ALKÜ Fen Bilimleri Dergisi 2019, Özel Sayı (NSP 2018): 45-51

First Forbidden β-Decay Logft Values of Neutron Rich Te Isotopes

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Abstract

Neutral tellurium detected in three metal-poor stars enriched by products of r-process nucleosynthesis by using near-ultraviolet spectra obtained with the Space Telescope Imaging Spectrograph on board the Hubble Space Telescope. Tellurium (Te, Z=52) was found at the second r-process peak (A≈130) associated with N=82 neutron shell closure [1]. The beta decay rates of unstable isotopes may be drastically changed under stellar conditions. In this work, we have been investigated the first forbidden (|ΔJ|=0, 1 and 2) transitions strength of some neutron rich Tellurium isotopes. The theoretical framework is based on a proton neutron quasiparticle random phase approximation (pn-QRPA) in the particle-hole (ph) channel. The transition probabilities in the Woods-Saxon potential basis have been calculated within the ξ-approximation [2]. The calculated first forbidden β-decay logft values are in better agreement with experimental data [3].

Keywords: First Forbidden, β-Decay, Logft

1. INTRODUCTION

For the physical condition prediction many supernova models fails to explain successfully the r-process nucleosynthesis. Sound knowledge of many properties of neutron rich nuclei are required for the description of production of r-process isotopes. The weak decay processes are important elements in astrophysical measures. Due to weak interactions core collapse of massive stars activates the supernova explosions. In neutronization of stellar core via electron capture by nuclei and by free protons weak interactions plays an important role. In evolution of massive this process effects the creation of heavier elements beyond iron in late phase of evolution in r-process. In supernova outburst weak rates determines the mass of the core and estimates the strength of the shock waves produced [4,5]. To study the r-process better understanding of β-decay properties of neutron rich nuclei is obligatory. In astrophysical environment it is believed that r-process take place at neutron density greater than $10^{20} cm^{-3}$ and temperature greater than $10^{9} k$ and the neutrons are captured rapidly than competing the β-decay and at small neutron separation energy of $\lesssim 3MeV$ the r-process goes through the neutron rich domain. Weak processes explanation is an open question for structure based nuclear theories and is important to explore the physics beyond the standard model [6]. Better agreement of estimation of beta decay half-lives is a challenging task in nuclear physics. Various models are used for the calculations of beta decay half-lives. Gross Theory statistical in nature was used by Tkahashi et al. for the calculations of these rates [7]. But it was realized the need of microscopic model and then for the calculations of weak rates of large scaled r-process uses two different microscopic models have been used, the Shell model and the quasiparticle random phase approximation (QRPA). In the studies of β-decay the proton-neutron QRPA theory has been widely used. In pn-QRPA a quasiparticle basis using pairing interaction is constructed and Schematic Gamow-Teller (GT) residual interaction is solved using RPA equation. Sorensen and Halbleib [8] modifying the RPA model for the calculations of relevant transitions developed this model. Later on this model was extended for deformed nuclei as well by many authors for deformed [9-17] and for spherical nuclei [18-21]. For allowed weak rates, from atomic number 6-114 microscopic calculations were carried out by [22]. Both beta decay [23] and electron capture [24] were studied for the calculations of β-decay properties for the nuclei far from the line of stability. Under terrestrial conditions the same model was used for the calculations of unique first-forbidden (U1F) transitions ($|\Delta J| = 2$) by [25,26]. It was shown that for nuclei near to stability and near to magic nuclei U1F transition has more contribution to the total strength (Fig. 9 and table IX of [25]). Nabi and Klapdor used this model for the first time in stellar matter to calculate weak interaction rates [27-29]. Nabi and Stoica modifies the same code for the calculations of U1F rates in stellar environment [30]. Borzov further modified the QRPA based on Fayans energy function for calculations of role allowed and first-forbidden (FF) in the half-lives [4]. Large scale shell model is used for half-lives of r-processed waiting point nuclei which includes FF contributions.

Suhonen studied the ground state transition in allowed and FF β-decays [31]. Civitarese et al. studied dependence of spin-isospin on even-even, odd-odd and FF β-decay transitions for $|\Delta J| = 0, 2$ [32]. Çakmak et al. used QRPA model for spherical nuclei in the mass range 90-214 to study the $0^+ \leftrightarrow 0^-$ transitions [33] and the results were in better agreement with the experimental results and previous calculations. Thus is concluded that in total decay rates the FF transitions plays an important role and must take into account for the calculation of half-lives.

In this paper, we studied the FF transitions of Te isotopes by using pn-QRPA for spherical nuclei in the Woods-Saxon potential. For the calculations of FF transitions we considered the particle-hole term of the effective interactions of the β-decay.

2. THE pn-QRPA (WS) FORMALISM

FF transitions log*ft* values have been calculated using spherical schematic model within the framework of pn-QRPA (WS) method. The Woods-Saxon potential with Chepurnov parametrization has been used as a mean field basis in numerical calculations. The eigenvalues and eigenfunctions of the Hamiltonian with separable residual GT effective interactions in particle-hole (ph) channel were solved within the framework of pn-QRPA model.

The model Hamiltonian which generates the spin-isospin dependent vibrations modes with $\lambda^{\pi} = 0^-, 1^-$ and 2^- on odd-odd nuclei in quasi boson approximation is given as

$$\hat{H} = \hat{H}_{sqp} + \hat{h}_{ph} \tag{1}$$

The single quasi-particle Hamiltonian of the system is given by

$$\hat{H}_{sqp} = \sum_{j_r} \varepsilon_{j_r} \alpha^{\dagger}_{j_r m_r} \alpha_{j_r m_r (\tau=p,n)} \tag{2}$$

where ε_{j_r} and $\alpha^{\dagger}_{j_r m_r}$ ($\alpha_{j_r m_r}$) are the single quasi-particle energy of the nucleons with angular momentum j_r and the quasi-particle creation (annihilation) operators, respectively.

The $\hat{h}_{ph}$ is the spin-isospin effective interaction Hamiltonian which generates $0^-, 1^-, 2^-$ vibration modes in particle-hole channel and given as

$$\hat{h}_{ph} = 2\chi_{ph}\sum_{\mu} T^{+}_{\lambda\mu}T^{-}_{\lambda\mu}\ , \qquad T^{-}_{\lambda\mu} = \left(T^{+}_{\lambda\mu}\right)^{+} \tag{3}$$

where $T^{\pm}_{\lambda\mu}$ is the first forbidden beta decay operator and χ_{ph} is particle-hole effective interaction constant.

$$T^{\pm}_{\lambda\mu} = \begin{cases} g_V \sum_k t_{\pm}(k) r_k Y_{1\mu}(\theta_k, \varphi_k) & \text{for dipole interactions} \\ g_A \sum_k t_{\pm}(k) r_k \left[Y_{1\mu}(\theta_k, \varphi_k), \sigma_1(k)\right]_{\lambda} & \text{for spin} - \text{dipole interactions} \end{cases} \tag{4}$$

where λ is 0, 1 and 2 values. The beta decay operator in quasiparticle space is written as follows

$$T^{+}_{\lambda\mu} = \sum_{p,n}\left[\bar{b}_{pn}C^{+}_{np}(\lambda,\mu) + (-1)^{\lambda+\mu+1} b_{pn}C_{np}(\lambda,-\mu)\right] + \sum_{p,n}\left[d_{pn}D^{+}_{np}(\lambda,\mu) + (-1)^{\lambda+\mu}\bar{d}_{pn}D_{np}(\lambda,-\mu)\right] \tag{5}$$

The $C^{+}_{np}(\lambda,\mu)$ and $D^{+}_{np}(\lambda,\mu)$ are the quasi boson operators and given as

$$C^{+}_{np}(\lambda,\mu) = \sqrt{\frac{2\lambda+1}{2j_n+1}} \sum_{m_n, m_p} (-1)^{j_p - m_p}\left(j_p m_p \lambda\mu / j_n m_n\right)\alpha^{+}_{j_n m_n}\alpha^{+}_{j_p - m_p}$$

$$D^{+}_{np}(\lambda,\mu) = \sqrt{\frac{2\lambda+1}{2j_n+1}} \sum_{m_n, m_p}\left(j_p m_p \lambda\mu / j_n m_n\right)\alpha^{+}_{j_n m_n}\alpha_{j_p m_p} \tag{6}$$

The Hamilton operator of even mass nuclei and the effective interactions in the particle-hole channel are given

$$H = H_0 + h_{CD} \tag{7}$$

$$h_{CD} = 2\chi_{ph}\sum_{\mu}\left[T^{+}_{\lambda\mu}(C)T^{-}_{\lambda\mu}(D) + T^{+}_{\lambda\mu}(D)T^{-}_{\lambda\mu}(C)\right] \tag{8}$$

The equation of motion the pn-QRPA may be written and

$$\left[\mathrm{H}, \Omega^{j+}_{j_k m_k}\right] = W^{j}_{j_k}\Omega^{j+}_{j_k m_k} \tag{9}$$

The secular equation for the ω^{k}_{n} energies is found as follows

$$W^{j}_{j_k} - \varepsilon_{j_k} = \frac{2\lambda+1}{2j_k+1}\sum_{i, j_\nu}\frac{\left\{2\chi_{ph}\left[M^{+}_{i}(0^{+}\to\lambda^{-}_{i})d_{\nu k} + M^{-}_{i}(0^{+}\to\lambda^{-}_{i})\bar{d}_{\nu k}\right]\right\}^{2}}{W^{j}_{j_k} - \omega_i - \varepsilon_{j_\nu}} \tag{10}$$

2.1 Investigation Of The Beta Moment Matrix Elements

The relativistic beta moment matrix elements has an important role when the non relativistic beta moment matrix elements is small, depending on microscopic structure of the states.

The relativistic and non-relativistic matrix elements, respectively, for $\lambda^{\pi} = 0^{-}$ are given by

$$M^{\mp}(\rho_A, \lambda = 0) = \frac{g_A}{\sqrt{4\pi c}} \sum_k t_{\mp}(k)(\sigma_k . \vartheta_k), \quad (11a)$$

$$M^{\mp}(\rho_A, \kappa = 1, \lambda = 0) = g_A \sum_k t_{\mp}(k) r_k \{Y_1(r_k)\sigma_k\}_o, \quad (11b)$$

The relativistic and the non-relativistic matrix elements, respectively, $\lambda^{\pi} = 1^{-}$ are given by

$$M^{\mp}(j_v, \kappa = 0, \lambda = 1, \mu) = \frac{g_A}{\sqrt{4\pi c}} \sum_k t_{\mp}(k) r_k (\vartheta_k)_{1\mu}, \quad (12a)$$

$$M^{\mp}(\rho_v, \lambda = 1, \mu) = g_A \sum_k t_{\mp}(k) r_k Y_{1\mu}(r_k), \quad (12b)$$

$$M^{\mp}(j_v, \kappa = 1, \lambda = 1, \mu) = g_A \sum_k t_{\mp}(k) r_k \{Y_1(r_k)\sigma_k\}_{1\mu}, \quad (12c)$$

The non-relativistic matrix element for $\lambda^{\pi} = 2^{-}$ is given as

$$M^{\mp}(j_A, \kappa = 1, \lambda = 2, \mu) = g_A \sum_k t_{\mp}(k) r_k \{Y_1(r_k)\sigma_k\}_{2\mu}, \quad (13)$$

Also, the transition probabilities $B(\lambda^{\pi} = 0^{-}, 1^{-}, 2^{-}; \beta^{\mp})$ are given by

$$B(\lambda^{\pi} = 0^{-}, \beta^{\mp}) = \left| \langle 0_i^{-} \left\| M^0_{\beta^{\mp}} \right\| 0^{+} \rangle \right|^2$$

$$M^0_{\beta^{\mp}} = \mp M^{\mp}(\rho_A, \lambda = 0) - i \frac{m_e c}{\hbar} \xi M^{\mp}(\rho_A, \kappa = 1, \lambda = 0) \quad (14)$$

$$B(\lambda^{\pi} = 1^{-}, \beta^{\mp}) = \left| \langle 1_i^{-} \left\| M^1_{\beta^{\mp}} \right\| 0^{+} \rangle \right|^2$$

$$M^1_{\beta^{\mp}} = M^{\mp}(j_v, \kappa = 0, \lambda = 1, \mu) \pm i \frac{m_e c}{\sqrt{3}\hbar} M^{\mp}(\rho_A, \lambda = 1, \mu) + i\sqrt{\frac{2}{3}} \frac{m_e c}{\hbar} \xi M^{\mp}(j_A, \kappa = 1, \lambda = 1, \mu) \quad (15)$$

$$B(\lambda^{\pi} = 2^{-}, \beta^{\mp}) = \left| \langle 2_i^{-} \left\| M^2_{\beta^{\mp}} \right\| 0^{+} \rangle \right|^2,$$

$$M^2_{\beta^{\mp}} = M^{\mp}(j_A, \kappa = 1, \lambda = 2, \mu) \quad (16)$$

In eqs. (14) and (15), the upper and lower signs refer to β^{-} and β^{+}decays, respectively.

The ft values are given by the equation

$$(ft)_{\beta^{\mp}} = \frac{D}{\left(\frac{g_A}{g_v}\right)^2 4\pi B(I_i \to I_f, \beta^{\mp})} \quad (17)$$

where

$$D = \frac{2\pi^3 \hbar^2 ln^2}{g_v^2 m_e^5 c^4} = 6250 sec. \qquad \frac{g_A}{g_v} = -1.254.$$

3. RESULT AND COMPARISON

The FF β-decay logft values of tellurium (Te) isotopes calculated within pn-QRPA (WS) formalism is shown in table 1. Only particle-hole interaction strength was considered for FF calculations within the pn-QRPA(WS) formalism. A quenching factor wasn't applied for all pn-QRPA(WS) calculations.
The pairing correlation constants were taken a $C_p = C_n = 12/\sqrt{A}$. The strength parameters of the effective interaction are $\chi_\beta = 30A^{-5/3} MeV fm^{-2}$, $\chi_\beta = 55A^{-5/3} MeV fm^{-2}$ and $\chi_\beta = 99A^{-5/3} MeV fm^{-2}$ for rank0, rank1 and rank2, respectively. As seen from Table 1, calculated the logft values of the $0^+ \leftrightarrow 0^-$ transitions for Te-119 and Te-133 isotopes, the $0^+ \leftrightarrow 1^-$ transitions for Te-119, Te-129 and Te-133 isotopes and the $0^+ \leftrightarrow 2^-$ transitions for Te-119, Te-121, Te-127, Te-129, Te-131 and Te-133 isotopes. The calculated logft values are also compared with experimental data. The data of ref. [3] has been used for experimental logft values. It can be seen that the pn-QRPA(WS) model calculates logft values in better agreement with the measured logft values. The FF contributions to the total beta decay half-lives are very important to understand stellar beta decay rates. When the Te isotopes becomes more and more neutron-rich, the phase space enhancement for U1F ($\Delta J = 2$) transitions decreases but still is bigger than the phase space for allowed transitions and lead to a significant U1F contribution to the total β-decay rates.

Table 1. The logft values of some neutron rich Te isotopes for FF beta transitions.

Transitions	Logft					
	Δ*J*= 0		**Δ*J*= 1**		**Δ*J*= 2**	
	Exp.[a]	**pn-QRPA (WS)**	**Exp.[a]**	**pn-QRPA (WS)**	**Exp.[a]**	**pn-QRPA (WS)**
${}^{119}_{52}Te + e^- \to {}^{119}_{51}Sb + \nu$	6.9	6.79	7.01	6.87	9.39	9.21
${}^{121}_{52}Te + e^- \to {}^{121}_{51}Sb + \nu$	-	-	-	-	9.7	9.65
${}^{127}_{52}Te \to {}^{127}_{53}I + e^- + \bar{\nu}$	-	-	-	-	10.21	9.87
${}^{129}_{52}Te \to {}^{129}_{53}Sb + e^- + \bar{\nu}$	-	-	9.91	9.47	10.14	9.92
${}^{131}_{52}Te \to {}^{131}_{53}I + e^- + \bar{\nu}$	-	-	-	-	10.78	10.3
${}^{133}_{52}Te + e^- \to {}^{133}_{51}Sb + \nu$	7.67	7.52	6.99	6.81	9.9	9.84

[a] The experimental values are taken from B. Singh et al. [3].

4. ACKNOWLEDGEMENTS

This study within the research project MEV 2018-300 (No:9-5 (Ph-1-MG-7) PAK-TURK) coded is supported by The Council of Higher Education of Turkey (CoHE) and the Higher Education Commission of Pakistan (HEC).

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