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Large deflection analysis of prismatic cantilever beam comparatively by using combing method and iterative DQM

Birleşim metodu ve iteratif DQM ile prizmatik ankastre kirişlerde karşılaştırmalı büyük sehim analizi

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Birleşim Metodu ve İteratif DQM ile Prizmatik Ankastre Kirişlerde Karşılaştırmalı Büyük Sehim Analizi

Araştırma Makalesi / Research Article

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ÖZ

Prizmatik ve prizmatik olmayan ankastre kirişlerin genel yükleme koşullarındaki büyük sehim problemi için tam anlamıyla analitik çözüm bulunmamaktadır. Prizmatik olmayan bir ankastre kirişin ele alındığı durumlarda ise büyük sehim probleminin zorluğu daha da artmaktadır. Bu çalışmada, İteratif Diferansiyel Quadrature Metodunun (I-DQM) ve Birleşim Metodunun (BM) karşılaştırılması yapılmıştır. İki yöntemle ayrı ayrı prizmatik ankastre kirişlerde büyük sehim probleminin sayısal çözümü gerçekleştirilmiştir. Dado ve Sadder (2005) tarafından geşitirlen yarı-analitik yöntem ile karşılaştırıldığında elde edilen sonuçlar her iki yöntemin de ele alınan problemin çözümünde oldukça etkili olduğunu göstermiştir. Bununla birlikte I-DQM'in BM'ye daha hassas ve geniş aralıklı bir çözüm sağladığı ortaya koyulmuştur.

Anahtar Kelimeler: Büyük sehim, iterarif diferansiyel quadrature metodu, birleşim metodu, nonlineer simülasyon, prizmatik ankastre kiriş.

Large Deflection Analysis of Prismatic Cantilever Beam Comparatively by Using Combing Method and Iterative DQM

ABSTRACT

There is no exactly analytical solution for the large deflection problem of prismatic cantilever beams under general loading conditions. In the case of considering a non-prismatic cantilever beam, the difficulty of the larger deflection problem is increased. In this study, the comparison of the Iterative Differential Quadrature Method (I-DQM) and the Combining Method (CM) was performed. Numerical solution of the large deflection problem was separately performed with both the I-DQM and the CM for prismatic cantilever beams. The obtaining results show that both of these methods gave more accurate solutions compared with a reliable semi-analytic method which was introduced by Dado and Sadder (2005). Besides, it was demonstrated that the I-DQM provided a more wide-range solution than the CM.

Keywords: Large deflection, iterative differential quadrature method, combining method, nonlinear simulation, cantilever prismatic beam.

1. INTRODUCTION

In recent years, the determination of large deflection behaviour of prismatic and non-prismatic cantilever beams has become a very important issue especially for civil, mechanical, aero plane and biomedical engineering. Beams and columns with varying strength values were used uniformly for distributing the strength and mass of complex structures. Besides, tiny cantilever beams were used for providing some functional properties to system in many special fields [1].

There is no exactly analytical solution for the large deflection problem of prismatic and non-prismatic cantilever beams under general loading conditions. Thus, numerical methods ensured proper approximations for the solution of large deflection problems [2]. According to literature review, researchers performed numerical, approximately analytical, and semi-analytical solutions for the large deflection problem of many kinds of cantilever beams. However, some of these studies weren't provide proper accuracy for the solution of problem. Also, some of them ensured very high accuracy but algorithms of these studies caused very high calculation time or difficulty for using a daily solution of the large deflection problem [3-22].

Navaee and Elling investigated equilibrium configuration of a prismatic cantilever beam for different loading conditions using the Elliptic Integral Method. Results of the investigation showed that there was more than one equilibrium configuration for flexible beams [23]. Faulkner et al., developed a New Segmental Shooting Technique for the solution of large deflection

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problem. The procedure used to solve the problem provided a solution for different boundary conditions. The proposed method was considered the large deflection problem as small deflection by dividing the beam into small pieces. The problem that was converted to the initial value problem was solved by the Shooting Technique [24]. Dado and Al-Sadder solved the large deflection problem of cantilever beams using a polynomial formulating the rotating angle of beam. Sadder et al. developed an algorithm for the solution of non-prismatic beams large deflection problem using Finite Difference Method [1]. Tolou and Herder carried out a semi-analytical solution procedure using the Adomian Decomposition Method for the large deflection problem of cantilever under point load. Results of the solutions proved that the solution procedure suitable for obtaining problem [17]. Batista obtained an analytical solution of equilibrium configuration of the large deflection problem of cantilever beams using Jacobi Elliptic Function [19].

In this study, comparison of the I-DQM and the CM is performed. With the intent to compare of the I-DQM and the CM, the large deflection analysis of prismatic cantilever beam was derived. The obtaining results proved that both methods were suitable for the nonlinear numerical solution. Besides, the I-DQM was ensured more accurate results than the CM.

2. WEIGHTING COEFFICIENT AND ITERATIVE DQM

Bellman et al. introduced the DQM for the numerical solution of differential equation in 1971. Most important parameter of this method is the calculation of weighting coefficients. Quan and Chan used Lagrange Polynomial as a test function for the calculation of weighting coefficients. Shu and Richards developed an algebraic formula for calculation of weighting coefficients using both approximations of Bellman et al. and Quan and Chan. This new method was called as Generalized DQM (GDQM) [25-29].

According to GDQM rth order derivative of f(x) is given in Equation 1.

$$\frac{d^r f(x_i)}{dx^r} = \sum_{j=1}^{N} a_{ij}^{(r)} f(x_j) \rightarrow i = 1,2,\dots,N \tag{1}$$

Here f(x) is a function of x which is described in $x\in[a,b]$. Besides, f(xi) shows the numerical values of f(x) for a certain value of xi ((i = 1,2, ... N). In the Eq. 1 a_ij^((r)) shows the weighting coefficient of DQM for rth order derivative. Calculation of weighting coefficient is presented in Eq. 1 and 2 by using Lagrange Interpolation Function in the following terms.

$$l_j(x) = \frac{\phi(x)}{(x - x_j)\phi^{(1)}(x_j)} \rightarrow j = 1,2,\dots,N \tag{2}$$

$$\phi(x) = \prod_{m=1}^{N} (x - x_m)\,; \quad \phi^{(1)}(x_j) = \frac{d\phi(x_j)}{dx} = \prod_{m=1, m\neq 1}^{N} (x_j - x_m) \tag{3}$$

$$a_{ij}^{(1)} = \frac{dl_j(x_i)}{dx} = \frac{\phi^{(1)}(x_i)}{(x_i - x_j)\phi^{(1)}(x_j)}, i,j = 1,2,\dots,N, i \neq j \tag{4}$$

$$a_{ii}^{(1)} = -\sum_{j=1, i\neq j}^{N} a_{ij}^{(1)}, \qquad i = 1,2,\dots,N \tag{5}$$

Similarly,

$$a_{ij}^{(r)} = \frac{d^r l_j(x_i)}{dx^r} = r\left(a_{ii}^{(r-1)} a_{ij}^{(1)} - \frac{a_{ij}^{(r-1)}}{(x_i - x_j)}\right), i,j = 1,2,\dots,N, i \neq j\,, r \geq 2 \tag{6}$$

$$a_{ii}^{(r)} = \frac{d^r l_j(x_i)}{dx^r} = -\sum_{j=1, i\neq j}^{N} a_{ij}^{(r)}, \qquad i = 1,2,\dots,N \tag{7}$$

$$a_{ii}^{(r)} = \sum_{k=1}^{N} a_{ik}^{(r-1)} a_{ik}^{(1)}, \qquad i,j = 1,2,\dots,N\ ,, r \geq 2 \tag{8}$$

The weighting coefficients computed using Equations from 2 to 8 described the matrix of $[A^{(r)}]$ given in Equation 9.

$$[A^{(r)}] = \left(\frac{d}{dx}\right)^r = \frac{d^r}{dx^r} = \frac{d^{r-1}}{dx^{r-1}}\frac{d}{dx} = \frac{d}{dx}\frac{d^{r-1}}{dx^{r-1}} = \begin{bmatrix} a_{11}^{(r)} & a_{12}^{(r)} & \dots & a_{1N}^{(r)} \\ a_{21}^{(r)} & a_{22}^{(r)} & \dots & a_{2N}^{(r)} \\ \vdots & \vdots & \vdots & \vdots \\ a_{N1}^{(r)} & a_{N2}^{(r)} & \dots & a_{NN}^{(r)} \end{bmatrix} \tag{9}$$

Currently, many of researchers used the GDQM with intent to solve nonlinear engineering problems [30-34]. When the problem has multiple boundary conditions at the same point, the DQM could not provide a numerical solution. With the intent to avoid this situation, researchers developed a lot of methods such as δ approximation. [35-39]. One of the most effective usages of the DQM is considered the method as an iterative scheme. Thus, iteration procedure provides the multiple boundary conditions at the same point. In this study, the Newton-Raphson Iteration Method was used as the iteration procedure, given simply in Eq. 10.

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \qquad (10)$$

3. COMBINING METHOD

One of the approaches used to overcome the difficulties encountered when using the DQM combined the DQM with another method [40-42].

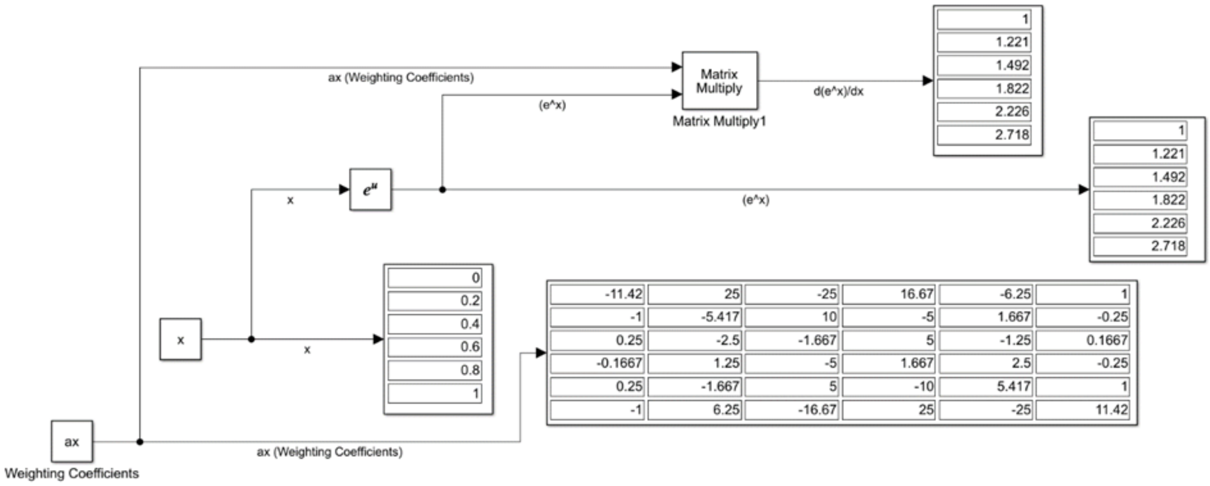

Figure 1. . Derivative of e^x by using CM

Girgin et al. developed the Combining Method (CM) which combined the DQM and Simulation Technique (ST) for providing applying multiple boundary conditions at the same point. ST was used for the solution of Ordinary Differential Equations (ODE) and automatic control problems. With this purpose, a lot of software was used such as Matlab/Simulink, Dymola, AMESim, and so on. Although the ST is a good method for the solution of ODE, the boundary conditions cannot be entered into the system. So, the CM was ensured as a new approach for the solution of boundary conditions problem and resolved the weakness of both DQM and ST [43-45]. Simplest form of the CM with n=6 grids is shown in Figure 1 for the derivative of ex. Also, Figure 2 shows that how to solve the given ODE in Equation 11 using the CM with n=6 grids.

$$\frac{dy}{dx} - y = 0\,, y(1) = e \qquad (11)$$

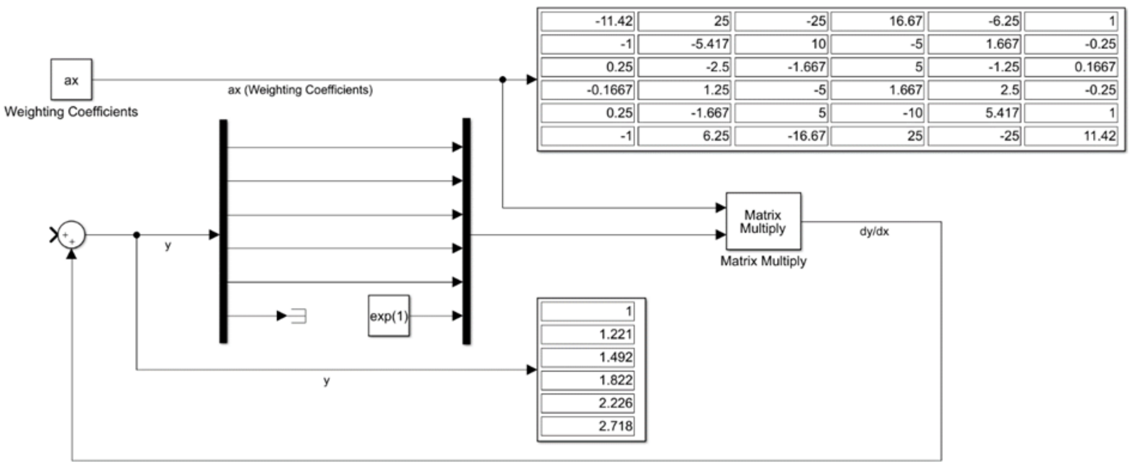

Figure 2. . Simple ODE solution using CM for Equation 11

4. COMPARISON OF I-DQM AND CM

The large deflection problem of prismatic cantilever beam considered for the purpose of comparison of I-DQM and CM. Equation 12 is determined large deflection behaviour of prismatic cantilever beam [2].

$$EI(s)\frac{d^2\theta}{ds^2}+E\frac{dI(s)}{ds}\frac{d\theta}{ds}+\left[\int_{s=s}^{s=L} q_y(s)ds+F_y\right]cos\theta-\left[\int_{s=s}^{s=L} q_x(s)ds+F_x\right]sin\theta=0 \quad (12)$$

The large deflection problem of prismatic cantilever beam investigated for two different loading conditions. Here, the elasticity modulus of the beam material and inertia of the beam are expressed as E and I(s), respectively. The distributed loads in the x and y directions and point loads from the free end are expressed by q_y, q_x, F_x and F_y, in Equation 11. The moment applied from the free end of the built beam obtained from the differential terms of Equation 11 as known from the theory of elasticity. In this study, EI(s) expression considered to be equal to one cause of the obtaining the large deflection problem discussed for a prismatic cantilever beam. The slope and deflection values are zero at the point of the cantilever support based on the theory of elasticity. These values are considered as basic boundary conditions of the large deflection problem. The I-DQM algorithms which are given in Eq. 13, 14 and 15 provide the solution of the large deflection problem.

$$f=EI(s)\cdot A^{(2)}\cdot\theta+E\frac{dI(s)}{ds}A^{(1)}\cdot\theta+\left[\int_{s=s}^{s=L} q_y(s)ds+F_y\right]cos\theta-\left[\int_{s=s}^{s=L} q_x(s)ds+F_x\right]sin\theta \quad (13)$$

$$f=\boldsymbol{EI(s)}\cdot A^{(2)}\cdot\theta+\boldsymbol{E}\frac{\boldsymbol{dI(s)}}{\boldsymbol{ds}}A^{(1)}\cdot\theta+\left[\int_{\boldsymbol{s=s}}^{\boldsymbol{s=L}} \boldsymbol{q_y(s)ds+F_y}\right]\boldsymbol{cos\theta}-\left[\int_{\boldsymbol{s=s}}^{\boldsymbol{s=L}} \boldsymbol{q_x(s)ds+F_x}\right]\boldsymbol{sin\theta} \quad (14)$$

$$\theta(s)=\theta(s)-\frac{f}{f'} \quad (15)$$

Here, “A” shows matrix of the weighting coefficients and bold terms shows Jacobians in the Newton Raphson Method.

The CM block diagram which gives the solution of the large deflection problem is given in Figure 3.

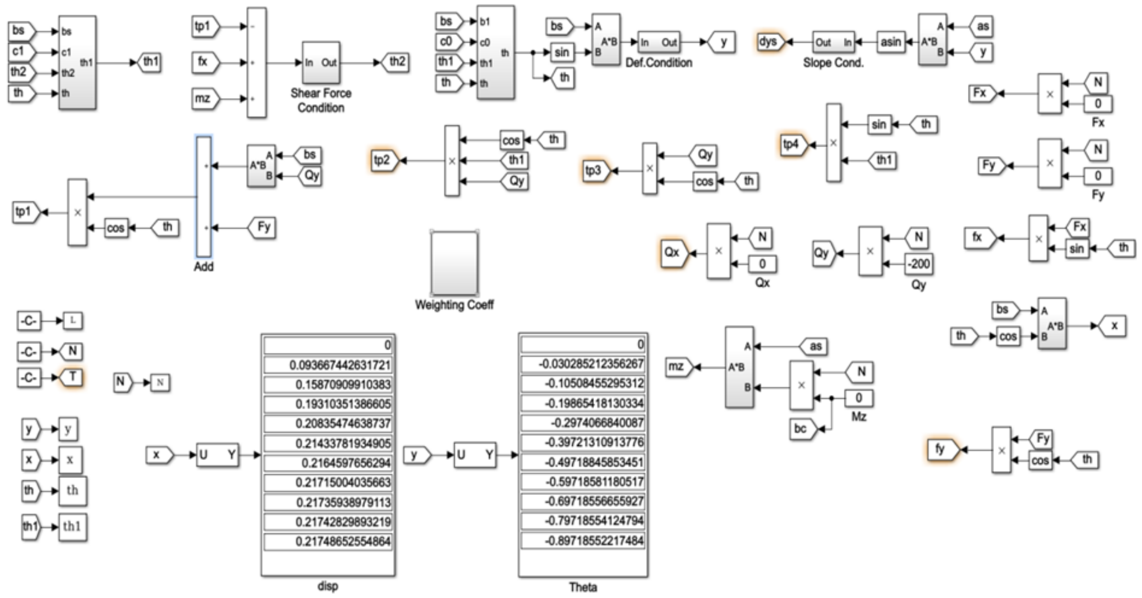

Figure 3. . Block Diagram of Solving the Large Deflection Problem by the CM

Problem 1:

Firstly, considered large deflection problem is shown in Figure 4. In this situation cantilever beam is discussed only under uniformly distributed load.

The solution obtained using I-DQM was considered for different values of the uniformly distributed load such as $q_y(s)$= -4, -10, -20, -40, -100, -150, -200, and -1000.

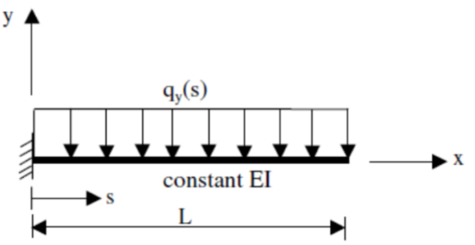

Figure 4. Cantilever Prismatic Beam under Uniformly Distributed Load.

The weighting coefficients matrix used while performing the solution was calculated by using 6 uniformly distributed grid points. Iteration number of the I-DQM was determined as 5. Results of numerical study provided reliable solution for large deflection problem even with very small CPU times (Figure 5). Compared to the other results in the literature, the I-DQM was ensured to be very successful in numerical results for the large deflection problem. Similarly, numerical simulation of the large deflection problem was performed by using CM for the Problem 1 with different distributed loads such as qy(s)= -4, -10, -20, -40, -100,-150 and -200 (Figure 6). 21 uniformly distributed grid points were used for computing the weighting coefficients of the CM. The obtaining results was shown that CM was reliable for solution of the large deflection problem.

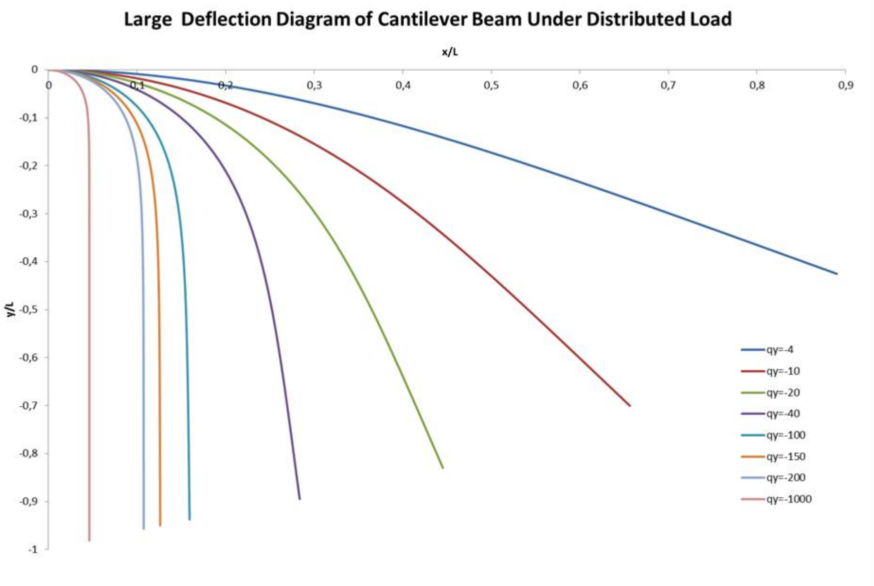

Figure 5. Large Deflection Diagram of Cantilever Beam under Distributed Load with I-DQM.

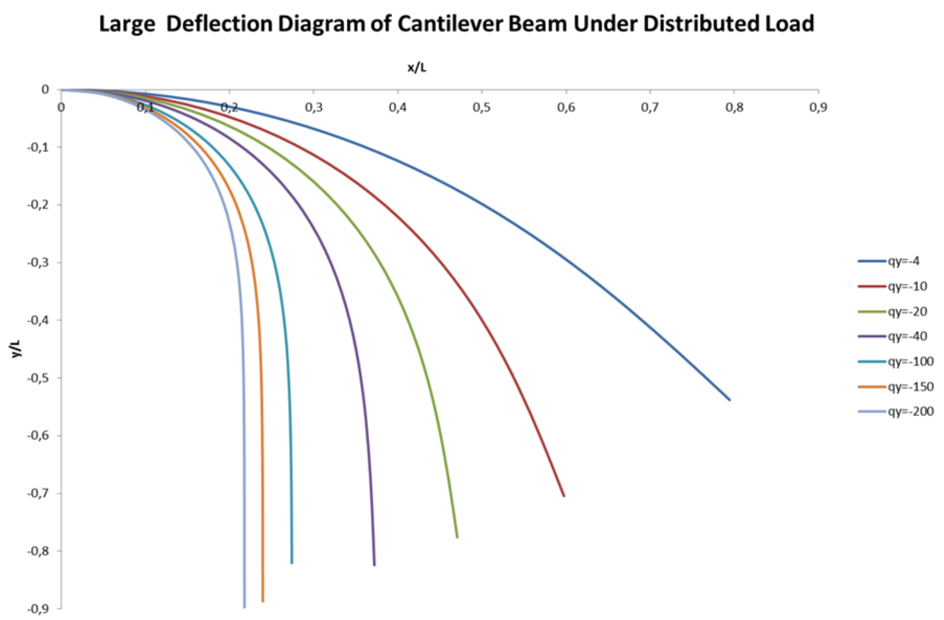

Figure 6. Large Deflection Diagram of Cantilever Beam under Distributed Load with CM

Dado and Al-Sadder developed a new semi-analytical method for solution of large deflection problem. This study is one of the most accurate solution procedures for the large deflection problem. Thus, this study was used as a reference to measure the reliability of the solutions provided by the I-DQM and the CM. Figure 7 shows that the solution of the problem could be performed up to maximum distributed load value of qy = -100 [2]. The results obtained using I-DQM provided a reliable solution up to the maximum distributed load qy = -1000. Also, the CM is ensured that a proper solution up to the maximum distributed load qy = -200. Hence, it was seen that both methods were effective for solving the large deflection problem of cantilever prismatic beams. Besides, the I-DQM was provide more accuracy than other studies.

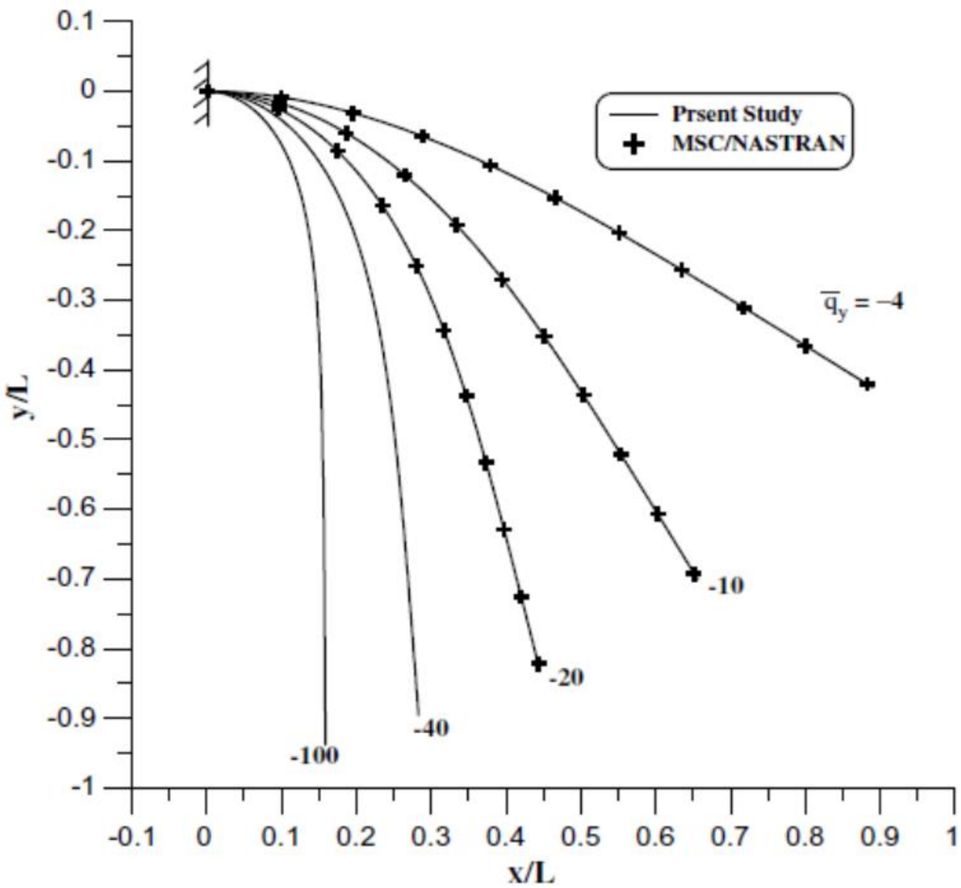

Figure 7. Results of the Problem 1 in the Study by Dado and Al-Sadder (2005).

Problem 2:

The second problem, solved by using I-DQM and CM, is shown in Figure 8. In this situation, the large deflection problem is considered under free end forces and moment.

When the solutions using the I-DQM were compared with other results in the literature, it is clearly seen that the method is more successful than other studies according to the deflection diagrams shown in Figure 9. The results show that the large deflection problem can be solved using the I-DQM until to Fx = -18, Fy = 104, Mz = -18. As in the first problem discussed, it was observed that the problem of large deflection was solved in very short CPU times which can be measured in milliseconds using 5 iterations and number of girds n =6.

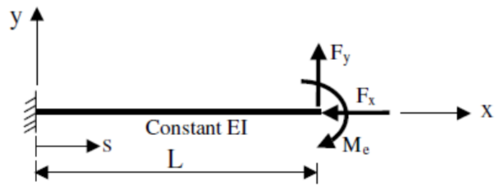

Figure 8. Cantilever Prismatic Beam under the Effect of Fx Fy and Me

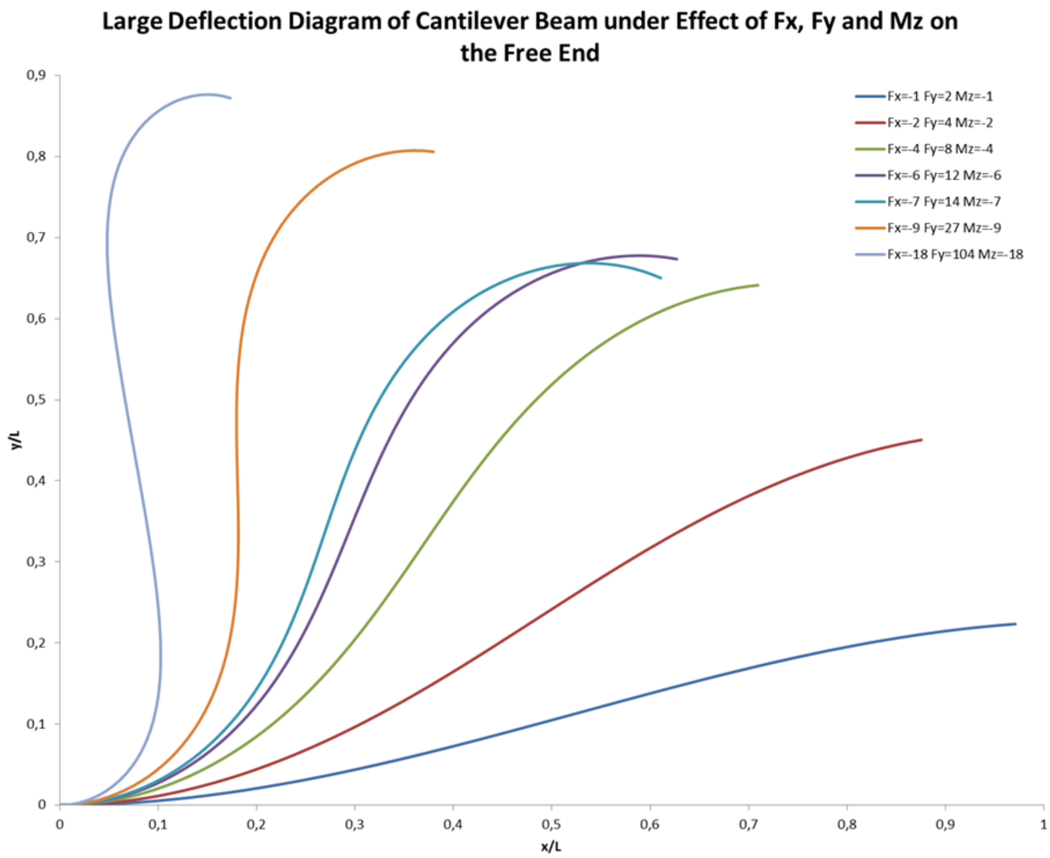

Figure 9. Large Deflection Diagram of Cantilever Beam for the Problem 2 with I-DQM

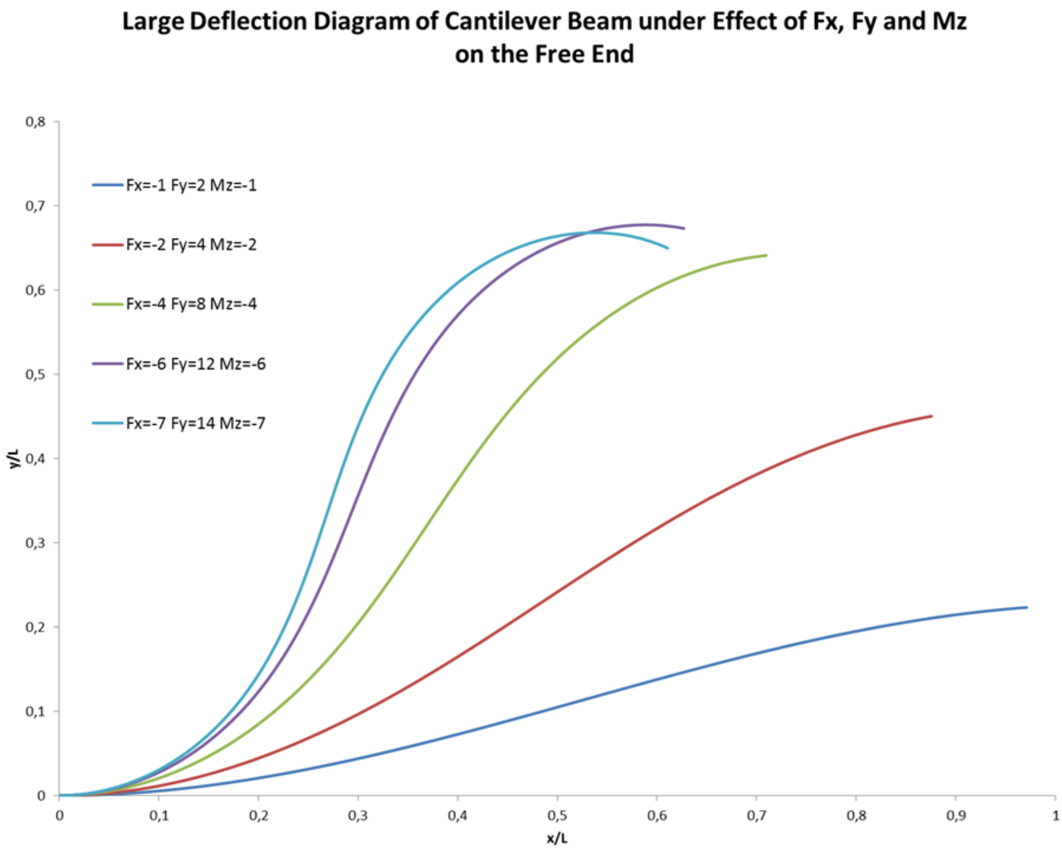

Figure 10. Large Deflection Diagram of Cantilever Beam for the Problem 2 with the CM

The deflection diagrams of the solutions for the problem 2 performed using the CM are given in Figure 10. According to this, reliable results can be obtained up to Fx = -7, Fy = 14, and Mz = -7 using 21 grids. It was found that the method did not achieve sufficient accuracy at higher load values.

It is clear that both methods are significantly successful (Figure 11) compared to other results in the literature. Reliable results can be obtained up to Fx = -7, Fy = 14, and Mz = -7 in the literature for solution of the Problem 2 [2]. Present study ensured wide range results using the I-DQM for the Problem 2. Also, the CM provided same accuracy with literature for the solution of the Problem 2.

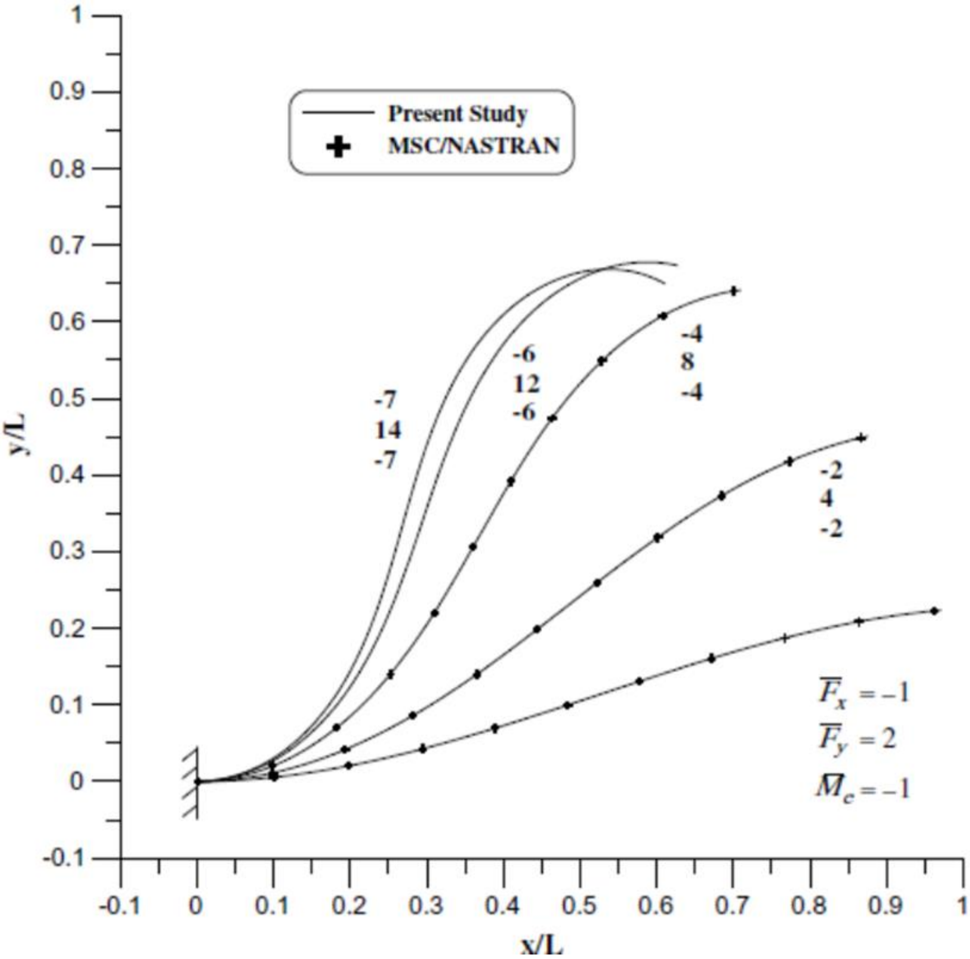

Figure 11. Results of the Problem 2 in the Study by Dado and Al-Sadder (2005)

6. CONCLUSIONS

In this study, the I-DQM and the CM were compared. Numerical solution of the large deflection problem was performed for prismatic cantilever beams with the I-DQM and the CM. The solutions performed using the CM ensured similar results with other studies in the literature. However, the solutions obtained using the I-DQM provided very high accuracy and a wide range of results at very low CPU times. Consequently, both of the methods seem to give reliable results for the structural large deflection problem.

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