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Research Article

Comparison of Dynamic Characteristics of Prestressed and Reinforced Concrete Beams

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ABSTRACT

To determine the dynamic characteristics of bridges built with prestressed and reinforced concrete beams, the dynamic properties of such beams should be known. Prestressing force is applied to the prestressed concrete (PSC) beam, unlike reinforced concrete (RC) beam. In this study, it is aimed to compare the dynamic properties of PSC and RC beams with the same material, section properties and effective span length. Dynamic properties such as the mode shapes and periods of the PSC and RC beam were determined by means of the formulation found in the literature and a computer program that uses the finite element method. For this study I-beam with 0.90m height and 15m effective span length was selected as an example. The selected beam was considered separately as PSC and RC. In the PSC beam, eight low-relaxation Grade270 prestressing strand with 15mm (0.6 in.) diameter were used, unlike reinforced concrete beams. Three dimensional finite element models (FEM) of PSC and RC beam were constituted using SAP2000 software. At the end of the study, mode shapes and periods of PSC and RC beams obtained from analytical prediction by formulation and numerical by FEM were compared with each other.

Keywords: *Prestressed beam, Reinforced beam, Natural frequency, Mode shapes, Prestressing force*

Öngerilmeli ve Betonarme Kirişlerin Dinamik Karakteristiklerinin Karşılaştırılması

ÖZET

Öngerilmeli ve betonarme kirişler kullanılarak inşa edilen köprülerin dinamik karakteristiklerini belirlemek için köprülerde kullanılan kirişlerin dinamik özelliklerinin bilinmesi gerekmektedir. Öngerilmeli kirişlere, betonarme kirişlerden farklı olarak öngerilme kuvveti uygulanmaktadır. Bu çalışmada malzeme, kesit özellikleri ve efektif açıklık değerleri aynı olan öngerilmeli ve betonarme kirişlerin dinamik karakteristiklerinin karşılaştırılması amaçlanmıştır. Öngerilmeli ve betonarme kirişlerin dinamik karakteristiklerinden mod şekilleri ve periyodların belirlenmesinde, literatürde bulunan bağıntılar ve sonlu elemanlar yöntemine dayalı bilgisayar programı kullanılmıştır. Bu çalışma için 15m uzunluğa 0.90m yüksekliğe sahip I kiriş örnek olarak seçilmiştir. Seçilen kiriş, öngerilmeli ve betonarme olarak ayrı ayrı dikkate alınmıştır. Öngerilmeli kirişte, betonarme kirişten farklı olarak 15mm (0.6in.) çaplı sekiz tane 270K sınıfı öngerme kablosu kullanılmıştır. Kirişlerin sonlu eleman modelleri

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SAP2000 programı kullanılarak oluşturulmuştur. Çalışmanın sonunda öngerilmeli ve betonarme kirişlerin analitik ve sayısal olarak elde edilen dinamik karakteristikleri birbiriyle karşılaştırılmıştır.

Anahtar Kelimeler: *Öngerilmeli kiriş, Betonarme kiriş, Doğal frekans, Mod şekilleri, Öngerme kuvveti*

I. INTRODUCTION

Prestressed and reinforced concrete beams are widely used in the construction of bridges around to world owing to advantages of these beams such as, high stability, lower construction and maintain cost and serviceability. Bridges are indispensable components of transportation network and their construction cost is very high from other components. Damaging of bridges cause disconnection of roads also loss of life and property for these reasons, understanding of real structural behavior of bridges is becoming important for engineers [1-3]. The growing interest of the scientific society on the structural health of the structure, it is very important to determine the modal parameters. Natural frequencies and mode shapes are important dynamic characteristics of structures. To determine these parameters of bridges built with prestressed concrete (PSC) or reinforced (RC) beams, the dynamic properties of such beams should be known in order to determine dynamic behavior and current state of the bridge for damage detection. PSC beams are subjected to prestressing force different from RC beams. Dynamic behavior of RC beams are available in many textbooks but the effect of prestressing force on dynamic behavior of PSC beam has been debated topic and still there is no agreement about how prestressing force affect the dynamic behavior of PSC girder. The natural frequency of a simply supported RC beam is

$$w_n = \frac{n^2 x \pi^2}{l^2} \sqrt{\frac{EI}{m}} \tag{1}$$

where *EI* is the flexural rigidity of the girder, *l* is the length of the beam, *m* is the mass per unit length of the beam, *n* is the mode number. Eqs. (1) derives from the dynamic analysis of simple supported beam with Euler-Bernoulli equations. Determination of the natural frequencies of simply supported RC beam with Eqs. (1) is acceptable in literature. However there is a contradictory result reported on determination of natural frequencies of simply supported PSC beam. Some report state that prestressing force affects the dynamic behavior of PSC girder. Tse et al. [4], Saidi et al. [5] and Chang and Yung [6] emphasized that compression softening, is a gradual decrease of mechanical resistance due to a continuous increase of deformation forced upon a concrete, affect the dynamic characteristic of prestressed concrete beam and the higher level of prestressing force causes to reduce the natural frequencies of the beam. Grace and Ross [7] pointed out that the modal parameters especially natural frequencies of girder is affected from the level of prestress force and location of prestressing tendon at the bottom flange of the girder. However modal shapes are less affected such parameters. Miyamoto et al. [8] struggled to determine natural frequencies and mode shapes prestressed beam which was strengthened with external tendons. They have concluded that natural frequencies shows decreasing tendency when the magnitude of prestressing force boost. The natural frequency of a simply supported axially compressed beam is [9]

$$w_n = \frac{n\pi}{l}\sqrt{\frac{1}{m}\left[EI\left(\frac{n\pi}{l}\right)^2 - N\right]} \tag{2}$$

where N is the axial compressive force. Eqs. (2) indicates that when the amount of the prestress force rise, the natural frequency of axially compressed simple supported tend to decreases. As seen from many reports, the prestressing force has effect on the natural frequencies of PSC beam, whereas some report state that the prestressing force has no effect (Deak [10], Hamed and Frostig [11], Pavic et al. [12], Noble et al. [13], Fengge and Rong [14]).

The main goal of the paper is to describe and compare the dynamic behavior of the PSC and RC beam. To achieve this goal, dynamic properties such as the mode shapes and periods of the PSC and RC beam were determined by means of the formulation found in the literature and a computer program that uses the finite element method. For this purpose I-beam with 0.90m height and 15m effective span length was selected as an example. The selected beam was considered separately as PSC and RC. In the PSC beam, eight low-relaxation Grade270 prestressing strand with 15mm (0.6in.) diameter were used, unlike reinforced concrete beams. Three dimensional finite element models (FEM) of PSC and RC beam were constituted using SAP2000 software [15]. To obtain dynamic characteristic of beams, only linear modal analysis was performed for RC beam, but both linear and nonlinear modal analysis performed for PSC beam to determine the effect of prestressing force. For this reason geometric nonlinearity was taken into account in prestress loads case and stiffness at the end of this case was used in nonlinear modal analyses of PSC beam model.

II. PRESTRESSED AND REINFORCED CONCRETE BEAM MODELS

Simply supported PSC and RC beam with 90cm height and 15m effective span length was selected as an application (Fig.1 and Table 1). The ultimate strength of concrete (f_c) was taken as 40MPa. Prestressing force was applied to the PSC beam, unlike RC beam. For this reason, in the PSC beam eight low-relaxation Grade 270 prestressing strand 15mm (0.6 in.) in diameter were used, unlike RC beams. Strands layout along the girder length was assumed as linear. The distance between strands (6 cm) was selected according to AASHTO LRFD [16]. The modulus of elasticity, passion ratio and density of concrete and strand are shown in Table 2.

***Table 1.** Parameters of girder*

Cross-Sectional Dimensions (cm)								
A	B	C	D	E	F	G	H	J
90	50	15	80	10	7.5	50	7.5	15

***Table 2.** Material properties considered in the numerical analysis*

Material	**Modulus of Elasticity (MPa)**	**Poisson's Ratio**	**Density (kg/m^3)**
Concrete	33836	0.2	2500
Strand	193053	0.3	7850

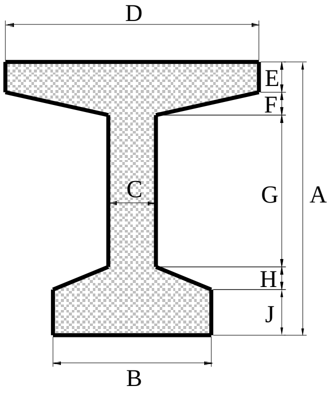
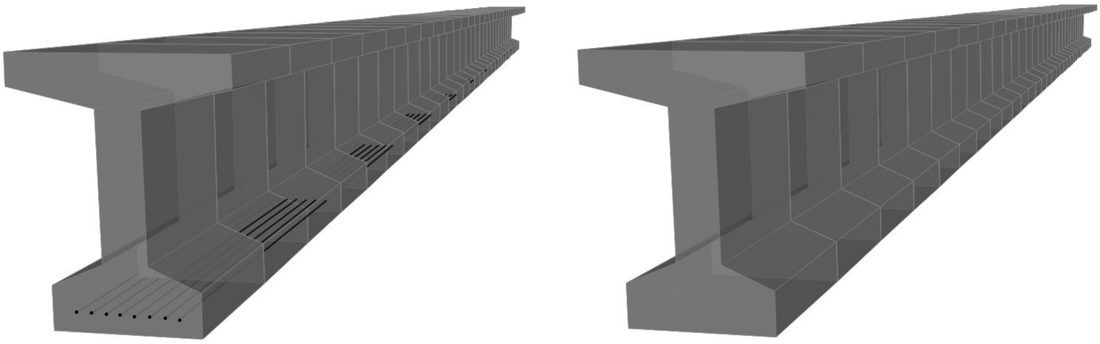


Figure 1. *Cross-section of the investigated beam*

III. FINITE ELEMENT MODELING

The three dimensional (3D) FEM of the PSC and RC beams were created by using the finite element analysis software [13] in order to obtain dynamic characteristic. 3D FEM of beams are given in Fig. 2. To obtain dynamic characteristic of beams, only linear modal analysis was performed for RC beam, but both linear and nonlinear modal analysis performed for PSC beam to determine the effect of prestressing force. For this reason geometric nonlinearity was taken into account in prestress loads case and stiffness at the end of this case was used in nonlinear modal analyses of PSC beam model. The PSC beam model consists of 15 frame elements and 8 tendons whereas RC beam model only consists of 15 frame elements. The beam and strands were represented with frame and tendon elements, respectively. Prestressing force was calculated as 195.51 kN when prestressing losses were not taken into consideration. As a boundary condition, the left and right hand supports were selected as pinned and roller, respectively.



(a) PSC beam

(b) RC beam

Figure 2. *Finite element model of **(a)** PSC beam and **(b)** RC beam*

IV. NUMERICAL RESULTS

In this section, dynamic characteristic such as periods and mode shapes obtained from formulation found in the literature and 3D FEMs of PSC and RC beam presented with detail. The value of periods of RC beam obtained from analytical prediction by formulation and numerically by linear modal analysis of FEM is given in Table 3. It is seen that the periods of RC beam obtained from analytically and numerically are close to each other. The first four mode shapes and periods of the RC beam obtained from the FEM modal analyses are given in Fig 3. The first four periods of RC beam obtained from analytical and numerical were attained which range between 0.261 and 0.030sn, 0.268 and 0.036sn, respectively. The four main vibration modes of beam: transverse, vertical, transverse and vertical modes, respectively.

Table 3. *Periods of first four mode of RC beam*

Techniques	**Periods [sn]**			
	1st Mode	**2st Mode**	**3st Mode**	**4st Mode**
Analytical	0.261	0.121	0.065	0.030
Numerical	0.268	0.124	0.072	0.036

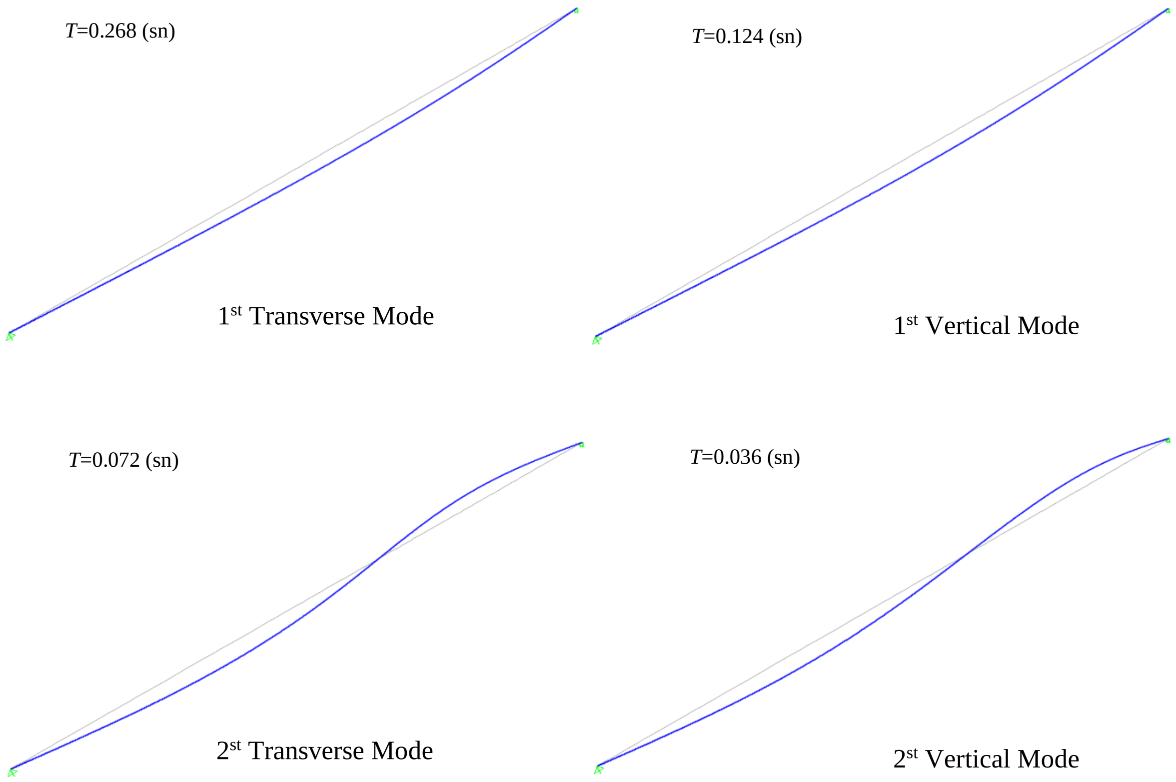
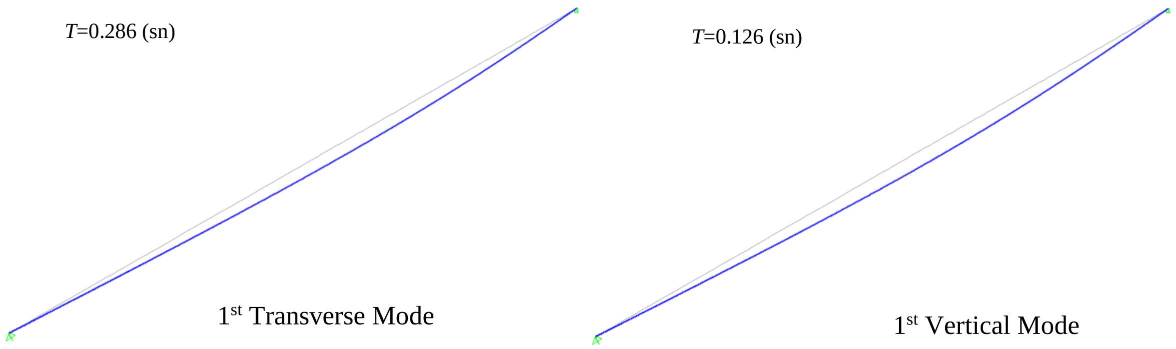


Figure 3. *Mode shapes of the RC girder from the FEM*

The periods of PSC beam obtained from analytical prediction by formulation and numerically by linear and nonlinear modal analysis of FEM are given in Table 4. It is seen that the period of PSC beam obtained from the linear modal analysis is same as the period of RC beam. However when the geometric nonlinearity was taken into account in prestress loads case and stiffness at the end of this case was used in nonlinear modal analyses the periods of PSC beam is increased and the values was getting closer that obtained from analytically. The mode shapes of PSC and RC beam obtained from all case of FEM are equal to each other. The result of FEM shows that modes shapes of PSC beams were not affected from prestressing force but periods were affected. The maximum effect of prestressing force is seen in the first period of PSC beam. The first four mode shapes and periods of the PSC beam obtained from the nonlinear modal analysis of FEM are given in Fig 4. The first four periods of PSC beam obtained from nonlinear modal analysis of FEM were attained which range between 0.289 and 0.036sn. The four main vibration modes of beam: transverse, vertical, transverse and vertical modes, respectively same as RC beam.

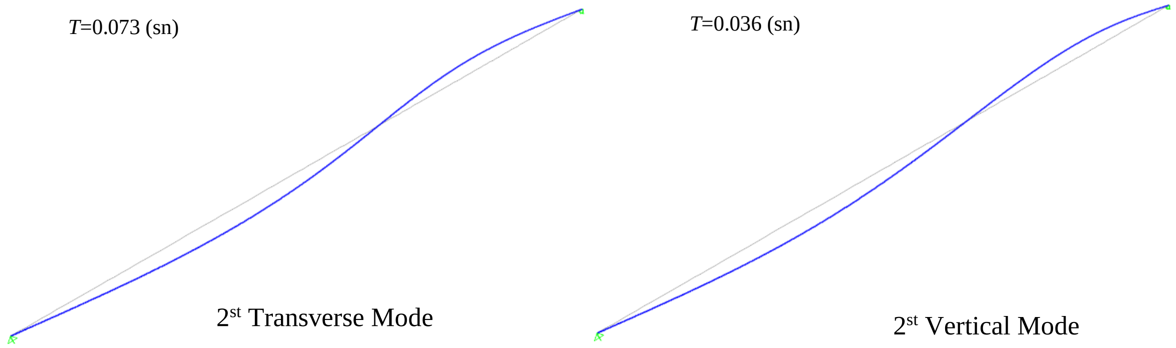
Table 4. *Periods of first four mode of PSC beam*

Techniques	**Periods [sn]**			
	1st Mode	**2st Mode**	**3st Mode**	**4st Mode**
Analytical	0.286	0.123	0.066	0.030
Numerical (Linear modal analysis)	0.268	0.124	0.072	0.036
Numerical (Nonlinear modal analysis)	0.289	0.126	0.073	0.036



1st Transverse Mode

1st Vertical Mode



2st Transverse Mode

2st Vertical Mode

Figure 4. *Mode shapes of the PSC girder from the FEM (nonlinear modal analysis)*

V. CONCLUSION

The aim of this study is to describe and compare the dynamic behavior of the prestressed concrete (PSC) and reinforced concrete (RC) beam. To achieve this aim, dynamic properties such as the mode shapes and periods of the PSC and RC beam were determined by means of the formulation found in the literature and a computer program that uses the finite element method. For this purpose I-beam with 0.9m height and 15m effective span length was selected as an example. The selected beam was considered separately as PSC and RC. In the PSC beam, eight low-relaxation Grade270 prestressing strand with 15mm (0.6 in.) diameter were used, unlike RC beams. Three dimensional finite element models of PSC and RC beam were created with SAP2000 software. To obtain dynamic characteristic of beams, only linear modal analysis was performed for RC beam, but both linear and nonlinear modal analysis performed for PSC beam to determine the effect of prestressing force. For this reason geometric nonlinearity was taken into account in prestress loads case and stiffness at the end of this case was used in nonlinear modal analyses of PSC beam model. At the end of the study, mode shapes and periods of PSC and RC beams obtained from analytical prediction by formulation and numerical by FEM are compared with each other. The main conclusions obtained from this study are:

- The periods of RC beam obtained from analytically and numerically are close to each other.
- The mode shapes of PSC and RC beam obtained from FEM linear and nonlinear modal analysis are overlap with each other. This result indicates that mode shapes of beam are not affected on prestressing force.
- The periods of PSC beam obtained from the linear modal analysis is same as the periods of RC beam. However when the geometric nonlinearity was taken into account in prestress loads case and stiffness at the end of this case was used in nonlinear modal analyses the periods of PSC beam is increased and the values was getting closer that obtained from analytical prediction suggested by Timoshenko et al., (1974).
- The maximum effect of prestressing force is seen in the first period of PSC beam.

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