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A Fuzzy Approach to Bi-Objective Newsboy Problem

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Abstract

The single period stochastic inventory problem is referred to as "Newsboy" or "Newsvendor" problem in the literature, and can be expressed as determining optimal order quantities under stochastic demand. Due to single order opportunity, the order quantity should be determined for the entire period. Most studies use the expected profit as the performance measure in the solution of the Newsboy problem. However, managers and decision makers may be more concerned with a probability level to achieve a specific target profit as another performance measure beside the expected profit. These two performance measures conflict with each other, and cannot be optimized simultaneously. Therefore, a Fuzzy Bi-Objective Programming model has been developed to find a good compromise solution to the problem. Two cases have been presented to show use of the proposed model under uniform and exponential demand.

Keywords: Programming; Optimization; Newsboy; Stock Control; Inventory; Fuzzy

İki Amaçlı Gazeteci Çocuk Problemine Bulanık Bir Yaklaşım

Özet

Literatürde "Newsboy" veya "Newsvendor" problemi olarak değinilen Tek Devrelik Stokastik Stok Problemi (TDSSP), rassal talep altında bir ürünün en iyi sipariş miktarını belirlemek olarak ifade edilebilir. Tek seferlik sipariş fırsatı olduğu için bütün devrenin sipariş miktarı belirlenmelidir. Çoğu çalışmada TDSPP'nin çözümünde beklenen kâr performans ölçüsü olarak kullanılır. Bununla birlikte yöneticiler ve karar vericiler beklenen kâr yanında başka bir performans ölçüsü olarak belirli bir hedef kar düzeyine ulaşma olasılığı ile daha fazla ilgilenebilirler. Bu iki performans ölçüsü birbiriyle çatışır ve genellikle eş anlı olarak eniyilenemez. Bu yüzden bu çalışmada probleme iyi bir uzlaşık çözüm bulmak için bir bulanık çok amaçlı programlama modeli geliştirilmiştir. Düzgün ve üstel dağılan talep altında önerilen modelin kullanımını göstermek için açıklayıcı iki örnek sunulmuştur.

Anahtar kelimeler: Programlama; Optimizasyon; Gazeteci Çocuk; Stok Kontrol; Envanter; Bulanık

1. Introduction

Due to rapid technological development and global competition, product life cycles are getting shorter and shorter especially for high technology products[1,2]. As downward trend in product life cycles continues, inventory problem for single period products will gain more importance[3]. In the classic Single Period Problem (SPP) the order quantity that maximizes the expected profit is determined under stochastic demand. In the SPP model any inventory left at the end of the period is either salvaged or disposed of. If the order quantity is smaller than the realized demand, some quantity of profit is forgone. The fashion and sporting goods industries are areas that the SPP is often employed to support decision making, both at the manufacturing and at retail level[4]. The SPP can also be used in managing capacity and evaluating advanced booking of orders in service industries such as airlines and hotels[5].

Most studies use the expected profit as the performance measure in the solution of the Newsboy problem. However, managers and decision makers may be more concerned with a probability level to achieve a specific target profit as another performance measure beside the expected profit. This probability level is called as the satisfaction probability. It is defined as the probability to exceed target profit level. For previous studies with respect to the satisfaction probability refer to [6–18].

The study has considered two conflicting performance measures (objectives) together in determining the optimal order quantity. The first one is the expected profit, and the second is the satisfaction probability. It is almost impossible to obtain an order quantity that optimizes both performance measures. Therefore, a Fuzzy Bi-Objective Programming (FBOP) model has been developed to find a good compromise solution to the problem. Two cases have been presented to show use of the proposed model under uniform and exponential demand distributions.

2. Analytic Model

2.1. Problem Definition

If unit selling price is denoted by R, unit cost by C, unit surplus cost incurred at the end of the period for each unit left by H, the period's demand that is a random variable with density function g(.) and cumulative distribution function G(.) by X, the order quantity to be obtained at the start of the selling period by Q, and if no penalty is incurred for each unit short, then the profit to be gained at the end of the selling period is as the following [18]:

$$Y(Q,X) = (R+H)A - (C+H)Q \quad (1)$$

A, the minimum of demand and the order quantity, represents realized sales. As the profit to be obtained for a specific order quantity will change depending on A, a random variable, the profit function given with (1) would also be a random variable.

2.2. Expected Profit Optimization

If the expected value of realized sales A=min (Q, D) is denoted by E[A], then the expected profit at the end of the period would be

$$EP(Q) = E[Y(Q,X)] = (R+H)E[A] - (C+H)Q \quad (2)$$

If density function of demand is denoted by g(x),then the expected value of A is

$$E[A] = \int_{-\infty}^{Q} xg(x)dx + \int_{Q}^{\infty} Qg(x)dx \quad (3)$$

The order quantity to maximize the expected profit is

$$Q_1^* = G^{-1}\left(\frac{R-C}{R+H}\right) \quad (4)$$

where G^{-1} is the inverse function of G [9].

2.3. Satisfaction Probability Optimization

Beside the expected profit maximization, maximizing the probability to exceed a given target profit level is also important. This probability is called as satisfaction probability[19].

The probability of random profit for Q to exceed target profit level T is denoted by $\theta = \mathrm{P}\{\mathrm{Y}(\mathrm{Q},\mathrm{X}) \geq \mathrm{T}\}$ and given with

$$\theta = \begin{cases} 0, \ Q < \frac{T}{R-C} \\ 1 - G(D_T), \ Q \geq \frac{T}{R-C} \end{cases} \quad (5)$$

where $x_T = \frac{[T+(C+H)Q]}{R+H}$ represents the actual demand required to achieve T [9].

The maximum value of θ is obtained at the optimal order quantity

$$Q_2^* = \frac{T}{R-C} \quad (6)$$

and the maximum value of θ is denoted by

$$\theta^* = P\{Y(Q_2^*, X) \geq T\} \quad (7)$$

3. Fuzzy Bi-Objective Programming Model

When the expected profit and the satisfaction probability are determined as two objectives for a SPP, the Bi-Objective SPP given below is obtained.

$$\max_{Q \geq 0} \mathrm{EP}(Q)$$
$$\max_{Q \geq 0} P\{Y(Q,X) \geq T\} \quad (8)$$
$$Q \geq \frac{T}{(R-C)}$$

The two objectives in the problem conflict with each other, and generally cannot be optimized simultaneously. For this reason a compromise solution to the problem is obtained from the solution of the following FBOP model

$$EP(Q) \cong EP(Q_1^*) \quad (9)$$
$$P\{Y(Q,X) \geq T\} \cong \theta^* \quad (10)$$
$$Q \geq \frac{T}{(R-C)} \quad (11)$$

The fuzzy equality numbered with (9) in the model can be interpreted as approaching optimal value of the expected profit as much as possible, and the fuzzy equality numbered with (10) can be commented as getting optimal value of the satisfaction probability as near as possible. The inequality numbered with (11) denotes crisp constraint on the lower bound of the order quantity. This constraint states that it is not possible to reach target profit level with orders smaller than T/(R-C).

The membership function for the expected profit function can be stated as

$$\mu_{EP}(Q) = \begin{cases} \frac{EP(Q)-EP_{Min}}{EP(Q_1^*)-EP_{Min}}, & Q^L < Q < Q^U \\ 0, & otherwise \end{cases} \quad (12)$$

This function gives the degree of approaching optimal value of the expected profit by any order quantity. The membership function for the expected profit function is derived from the normalization of the expected profit function.

Q^L and Q^U respectively denote the lower and the upper bound on the order quantity. Minimum of $EP(Q^L)$ and $EP(Q^U)$ is determined as the EP_{Min} value. In case of demand taking infinite positive values, Q^U replaces with Q' value determined by the rule below

$$Q' = \sup(Q: EP(Q) \geq 0) \quad (13)$$

In the membership function of satisfaction probability, Q^U does not change, stays the same. The membership function for the satisfaction probability function is

$$\mu_\theta(Q) = \begin{cases} \frac{[1-G(x_T)]-\theta^L}{\theta^*-\theta^L}, & \frac{T}{(R-C)} \leq Q \leq Q^U \\ 0, & otherwise \end{cases} \quad (14)$$

where θ^L denotes the lower bound on the satisfaction probability.

This function denotes the degree of coming close to optimal value of the satisfaction probability by any order quantity. The membership function of the satisfaction probability function is obtained from the normalization of the satisfaction probability function.

4. Solution of The Model

Using the definition of the fuzzy decision proposed by [20], the fuzzy decision set for the problem can be stated as

$$\mu_D(Q) = \min[\mu_{EP}(Q), \mu_\theta(Q)] \quad (15)$$

The optimum decision is element(s) with the highest membership degree in the fuzzy decision set, and is a solution to the following problem

$$\max_{Q\geq 0}[\mu_D(Q)] = \max_{Q\geq 0}[\min[\mu_{EP}(Q), \mu_\theta(Q)]] \quad (16)$$

Consequently, the model given with (9)-(11) transforms to the following optimization model

$$\begin{aligned} & max\ \alpha \\ & \text{s.t.} \\ & \alpha \leq \mu_{EP}(Q) \\ & \alpha \leq \mu_\theta(Q) \\ & Q \geq \frac{T}{(R-C)} \end{aligned} \quad (17)$$

Substituting the membership functions (12) and (14) into the model, the following non linear programming model is obtained:

$$\begin{aligned} & max\ \alpha \\ & \text{s.t.} \\ & \alpha \leq \frac{EP(Q)-EP_{Min}}{EP(Q_1^*)-EP_{Min}} \\ & \alpha \leq \frac{[1-G(x_T)]-\theta^L}{\theta^*-\theta^L} \\ & Q \geq \frac{T}{(R-C)} \end{aligned} \quad (18)$$

where α takes a value between 0 and 1, showing degree of compromise between the two objectives. The aim is to maximize α.

5. The Model When Demand has Uniform Distribution

When demand is uniformly distributed $X \sim U(a,b)$, the probability density function and distribution function of demand could be shown respectively as

$$g(x) = \begin{cases} \frac{1}{b-a}, & a \le x \le b \\ 0, & otherwise \end{cases} \quad (19)$$

and

$$G(x) = \begin{cases} \frac{x-a}{b-a}, & a \le x \le b \\ 0, & otherwise \end{cases} \quad (20)$$

From (2) the expected profit at the end of the period would be

$$EP(Q) = \frac{R+H}{b-a}\left(-\frac{a^2}{2} + Q\left(b - \frac{Q}{2}\right)\right) - (C+H)Q \quad (21)$$

The order quantity to maximize the expected profit is

$$Q_1^* = a + \frac{(R-C)(b-a)}{R+H} = b - \frac{(C+H)(b-a)}{R+H} \quad (22)$$

As a result of analyzing the expected profit function given with (21) by inventory parameters, the membership function of the expected profit function is defined with respect to the following rules:

If $(C+H)^2 = (R-C)^2$ then $EP(a) = EP(b)$

In this case

$$\mu_{EP}(Q) = \begin{cases} \frac{EP(Q)-EP(a\,(or)\,b)}{EP(Q_1^*)-EP(a\,(or)\,b)}, & a \le Q \le b \\ 0, & otherwise \end{cases} \quad (23)$$

If $(C+H)^2 > (R-C)^2$ then $EP(a) > EP(b)$

In this case

$$\mu_{EP}(Q) = \begin{cases} \frac{EP(Q)-EP(b)}{EP(Q_1^*)-EP(b)}, & a \le Q \le b \\ 0, & otherwise \end{cases} \quad (24)$$

If $(C+H)^2 < (R-C)^2$ then $EP(a) < EP(b)$

In this case

$$\mu_{EP}(Q) = \begin{cases} \frac{EP(Q)-EP(a)}{EP(Q_1^*)-EP(a)}, & a \le Q \le b \\ 0, & otherwise \end{cases} \quad (25)$$

The order quantity to maximize the satisfaction probability is given with (6). The maximum value of the satisfaction probability when demand is uniformly distributed is

$$\theta^* = \frac{b-x_T^*}{b-a} \quad (26)$$

where $x_T^* = \frac{[T+(C+H)Q_2^*]}{R+H}$.

Minimum value of the satisfaction probability $\theta^L = P\{Y(b,X) \ge T\}$ is

$$\theta^L = \frac{b-x_T'}{b-a} \quad (27)$$

where $x_T' = \frac{[T+(C+H)b]}{R+H}$.

Then, the membership function of the satisfaction probability function becomes

$$\mu_\theta(Q) = \begin{cases} \frac{(b-Q)}{b-[T/(R-C)]}, & \frac{T}{(R-C)} \le Q \le b \\ 0, & otherwise \end{cases} \quad (28)$$

Under uniformly distributed demand, the model given with (18) can be stated in two ways according to the following rules:

Model-1: $\{(C+H)^2 \ge (R-C)^2\}$

$$\begin{aligned} &max\ \alpha \\ &\text{s.t.} \\ &\alpha \le \frac{[(R+H)/(b-a)][Q(b-Q/2)-b^2/2]-(C+H)[Q-b]}{(C+H)^2(b-a)/2(R+H)} \\ &\alpha \le \frac{(b-Q)}{b-[T/(R-C)]} \\ &T/(R-C) \le Q \le b \end{aligned} \quad (29)$$

Model-2: $\{(C+H)^2 \le (R-C)^2\}$

$$\begin{aligned} &max\ \alpha \\ &\text{s.t.} \\ &\alpha \le \frac{[(R+H)/(b-a)][Q(b-Q/2)-a(b-a/2)]-(C+H)[Q-a]}{(R-C)^2(b-a)/2(R+H)} \\ &\alpha \le \frac{(b-Q)}{b-[T/(R-C)]} \\ &T/(R-C) \le Q \le b \end{aligned} \quad (30)$$

6. The Model When Demand Has Exponential distribution

In the case of exponential demand $(X \sim Exp(\lambda))$, probability density function and distribution function of demand could be given respectively with

$$g(x) = \begin{cases} \lambda\, exp(-\lambda x), & x > 0 \\ 0, & d.d. \end{cases} \quad (31)$$

and

$$G(x) = \begin{cases} 1 - \exp(-\lambda x), & x > 0 \\ 0, & d.d. \end{cases} \quad (32)$$

From the equality below

$$E[A] = \int_0^Q x\lambda \exp(-\lambda x)dx + \int_Q^\infty Q\lambda \exp(-\lambda x)\, dx \quad (33)$$

expected value of the realized sales is obtained as the following;

$$E[A] = \frac{1}{\lambda}[1 - \exp(-\lambda Q)] \quad (34)$$

Hence, the expected profit function can be expressed by

$$EP(Q) = (R + H)\frac{1}{\lambda}[1 - \exp(-\lambda Q)] - (C + H)Q \quad (35)$$

The order quantity that maximizes the expected profit and the maximum value of the expected profit are found respectively as

$$Q_1^* = -\frac{1}{\lambda}\ln\left(\frac{C+H}{R+H}\right) \quad (36)$$

and

$$EP(Q_1^*) = (R + H)\frac{1}{\lambda}\left[1 - \left(\frac{C+H}{R+H}\right)\right] + (C + H)\frac{1}{\lambda}\ln\left(\frac{C+H}{R+H}\right) \quad (37)$$

As a result of normalization of the expected profit function, membership function for the expected profit can be derived as the following:

$$\mu_{EP}(Q) = \begin{cases} \frac{EP(Q)-EP(0)}{EP(Q_1^*)-EP(0)}, & 0 \le Q < Q' \\ 0, & otherwise \end{cases} \quad (38)$$

or

$$\mu_{EP}(Q) = \begin{cases} \frac{EP(Q)-EP(Q')}{EP(Q_1^*)-EP(Q')}, & 0 \le Q < Q' \\ 0, & otherwise \end{cases} \quad (39)$$

With $EP(0) = 0$ and $EP(Q') = 0$, the membership function becomes

$$\mu_{EP}(Q) = \begin{cases} \frac{EP(Q)}{EP(Q_1^*)}, & 0 \le Q < Q' \\ 0, & otherwise \end{cases} \quad (40)$$

Q', the upper bound on the order quantity, is determined by (13).

The latest form of the membership function for the expected profit function is

$$\mu_{EP}(Q) = \begin{cases} \frac{(R+H)\frac{1}{\lambda}[1-exp(-\lambda Q)] - (C+H)Q}{(R+H)\frac{1}{\lambda}\left[1-\left(\frac{C+H}{R+H}\right)\right]+(C+H)\frac{1}{\lambda}ln\left(\frac{C+H}{R+H}\right)}, & 0 \le Q < Q' \\ 0, & otherwise \end{cases} \quad (41)$$

In the case of exponentially distributed demand, the satisfaction probability can be expressed by

$$\theta = \begin{cases} 0, & Q < \frac{T}{R-C} \\ 1 - G(x_T), & Q \ge \frac{T}{R-C} \end{cases} \quad (42)$$

or in detail

$$\theta = \begin{cases} 0, & Q < \frac{T}{R-C} \\ exp(-\lambda x_T), & Q \ge \frac{T}{R-C} \end{cases} \quad (43)$$

The maximum value of the satisfaction probability is gained at Q_2^*. It can be stated by

$$\theta^* = P\{Y(Q_2^*, X) \ge T\} \quad (44)$$

or more specifically

$$\theta^* = 1 - G(x_T^*) = \exp(-\lambda x_T^*) \quad (45)$$

where x_T^* is obtained from

$$x_T^* = \frac{[T+(C+H)Q_2^*]}{R+H} \quad (46)$$

From normalization of the satisfaction probability function, membership function for the satisfaction probability can be derived as the following:

$$\mu_\theta(Q) = \begin{cases} 0, & Q < \frac{T}{R-C} \\ \frac{[1-G(x_T)]-\theta^L}{\theta^*-\theta^L}, & Q \ge \frac{T}{R-C} \end{cases} \quad (47)$$

or

$$\mu_\theta(Q) = \begin{cases} 0, & Q < \frac{T}{R-C} \\ \frac{[1-G(x_T)]-\theta^L}{[1-G(x_T^*)]-\theta^L}, & Q \ge \frac{T}{R-C} \end{cases} \quad (48)$$

Because $\theta^L = \lim_{Q\to\infty} \theta = 0$, the membership function takes the following form:

$$\mu_\theta(Q) = \begin{cases} 0, & Q < \frac{T}{R-C} \\ \frac{\exp(-\lambda x_T)}{\exp(-\lambda x_T^*)}, & Q \ge \frac{T}{R-C} \end{cases} \quad (49)$$

As a result, the membership function for the satisfaction probability becomes

$$\mu_\theta(Q) = \begin{cases} 0, & Q < \frac{T}{R-C} \\ \exp(\lambda\left(\frac{C+H}{R+H}\right)\left[\frac{T}{R-C} - Q\right]), & Q \ge \frac{T}{R-C} \end{cases} \quad (50)$$

Under exponentially distributed demand, the model given with (18) can be expressed as the following:

$max\ \alpha$

s.t.

$$\alpha \leq \frac{(R+H)\frac{1}{\lambda}[1-\exp(-\lambda Q)] - (C+H)Q}{(R+H)\frac{1}{\lambda}\left[1-\left(\frac{C+H}{R+H}\right)\right]+(C+H)\frac{1}{\lambda}\ln\left(\frac{C+H}{R+H}\right)}$$

$$\alpha \leq \exp\left(\lambda\left(\frac{C+H}{R+H}\right)\left[\frac{T}{R-C} - Q\right]\right) \quad (51)$$

$$Q \geq \frac{T}{R-C}$$

7. Two Cases with Examples

Two cases below are to show use of the model for two distinct demand distributions. The function ”fmincon”, one of the optimization tools of MATLAB R2009b program, has been employed to solve developed models .

7.1. The case of uniformly distributed demand

For a hypothetical product with C=10, R=20, H=15 and T=150, and under uniformly distributed demand with a=10 and b=20, solution of the developed FBOP model has been obtained. The expected profit and the satisfaction probability as a function of the order quantity can be seen in the Figure 1 and the Figure 2, respectively. The Figure 3 shows the membership functions derived from the expected profit and the satisfaction probability functions, and the optimal solution point.

It can be said from the depiction of the optimal solution in Figure 3 that the best compromise solution to the problem is obtained on the order quantity of 15 units with the highest degree of compromise between the two objectives (0.91). That is, decision maker will achieve the highest degree of compromise between the expected profit and the probability of gaining at least 150 units by placing order of 15 units. However, since $(C+H)^2 \geq (R-C)^2$ condition holds, *Model − 1* is considered in solving the problem.

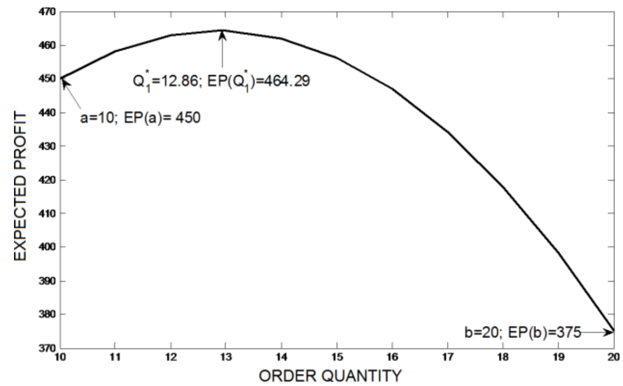

Fig. 1: The expected profit as a function of the order quantity.

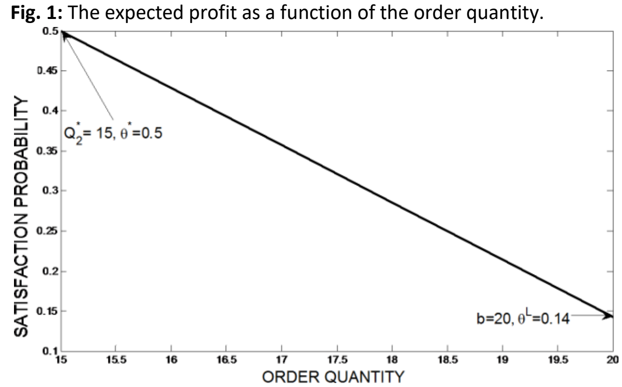

Fig. 2:.The Satisfaction probability function for the target profit value T=150 as a function of the order quantity.

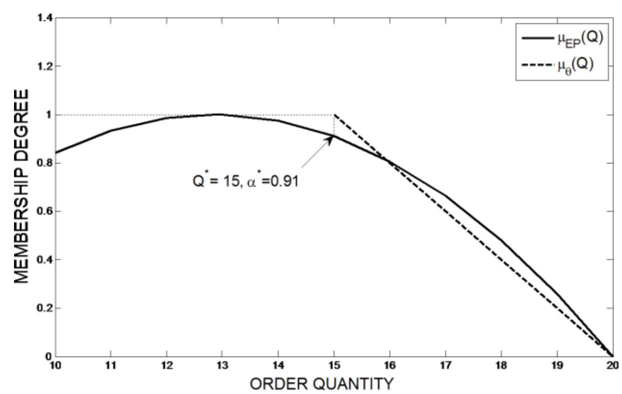

Fig. 3: The membership functions of the expected profit function and the satisfaction probability function, and the optimal solution point.

7.2. The case of exponentially distributed demand

A hypothetical product with C=10, R=20, H=15 , and two target profit levels have been considered. Under exponentially distributed demand (λ = 1/15), solutions of the developed FBOP models have been obtained. Figure 4 and Figure 5 display the expected profit and the satisfaction probability for T=25 as a function of order quantity, respectively. Figure 6 illustrates the membership functions obtained from the expected profit and the satisfaction probability functions for T=25 and T=50. The two best compromise solution points have also been indicated in Figure 6a and Figure 6b.

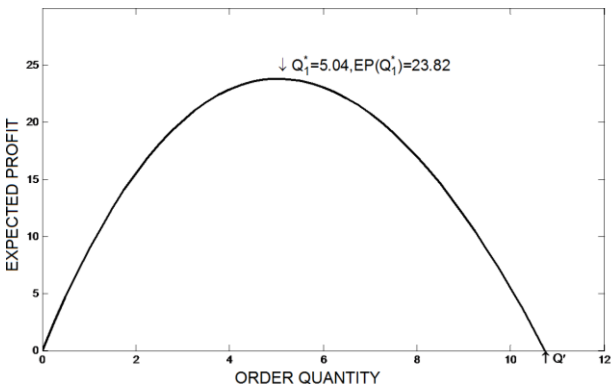

Fig. 4: The expected profit as a function of the order quantity.

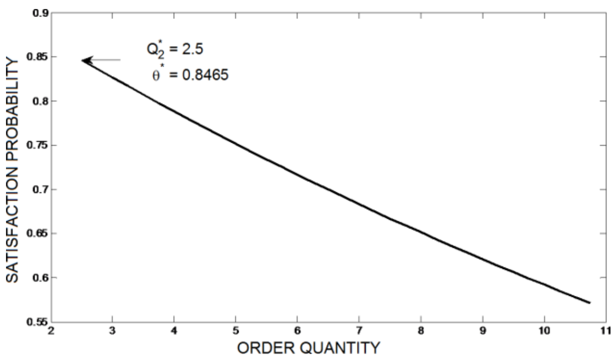

Fig. 5: The Satisfaction probability function for the target profit value T=25 as a function of the order quantity.

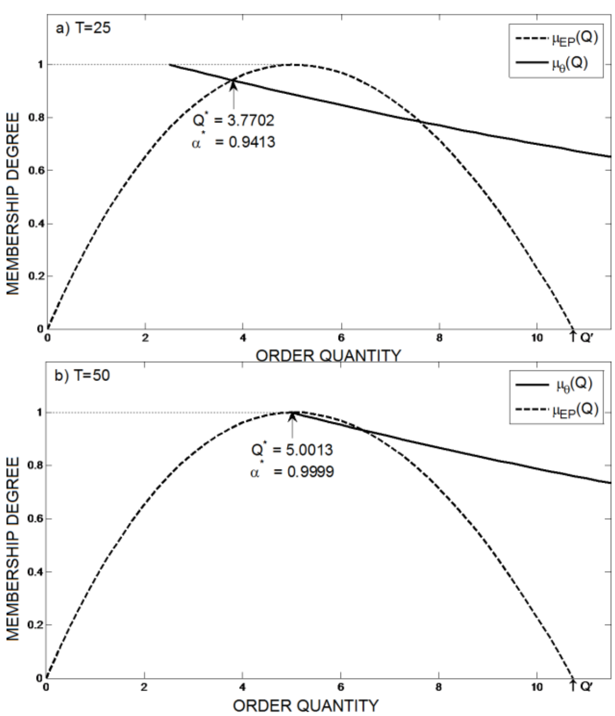

Fig. 6: The membership function for the expected profit function, the membership functions of the satisfaction probability functions for two target profit levels and the optimal order points: a) T=25, b) T=50

It can be stated from the pictures of the best compromise solutions in Figure 6a and 6b that the best compromise solutions to the two problems for T=25 and T=50 are acquired on the order quantity of 3.7702 and 5.0013 units, respectively with the highest degrees of compromise 0.9413 and 0.9999, respectively. That is, decision maker will get the highest degree of compromise between the expected profit and the probability of gaining at least 25 units by placing order of 3.7702 units, and the highest degree of compromise between the expected profit and the probability of gaining at least 50 units by placing order of 5.0013 units. Figure 6b also indicates that the individual best values of the two objectives are almost achieved by the given order quantity.

8. Conclusion

In the study, the probability of exceeding a target profit level (satisfaction probability) has been considered as a performance measure beside the expected profit. Membership functions have been derived from the performance measures by normalization. The membership functions derived denote the degree of approaching optimal value of the expected profit and the degree of approaching optimal value of the satisfaction probability for a target profit level. A FBOP model has been developed to get best compromise solution. This model has been solved based on the notion of decision making in a fuzzy environment proposed by Bellman and Zadeh.

Two cases have been presented to demonstrate use of the proposed model under uniform and exponential demand. Results indicated that the FBOP model produces valid compromise solutions and this attempt is successful. Demand distributions other than uniform and exponential can be used, and different objectives like service rate may be added to the model.

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