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Seiberg-Witten-Like Equations on 8-Manifolds without Self-Duality

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Abstract

In this paper, Seiberg−Witten−like equations without self−duality are defined on 8 −dimensional manifolds. Then, non−trivial and flat solutions are given to them on $\mathbb{R}^8$. Finally, on 8 −real−dimensional Kähler manifolds a global solution to these equation is obtained for a given negative and constant scalar curvature.
Keywords: Seiberg−Witten equations; $Spin$ and $Spin^c$ geometry; Curvature; Without self−duality.

1. Introduction

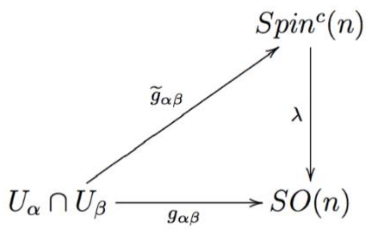
On the 4 −manifolds, Seiberg−Witten equations introduced by E. Witten are consisted of Dirac equation and Curvature equation [9]. These equations provide information about the topology and geometry of the 4 −manifolds [3,4,6,8,9]. To define these equations, one needs two entities as an $i\mathbb{R}$ valued connection 1 −form and a spinor field. Dirac equation can be defined on any manifold endowed with $Spin^c$ −structure. But, defining the curvature equation needs self−duality concept of two form. Since self−duality concept is meaningful only in 4 −dimension, generalized self−duality concept is given to define the curvature equation on a noun four− dimensional manifold. Accordingly, Seiberg−Witten equations are investigated up to 4 −dimensional manifolds by defining generalized self−duality concept [1,5]. On 8 −manifolds, Seiberg−Witten−like equations have been studied in [1,2,5] depending on the $Spin$ and $Spin^c$ −structure. In [1], the author defined Seiberg−Witten−like equations on the $Spin$ manifold with respect to the generalized self−duality concept and gives them local solutions. Then, in [2] these equations are constructed on the $Spin^c$ manifolds and non−trivial local solutions are given to them. Finally, the global solutions of these equations are given on the 8−manifolds endowed with $SU(4)$ −structure in [5].

The purpose of this paper is planning to give in two part. One of them is to write down Seiberg−Witten−like equations without using the self−duality concept on the 4 −manifolds and to show similarities with the classical Seiberg−Witten equations. The other one is to define these equations on the 8 −manifolds without using the self−duality concept and to obtain a non−trivial flat solution on the 8 −dimensional Riemannian manifolds. Also, to give them a global solution on the 8 −real−dimensional Kähler manifold for a given negative and constant scalar curvature.

2. Materials and Methods

2.1 $Spin^c$ −structure and Dirac operator

Suppose that M is an orientable Riemannian manifold. Hence, there exist an open covering $\{U_\alpha\}_{\alpha\in\mathcal{A}}$ of M with the transitions functions $g_{\alpha\beta}: U_\alpha \cap U_\beta \longrightarrow SO(n)$ for TM. If there exists another collection of transition functions $\tilde{g}_{\alpha\beta}: U_\alpha \cap U_\beta \longrightarrow Spin^c(n)$ such that the following diagram commutes



That is, $\lambda \circ \tilde{g}_{\alpha\beta} = g_{\alpha\beta}$ and the cocycle condition $\tilde{g}_{\alpha\beta}(x) \circ \tilde{g}_{\beta\gamma}(x) = \tilde{g}_{\alpha\gamma}(x)$ on $U_\alpha \cap U_\beta \cap U_\gamma \neq \emptyset$ is satisfied, then M is called $Spin^c$ manifold.

On a $Spin^c$ manifold, one can construct $P_{SO(n)}$, $P_{Spin^c(n)}$ and P_{S^1} principal bundles by using principal bundle construction lemma [7]. Also, by using P_{S^1} principal bundle one can construct determinant line bundle

$$\mathcal{L} \coloneqq P_{Spin^c(n)} \times_l \mathbb{C} = P_{S^1} \times_{U(1)} \mathbb{C}$$

where

$$l_{\alpha\beta} = l: U_\alpha \cap U_\beta \longrightarrow Spin^c(n).$$

Moreover, an associated complex vector bundle $\mathbb{S} = P_{Spin^c(n)} \times_{\kappa_n} \Delta_n$ can be constructed by considering spinor representations

$$\kappa_n: Spin^c(n) \longrightarrow Aut(\Delta_n)$$

where $\Delta_n = \mathbb{C}^{2^{\frac{n}{2}}}$. If the dimension of M is even, then $\mathbb{S}$ spinor bundle splits into two pieces $\mathbb{S} = \mathbb{S}^+ \oplus \mathbb{S}^-$ [4]. The sections of the complex vector bundle are called spinor fields. On the complex vector bundle $\mathbb{S}$ one can define Hermitian inner product as follows:

$$<,>: \Gamma(\mathbb{S}) \times \Gamma(\mathbb{S}) \longrightarrow \mathbb{C}$$
$$([p,\Psi],[p,\Phi]) \mapsto <\Psi,\Phi> = \overline{\Psi}\cdot\Phi. \quad (2.1)$$

By using Hermitian inner product defined in (2.1) one can associate each spinor Ψ to an endomorphism of $\mathbb{S}$ by the formula

$$\Psi\Psi^*: \mathbb{S} \longrightarrow \mathbb{S}$$
$$\tau \mapsto <\Psi,\tau>\Psi.$$

Following bundle homomorphisms are useful while studying on spinors. Extended map of κ_n is defined by

$$\kappa: TM \longrightarrow End(\mathbb{S}).$$

Some authors called the map κ a $Spin^c$ −structure on the manifold M [8].
The Clifford multiplication with X is defined

$$X\cdot\Psi := \kappa(X)(\Psi)$$

where $X \in \Gamma(TM)$ and $\Psi \in \Gamma(\mathbb{S})$.

A spinor covariant derivative operator ∇^A is obtained by using an $A: TP_{S^1} \longrightarrow i\mathbb{R}$, $i\mathbb{R}$ −valued 1 −form in the principal bundle P_{S^1} and Levi−Civita connection ∇ on M as follows

$$\nabla^A{}_X\Psi = \mathrm{d}\Psi(\mathrm{X}) + \frac{1}{2}\sum_{i<j}\omega_{ij}(X)e_i\cdot e_j(\Psi) + \frac{1}{2}A(X)(\Psi)$$

where $\Psi \in \Gamma(\mathbb{S})$ and $X \in \Gamma(TM)$.

Now we can define the Dirac operator locally as follows.

Definition 1: Let $e = \{e_1, e_2, \dots, e_n\}$ be any local orthonormal frame on $U \subset M$. Then the local expression of the Dirac operatör $D_A = \Gamma(\mathbb{S}) \rightarrow \Gamma(\mathbb{S})$ is

$$D_A\Psi = \sum_{i=1}^{n} e_i.\nabla^A_{e_i}\Psi$$

where $\Psi \in \Gamma(\mathbb{S})$ and $A \in \Omega^1(M, i\mathbb{R})$. Dirac operator decomposes into $D_A = D_A^+ \oplus D_A^-$ in the case of dimension of M is even.

By using κ, another bundle map ρ associated each 2 −form to an endomorphism of $\mathbb{S}$, can be defined on the orthonormal frame $\{e_1, e_2, \dots, e_n\}$ as follows

$$\rho: \Lambda^2(T^*M) \longrightarrow End(\mathbb{S})$$
$$\eta = \sum_{i<j}\eta_{ij}e_i \wedge e_j \mapsto \rho(\eta) = \sum_{i<j}\eta_{ij}\kappa(e_i)\kappa(e_j).$$

Also ρ can be extend to a complex valued 2 −forms [8] such that

$$\rho: \Lambda^2(T^*M)\otimes\mathbb{C} \longrightarrow End(\mathbb{S}).$$

Also ρ can be defined on the half spinor bundles $\mathbb{S}^{\pm}$. The half−spinor bundles $\mathbb{S}^{\pm}$ are invariant under $\rho(\eta)$ for all $\eta \in \Lambda^2(T^*M)$. That is,

$$\rho(\eta)(\Psi) \in \mathbb{S}^+, \forall\Psi \in \mathbb{S}^+$$
$$\rho(\eta)(\Psi) \in \mathbb{S}^-, \forall\Psi \in \mathbb{S}^-.$$

Then, we obtain the following maps by restriction
$\rho^+(\eta) = \rho(\eta)|_{S^+}$, $\rho^-(\eta) = \rho(\eta)|_{S^-}$. In this case

$$\rho^+: \Lambda^2(T^*M)\otimes\mathbb{C} \longrightarrow End(\mathbb{S}^+)$$

is expressed as follows:

$$\rho^+(\eta) = \rho^+\left(\textstyle\sum_{i<j}\eta_{ij}e_i \wedge e_j\right) = \textstyle\sum_{i<j}\eta_{ij}\kappa(e_i)\kappa(e_j).$$

Note that, the space of $i\mathbb{R}$ −valued 2 −forms $\Lambda^2(\mathrm{M}, \mathrm{i}\mathbb{R})$ is a sub bundle of $\Lambda^2(\mathrm{M}, \mathrm{i}\mathbb{R}) \times \mathbb{C}$. We consider the sub bunde $W = \rho^+(\Lambda^2(\mathrm{M}, \mathrm{i}\mathbb{R}))$ of $\mathrm{End}(\mathbb{S})$ to define curvature equation.

In order to be able to give a global solution for the Seiberg−Witten−like equation defined without self−duality, the manifold must be endowed with $SU(4)$ −structure. That guarantees the existence of a Hermitian metric compatible with the complex structure on a Hermitian manifold. On the Hermitian manifold one can construct canonical $Spin^c$ −structure and by using this structure spinorial bundle can be defined with a spinorial connection. Also, Dirac operator is associated with such a connection. As a result Seiberg−Witten−like equation without self−duality can be defined on such manifold and a global solution can be given to it.

In the following, before the global solution is given, a short brief of the Kähler manifolds is given.

2.2 Kähler Manifolds

On the 8 −manifolds endowed with $SU(4)$ −structure, there exists an almost complex structure satisfying $J: TM \longrightarrow TM, J^2 = -I_d$.

A smooth manifold endowed with an almost complex structure is called an almost complex manifold and denoted by (M, J).

The almost complex structure J acts on the space of 1 −forms as follows:

$$J: TM \longrightarrow \mathrm{TM}$$
$$\omega \mapsto J(\omega)(X) := \omega(JX)$$

where $\omega \in \Gamma(T^*M)$ and $X \in \Gamma(TM)$. Moreover, J acts on the complexification of the cotangent bundle of M as

$$J: T^*M \otimes_{\mathbb{R}} \mathbb{C} \longrightarrow J: T^*M \otimes_{\mathbb{R}} \mathbb{C}$$
$$\omega\otimes Z \mapsto J(\omega)\otimes Z.$$

Since $J^2 = -I_d$, $\pm i$ are eigenvalues of J. Then $T^*M \otimes_{\mathbb{R}} \mathbb{C}$ is the direct sum of

$$T^*M \otimes_{\mathbb{R}} \mathbb{C} = \Lambda^{1,0}(M) \oplus \Lambda^{0,1}(M)$$

where

$$\Lambda^{1,0}(M) = \{Z \in T^*M \otimes_{\mathbb{R}} \mathbb{C} \mid JZ = iZ\}$$
$$\Lambda^{0,1}(M) = \{Z \in T^*M \otimes_{\mathbb{R}} \mathbb{C} \mid JZ = -iZ\}.$$

The space of r −forms is given as:

$$\Lambda^r(M) = \sum_{a+b=r}\Lambda^{a,b}(M)$$

where
$\Lambda^{a,b}(M) = span\{x \wedge y | x \in \Lambda^a(\Lambda^{1,0}(M)), y \in \Lambda^b(\Lambda^{0,1}(M))\}$
is the space of (a, b) type complex forms. Finally, Kähler manifold is defined as follows.

Definition 2: Let (M, J) be an almost complex manifold. Then, a Riemannian metric g is called Hermitian metric if it is compatible with the almost complex structure J:

$$g(JX, JY) = g(X, Y)$$

where $X, Y \in \Gamma(TM)$.

The associated smooth $2-$form Φ defined by

$$\Phi(X,Y)=g(X,JY)$$

is called the Kähler $2-$form and satisfies $\Phi(JX,JY)=\Phi(X,Y)$. If Φ is closed then M is called Kähler Manifold and the metric on M is called a Kähler metric.

2.3 Dirac operator on the Kähler Manifolds

In this section, we talk about the canonical $Spin^c$ $-$structure of a Kähler manifold and its spinor bundle with associated connection. Since the structure group of any Kähler manifold of dimension n is $U(n)$, it admits a canonical $Spin^c$ $-$structure given by:

$$P_{Spin^c(n)}=P_{U(n)}\times_F Spin^c(2n)$$

where $F: U(n)\longrightarrow Spin^c(2n)$ is the lifting map [4]. The associated canonical spinor bundle then has the form:

$$\mathbb{S}\cong\Omega^{(0,*)}(M)$$

where $\Omega^{(0,*)}(M)$ is the direct sum of $\Omega^{(0,1)}(M)\oplus\Omega^{(0,2)}(M)\oplus...\oplus\Omega^{(0,i)}(M),\ i\in\mathbb{N}$. There are two ways to include a spinorial Levi$-$Civita connection on $\mathbb{S}$.

The first is obtained by the extension of the connection to forms and the latter is obtained via $Spin^c$ $-$ structure. In this work, we mainly focused on the canonical $Spin^c$ $-$structure with the following isomorphism:

$$\mathbb{S}\cong\Omega^{(0,*)}(M).$$

On this bundle, we described Dirac operator defined on $\mathbb{S}$ and we give the relation with the Dirac$-$type operator defined on $\Omega^{(0,*)}(M)$.

In the case of Kähler manifold endowed with a canonical $Spin^c$ $-$structure, there is a spinorial connection ∇^A on the associated spinor bundle $\mathbb{S}$ induced by an unitary connection $1-$form A on the determinant line bundle $\mathcal{L}$ together with the spinorial Levi$-$Civita connection ∇. Also, on the associated spinor bundle one can describe Dirac operator as follows:
Let $\{e_i\}$ $i=1,\dots,n$ be a local orthonormal frame on M. Then the Dirac operator D^A is given by:

$$D_A\Psi=\sum_{\mathrm{i}=1}^{n} e_i.\nabla^A_{e_i}\Psi.$$

Moreover, by considering Kähler manifolds with $\Omega^{(0,*)}(M)$ associated spinor bundle the Dirac type operator is defined as follows:
Let
$\bar{\partial}:\Omega^{0,r}(M)\longrightarrow\Omega^{0,r+1}(M),\ \bar{\partial}^*:\Omega^{0,r}(M)\longrightarrow\Omega^{0,r-1}(M),$
given by:
$\overline{\partial_0}=\sum_{i=1}^{n}\overline{Z_\iota}^*\wedge\nabla_{\overline{Z_\iota}},\ \ \overline{\partial_2}^*=-\sum_{i=1}^{n}\iota(\overline{Z_\iota})^*\wedge\nabla_{\overline{Z_\iota}}$ respectively, where ∇ is the extension of the Levi$-$Civita connection to $\Omega^{(0,*)}(M)$ and ι is the contraction operator.
Since $\mathbb{S}\cong\Omega^{(0,*)}(M)$, one has

$$D_{A_0}=\sqrt{2}\,(\overline{\partial_0}+\overline{\partial_2}^*)\qquad(2.3)$$

where A_0 is the Levi$-$Civita connection of the line bundle $L=\Lambda^2(M)$ of the canonical $Spin^c$ $-$structure.

2.4 Seiberg$-$Witten Equations Without Self$-$Duality on the $n-$ Manifolds

Definition 3: Let (M,g) be a $n-$dimensional $Spin^c$ manifold. Then Seiberg$-$Witten Like equations for the pair$(A,\Psi)$is given by

$$D_A\Psi=0,\ \text{Dirac Equation}$$

$$\rho^+(F_A)=\tfrac{1}{2}(\Psi\Psi^*)^+,\ \text{Curvature Equation}\qquad(2.4)$$

where F_A is the curvature of A and $(\Psi\Psi^*)^+$ is the orthogonal projection of $\Psi\Psi^*$ onto $W=\rho^+(\Omega^2(\mathrm{M},\mathrm{i}\mathbb{R}))$. In the local orthonormal frame $\{e_1,\dots,e_n\}$,

$$\begin{aligned}(\Psi\Psi^*)^+ &= Proj_W(\Psi\Psi^*)\\ &=\sum_{i<j}\frac{<\rho^+(e^i\wedge e^j),\Psi\Psi^*>}{<\rho^+(e^i\wedge e^j),\rho^+(e^i\wedge e^j)>}\rho^+(e^i\wedge e^j).\end{aligned}$$

3. Results and Discussion

In this section, we write down the Seiberg$-$Witten$-$Like equation on 4 and $8-$dimensional manifolds. Then we compare the solution of these equations with the solution of classical Seiberg$-$Witten equations on $\mathbb{R}^4$ [8,9]. Finally, we give a global solution to these equations on $8-$manifolds.

3.1 Seiberg$-$Witten$-$like equation on $\mathbb{R}^4$

In $M=\mathbb{R}^4$ case, the explicit form of the Dirac operator with respect to the $Spin^c(4)$ $-$structure is given as follows:

$$\frac{\partial\psi_1}{\partial x_1}x_4+A_1\psi_1=i\left(\frac{\partial\psi_1}{\partial x_2}+A_2\psi_1\right)+\frac{\partial\psi_2}{\partial x_3}+A_3\psi_2$$
$$+i\left(\tfrac{\partial\psi_2}{\partial x_4}+A_4\psi_2\right),$$

$$\frac{\partial\psi_2}{\partial x_1}+A_1\psi_2=-i\left(\frac{\partial\psi_2}{\partial x_2}+A_2\psi_2\right)-\frac{\partial\psi_1}{\partial x_3}-A_3\psi_1$$
$$+i\left(\tfrac{\partial\psi_1}{\partial x_4}+\tfrac{1}{2}A_4\psi_1\right).$$

The second equation of the Seiberg$-$Witten$-$like equations without self$-$duality is obtained as follows

$$F_{12}+F_{34}=-\frac{i}{2}(|\psi_1|^2-|\psi_2|^2),$$

$$F_{13}-F_{24}=\frac{1}{2}(\psi_1\overline{\psi_2}-\psi_2\overline{\psi_1}),$$

$$F_{14}+F_{23}=-\frac{i}{2}(\psi_1\overline{\psi_2}+\psi_2\overline{\psi_1}).$$

Notice that, in the case of $M=\mathbb{R}^4$ Seiberg$-$Witt-ten$-$like equations without self$-$duality coincide with the classical Seiberg$-$Witen equations [8,9].

In the following we define the Seiberg$-$Witten$-$like equations on $8-$manifolds.

3.2 Seiberg$-$Witten$-$ like equation on $\mathbb{R}^8$

By considering the following $Spin^c$ $-$structure [5]:

$$\kappa_8:\mathbb{R}^8\longrightarrow\mathbb{C}^{16},$$

$$\kappa_8(e_i) = \begin{bmatrix} 0 & \mu(e_i) \\ -\mu(e_i) & 0 \end{bmatrix} \quad (3.1)$$

where e_i, $i = 1, \dots, 8$ is the standard basis of $\mathbb{R}^8$, $\mu(e_1) = I_d$ is a 8×8 identity matrix , and for $i = 1, \dots, 8$ explicit form of $\mu(e_i)$ are given by

$\mu(2) = I_2 \otimes I_2 \otimes m_1$, $\mu(3) = I_2 \otimes I_2 \otimes m_2$,
$\mu(4) = iI_2 \otimes m_1 \otimes m_1 m_2$, $\mu(5) = iI_2 \otimes m_2 \otimes m_1 m_2$,
$\mu(6) = -m_1 \otimes m_1 m_2 \otimes m_1 m_2$, $\mu(7) = -m_2 \otimes m_1 m_2 \otimes m_1 m_2$,
$\mu(8) = -m_1 m_2 \otimes m_1 m_2 \otimes m_1 m_2$

where I_2 is a 2×2 identity matrix and

$$m_1 = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}, \quad m_2 = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}.$$

By using (3.1) one can obtain explicit form of (2.4) and the solution of these equation can be obtained by the following pair,

$$A = \sum_{i=1}^{8} -2ix_i \, dx^i$$

and

$$\Psi = \left(0,0,0, e^{\sum_{j=1}^{8} -\frac{1}{2}x_j^2}, 0,0, e^{\sum_{j=1}^{8} -\frac{1}{2}x_j^2}, 0\right).$$

Here (A, Ψ) is the local, non−trivial but flat $(ie. F_A = 0)$ solution of the Seiberg−Witten−like equation without self−duality with respect to $M = \mathbb{R}^8$.

In the next subsection, a global solution to the Seiberg−Witten−like equations without self−duality is given on 8 −real−dimensional Kähler Manifolds.

3.3 Seiberg−Witten−Like Equations on the 8 −Real−Dimensional Kähler Manifold

Let (M, g, J) be a 8 −real−dimensional Kähler manifold endowed with a canonical $Spin^c$ −structure and e_1, $e_2 = J(e_1)$, e_3, $e_4 = J(e_3)$, e_5, $e_6 = J(e_5)$, e_7, $e_8 = J(e_7)$, be a local orthonormal frame with the dual basis $\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$. Then the Kähler 2 −form has the form

$$\Phi = e_1 \wedge e_2 + e_3 \wedge e_4 + e_5 \wedge e_6 + e_7 \wedge e_8.$$

Under the action Φ, one gets the following decomposition

$$\mathbb{S} = \mathbb{S}_0 \oplus \mathbb{S}_1 \oplus \mathbb{S}_2 \oplus \mathbb{S}_3 \oplus \mathbb{S}_4,$$

where

$$\begin{aligned} \mathbb{S}_0 &= \{\Psi \in \mathbb{S} | \Phi\,\Psi = 4i\Psi\}, \\ \mathbb{S}_1 &= \{\Psi \in \mathbb{S} | \Phi\,\Psi = 2i\Psi\}, \\ \mathbb{S}_2 &= \{\Psi \in \mathbb{S} | \Phi\,\Psi = 0\}, \\ \mathbb{S}_3 &= \{\Psi \in \mathbb{S} | \Phi\,\Psi = -2i\Psi\}, \\ \mathbb{S}_4 &= \{\Psi \in \mathbb{S} | \Phi\,\Psi = -4i\Psi\}. \end{aligned}$$

Accordingly, $f: i^4 e_1 \cdot e_2 \cdot e_3 \cdot e_4 \cdot e_5 \cdot e_6 \cdot e_7 \cdot e_8 : \mathbb{S} \longrightarrow \mathbb{S}$ endomorphism, the complex spinor bundle $\mathbb{S}$ splits into

$$\mathbb{S} = \mathbb{S}^+ \oplus \mathbb{S}^-$$

where

$$\mathbb{S}^+ = \mathbb{S}_0 \oplus \mathbb{S}_2 \oplus \mathbb{S}_4 \cong \Lambda^{0,4}(M) \oplus \Lambda^{0,2}(M) \oplus \Lambda^{0,0}(M),$$
$$\mathbb{S}^- = \mathbb{S}_0 \oplus \mathbb{S}_3 \cong \Lambda^{0,3}(M) \oplus \Lambda^{0,1}(M).$$

Let Ψ_0 be a spinor in $\mathbb{S}_4 \cong \Omega^{0,0}(M)$ corresponding to constant function 1 in the chosen coordinates

$$\Psi_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$ By using Ψ_0, one has

$$\frac{(\Psi_0 \Psi_0{}^*)^+}{2} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{4} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} \end{bmatrix}.$$

Let A_0 be the connection on the S^1 −principal bundle P_{S^1} induced by means of the Levi−Civita connection ∇ in the line bundle $L = \Omega^{0,2}(M)$ of the canonical $Spin^c$ −structure [4]. Accordingly, the corresponding Dirac operator $D_{A_0}: \Gamma(\mathbb{S}^+) \longrightarrow \Gamma(\mathbb{S}^-)$ coincides with $\sqrt{2}(\overline{\partial_0} \oplus \overline{\partial_2}^*)$. Also, the curvature of the connection 1 −form A_0 is given by

$$F_{A_0} = i\rho_{ric} \quad (3.2)$$

where $\rho(X,Y) = (X,Y) = g(X, J \circ Ric(Y))$ and $Ric: TM \longrightarrow TM$ denotes the Ricci tensor. Since the almost complex structure J and the Ricci tensor Ric commute, one has

$$\begin{aligned} \rho_{ric} = & -R_{11} e_1 \wedge e_2 - R_{33} e_3 \wedge e_4 - R_{13}(e_1 \wedge e_4 - e_2 \wedge e_3) \\ & + R_{14}(e_1 \wedge e_3 - e_2 \wedge e_4) - R_{15}(e_1 \wedge e_6 - e_2 \wedge e_5) \\ & + R_{16}(e_1 \wedge e_5 + e_2 \wedge e_6) - R_{17}(e_1 \wedge e_8 - e_2 \wedge e_7) \\ & + R_{18}(e_1 \wedge e_7 + e_2 \wedge e_8) - R_{35}(e_3 \wedge e_6 - e_4 \wedge e_5) \\ & + R_{36}(e_3 \wedge e_5 + e_4 \wedge e_6) - R_{37}(e_3 \wedge e_8 - e_4 \wedge e_7) \\ & + R_{38}(e_3 \wedge e_7 + e_4 \wedge e_8) - R_{55} e_5 \wedge e_6 - R_{77} e_7 \wedge e_8 \\ & - R_{57}(e_5 \wedge e_8 - e_6 \wedge e_7) + R_{58}(e_5 \wedge e_7 - e_6 \wedge e_8). \end{aligned}$$

In the following a global solution is given for the appropriate Ricci tensor.

Theorem 1.
Let (M, g, J) be an 8 −real−dimensional Kähler manifold. Then for a given negative and constant scalar curvature s $(A_0, \Psi = \sqrt{-2s}\Psi_0)$ is the solution of the Seiberg−Witten−like equations without self−duality.

Proof. Since $\Psi = \sqrt{-2s}\,\Psi_0 \in \Omega^{0,0}(M)$ and Ψ is the spinor field corresponding to the constant function 1, by using (2.3), one gets $D_{A_0} \equiv 0$. Satisfying the curvature equation remains. To achive this, Ric must be taken as follows:

$$Ric = [R_{ij}]_{8\times 8} = \begin{cases} \frac{S}{8} & i = j \\ 0 & i \neq j, \end{cases}$$

where s is the negative and constant. By using Ric in (3.2), one gets $\rho^+(F_{A_0}) = i\rho^+(\rho_{ric})$ which means $\rho^+(F_{A_0}) = \frac{(\Psi\Psi^*)^+}{2}$.

4. Conclusion

We give a global solution to the Seiberg−Witten−like equations on 8 −real−dimensional Kähler manifolds for a given negative and constant scalar curvature.

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