in (33) and from (8), (10) we get

$$S(U,QY)=-2\alpha^2(n-1)S(U,Y)-\alpha^4(n-1)^2 g(U,Y). \tag{34}$$

Hence we have the following

Theorem 6. *In an n-dimensional ($n>3$) α-cosymplectic manifold M^n if the condition $P(X,Y)W_2=0$ holds then M^n is an Einstein manifold and the equation (34) is satisfied on M^n.*

Lemma 7. [6] *Let A be a symmetric $(0,2)$-tensor at point x of a semi-Riemannian manifold (M^n,g), $n>3$, and let $T=g\bar{\wedge}A$ be the Kulkarni-Nomizu product of g and A. Then, the relation*

$$T.T=k.Q(g,T), k\in R$$

is satisfied at x if and only if the condition

$$A^2=k.A+\lambda g, \lambda\in R$$

holds at x.

From Theorem 6 and Lemma 7 we get the following:

Corollary 8. *Let M be an n-dimensional ($n>3$) α-cosymplectic manifold satisfying the condition $P(X,Y).W_2=0$, then $T.T=k.Q(g,T)$, where $T=g\bar{\wedge}S$ and $k=-2\alpha^2(n-1)$.*

6. α-COSYMPLECTIC MANIFOLD SATISFYING $\tilde{Z}(X,Y).W_2 = 0$

In this section we consider a α-cosymplectic manifold M^n satisfying the condition

$$\tilde{Z}(X,Y).W_2 = 0.$$

This equation implies

$$\begin{aligned} &\tilde{Z}(X,Y)W_2(Z,U)V - W_2(\tilde{Z}(X,Y)Z,U)V \\ &-W_2(Z,\tilde{Z}(X,Y)U)V - W_2(Z,U)\tilde{Z}(X,Y)V = 0. \end{aligned} \tag{35}$$

Now $X = \xi$ in (35), we have

$$\begin{aligned} &\tilde{Z}(\xi,Y)W_2(Z,U)V - W_2(\tilde{Z}(\xi,Y)Z,U)V \\ &- W_2(Z,\tilde{Z}(\xi,Y)U)V - W_2(Z,U)\tilde{Z}(\xi,Y)V = 0. \end{aligned} \tag{36}$$

Using (17) in (36), we get

$$\begin{aligned} \{\alpha^2 + \frac{r}{n(n-1)}\}\{ &- g(Y,W_2(Z,U)V)\xi + g(W_2(Z,U)V,\xi)Y \\ &+ g(Y,Z)W_2(\xi,U)V - \eta(Z)W_2(Y,U)V \\ &+ g(Y,U)W_2(Z,\xi)V - \eta(U)W_2(Z,Y)V \\ &+ g(Y,U)W_2(Z,U)\xi - \eta(V)W_2(Z,U)Y\}. \end{aligned} \tag{37}$$

Taking the inner product with ξ and using (24) in (37), we have

$$\{\alpha^2 + \frac{r}{n(n-1)}\}g(Y,W_2(Z,U)V) = 0.$$

Again from (17) we have $\alpha^2 + \frac{r}{n(n-1)} \neq 0$. Hence we have

$$W_2(Z,U,V,Y) = 0.$$

From the proof of Theorem 2 and Theorem 5 we can say:

Theorem 9. *An n-dimensional ($n > 3$) α-cosymplectic manifold M satisfying the condition $\tilde{Z}(\xi,Y).W_2 = 0$ is an Einstein manifold and locally isometric to the hyperbolic space $H^n(-\alpha^2)$.*

7. α-COSYMPLECTIC MANIFOLD SATISFYING $C(X,Y).W_2 = 0$

In this section we consider a α-cosymplectic manifold M^n satisfying the condition

$$C(X,Y).W_2 = 0$$

Theorem 10. *Let M^n be an n-dimensional ($n > 3$) α-cosymplectic manifold satisfying the condition $C(X,Y).W_2 = 0$. Then M^n is an Einstein manifold.*

Proof. This equation implies

$$\begin{aligned} &C(X,Y)W_2(Z,U)V - W_2C(X,Y)Z,U)V \\ &- W_2(Z,C(X,Y)U)V - W_2(Z,U)C(X,Y)V = 0. \end{aligned} \tag{38}$$

Putting $X=\xi$ in (38), we have

$$C(\xi,Y)W_2(Z,U)V - W_2(C(\xi,Y)Z,U)V \\ - W_2(Z,C(\xi,Y)U)V - W_2(Z,U)C(\xi,Y)V = 0. \tag{39}$$

Using (18) in (39), we have

$$Ag(Y,W_2(Z,U)V)\xi - A\eta(W_2(Z,U)V)Y - BS(Y,W_2(Z,U)V)\xi + B\eta(W_2(Z,U)V)QY \tag{40}$$
$$-Ag(Y,Z)W_2(\xi,U)V + A\eta(Z)W_2(Y,U)V + BS(Y,Z)W_2(\xi,U)V - B\eta(Z)W_2(QY,U)V$$
$$-Ag(Y,U)W_2(Z,\xi)V + A\eta(U)W_2(Z,Y)V + BS(Y,U)W_2(Z,\xi)V - B\eta(U)W_2(Z,QY)V$$
$$-Ag(Y,V)W_2(Z,U)\xi + A\eta(V)W_2(Z,U)Y + BS(Y,V)W_2(Z,U)\xi - B\eta(V)W_2(Z,U)QY$$

respectively, where $A=\frac{\alpha^2(n-1)+r}{(n-1)(n-2)}$ and $B=\frac{1}{n-2}$. Taking the inner product with ξ and using (24), we obtain

$$Ag(Y,W_2(Z,U)V) - BS(Y,W_2(Z,U)V) = 0. \tag{41}$$

Thus M is an Einstein manifold. □

8. α-cosymplectic manifolds satisfying $\tilde{C}(X,Y).W_2=0$

In this section we consider a α-cosymplectic manifold M^n satisfying the condition

$$\tilde{C}(X,Y).W_2 = 0.$$

Theorem 11. *Let M be an n-dimensional $(n>3)$ α-cosymplectic manifold satisfying the condition $\tilde{C}(X,Y).W_2=0$. Then we get*

1) *if $b=0$, then M is an Einstein manifold and M is locally isometric to the hyperbolic space $H^n(-\alpha^2)$.*
2) *if $b\neq 0$, then M is an Einstein manifold.*

Proof. This equation implies

$$\tilde{C}(X,Y)W_2(Z,U)V - W_2(\tilde{C}(X,Y)Z,U)V \\ - W_2(Z,\tilde{C}(X,Y)U)V - W_2(Z,U)\tilde{C}(X,Y)V = 0. \tag{42}$$

Putting $X=\xi$ in (42), we have

$$\tilde{C}(\xi,Y)W_2(Z,U)V - W_2(\tilde{C}(\xi,Y)Z,U)V \\ - W_2(Z,\tilde{C}(\xi,Y)U)V - W_2(Z,U)\tilde{C}(\xi,Y)V = 0. \tag{43}$$

Using (19) in (43), we have

$$\begin{aligned} &K\{g(W_2(Z,U)V,\xi)Y - g(Y,W_2(Z,U)V)\xi - \eta(Z)W_2(Y,U)V \\ &+ g(Y,Z)W_2(\xi,U)V - \eta(U)W_2(Z,Y)V + g(Y,U)W_2(Z,\xi)V \\ &- \eta(V)W_2(Z,U)Y + g(Y,V)W_2(Z,U)\xi\} \\ &b\{S(Y,W_2(Z,U)V)\xi - g(W_2(Z,U)V,\xi)QY - S(Y,Z)W_2(\xi,U)V \\ &+ \eta(Z)W_2(QY,U)V - S(Y,U)W_2(Z,\xi)V + \eta(U)W_2(Z,QY)V \\ &- S(Y,Z)W_2(Z,U)\xi + \eta(V)W_2(Z,U)QY\} = 0, \end{aligned} \tag{44}$$

where $K = a\alpha^2 + b\alpha^2(n-1) + \frac{r}{n}(\frac{a}{n-1} + 2b)$. Taking the inner product with ξ and using (24) in (44), we have

$$Kg(Y,W_2(Z,U)V) - bS(Y,W_2(Z,U)V) = 0. \tag{45}$$

From this equation, if $b = 0$ then $W_2 = 0$ and if $b \neq 0$ then $S(Y,W_2(Z,U)V) = \frac{K}{b}g(Y,W_2(Z,U)V)$. Hence , the proof is completed. □

Corollary 12. *Let M be an n- dimensional $(n > 3)$ α-cosymplectic manifold satisfying the condition $\tilde{C}(\xi,Y).W_2 = 0$, then $T.T = kQ(g,T)$, where $T = g\bar{\wedge}S$ and $k = \frac{K}{b} - \alpha^2(n-1)$.*

Proof. If $b \neq 0$, then using $Z = V = \xi$ in (45) and from (1) and (10), we have

$$S(QY,U) = (\frac{K}{b} - \alpha^2(n-1))S(U,Y) + \alpha^2(n-1)\frac{K}{b}g(U,Y).$$

Hence, we have desired result from Lemma 7. □

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