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A New Power Allocation Method with a Nonlinear Cost Constraint in Stratified Random Sampling

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Abstract: When the population is heterogeneous, stratified random sampling is generally preferred for estimation the population parameters. There are a lot of sample allocation methods in stratified random sampling. However, most of sample allocation methods ignore the selection cost or assume the selection cost as equal in all strata. Bankier also suggested a power allocation method without considering the cost function [1]. However, in real life applications, it is very rare to come across such cases. Therefore, it would be more realistic to take the cost into account for allocation procedures. In this study, a new power allocation method is proposed by taking into account a non-linear cost function constraint in Bankier's method. The Neyman allocation and square root allocation results are also obtained by using this new allocation method. The performance of the proposed method is examined for different model parameters and their different cases using data from 2012 Structural Business Survey of TURKSTAT.

Tabakalı Tesadüfi Örneklemede Doğrusal Olmayan Maliyet Kısıtı Altında Yeni bir Güç Paylaştırma Yöntemi

Anahtar Kelimeler
Tabakalı tesadüfi örnekleme,
Neyman paylaştırma,
Optimum paylaştırma,
Doğrusal olmayan maliyet fonksiyonu,
Doğrusal olmayan programlama

Özet: Yığın heterojen olduğunda, yığın parametrelerini tahmin etmek için genellikle tabakalı tesadüfi örnekleme tercih edilir. Tabakalı tesadüfi örneklemede çok sayıda örnek paylaştırma yöntemi bulunmaktadır. Bununla birlikte, örnek paylaştırma yöntemlerinin çoğu tabakalardan birim seçme maliyetini ihmal etmekte ya da bütün tabakalar için eşit varsaymaktadır. Bankier da maliyet fonksiyonunu göz önüne almayan bir güç paylaştırma yöntemi önermiştir [1]. Bununla birlikte uygulamalarda, maliyetin olmadığı durumlarla karşılaşmak yok denecek kadar azdır. Bu yüzden, paylaştırma işlemi için maliyeti göz önüne almak daha gerçekçi bir yaklaşım olacaktır. Bu çalışmada, Bankier'ın modeline doğrusal olmayan maliyet kısıtını ekleyen yeni bir güç paylaştırma yöntemi önerilmiştir. Önerilen yöntemin performansı, farklı model parametreleri ve parametrelerin farklı durumları için 2012 TUİK Yapısal İş İstatistikleri verisi kullanılarak incelenmiştir.

1. Introduction

The main aim of sampling methods is increasing the precision of the estimator using prior information about the population. In practice, there exist several sampling methods implemented for this purpose. It is possible to increase precision with stratified random sampling by constituting homogenous strata when the population is heterogeneous. Equal, proportional, optimum, and Neyman allocation methods are the most popular allocation methods in stratified sampling [2]. Typically, it is assumed that the cost of the unit selection in each stratum is equal or so low that it can be ignored. A compromise among two or more allocation methods have been proposed especially for populations in which strata sizes and variances differ excessively. One of the most famous compromise strategies used in the design of several surveys is power allocation. Power allocation has been proposed by Carroll [3] and Felligi [4]. Bankier proposed a new power allocation method, which was derived from Neyman and equal allocation methods [1]. In this model, the selection cost for each stratum is assumed equal. Costa et al. proposed another

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allocation in which cost is ignored and equal and proportional allocation methods are used [5]. Longford studied an allocation method which minimizes both variance of the estimation of the strata means and variance of the estimation of the population mean [6]. Choudry et al. [7] compared the performance of their proposed allocation method with the Bankier, Costa et al., and Longford methods using real life data. Şahin Tekin et. al. [8] are proposed a new compromise allocation method with non-linear cost constraint using the model given by Costa et. al. [5].

In practice, a case of equal selection cost or ignorable cost in strata rarely occurs. For this reason, it would be more realistic approach to take the cost into consideration to determine the sample sizes of strata. Linear cost function in Eq.(1) is generally used if strata costs are taken into account.

$$t = t_0 + \sum_{h=1}^{L} t_h n_h \tag{1}$$

Here, t is defined as total cost for survey, t_0 is fixed cost, t_h is selection cost for one unit from stratum h, and n_h is the sample size of the stratum h ($h=1,2,\ldots,L$). It is very easy to determine the sample size n_h when the cost function is linear as in Eq.(1). This function is appropriate when the selection cost for one unit from stratum h is not significantly different. However, the selection of one unit from stratum h may not result in one unit increase in the cost function. For instance, in a rural area survey study, after the cost of transportation is disbursed, more than one unit can be observed without an additional cost. In this case, the selection cost of one sampling unit from stratum h would result in less than one unit increase in the cost function. In contrary, the selection cost of one sampling unit in stratum h may also result in more than one unit increase in the cost function. In such cases, cost function is in a non-linear form. Cochran [2], Bretthauer et al. [9], and Chernyak [10] have defined the non-linear cost function as given below:

$$t = t_0 + \sum_{h=1}^{L} t_h n_h^{\alpha} \tag{2}$$

Here, α indicates the effect on the cost function for the selection of one sampling unit from stratum h. If α is smaller than 1, the selection of one sampling unit from the strata affects the cost function less than one unit and if α is larger than 1, the selection of one sampling unit from the strata affects the cost function greater than one unit. Eqs.(1) and (2) have the same results when α is equal to 1. α is a positive value determined by the researcher. Since the selection cost of one unit differs between strata, it is suggested to use the cost function in Eq.(2).

In this study, a new power allocation model is proposed, which takes into account the non-linear cost function. This model using a non-linear cost constraint is a modified version of Bankier's power allocation method. The proposed power allocation method is obtained and presented in Section 2. By applying the proposed model to 2012 Structural Business Survey (SBS) data of TURKSTAT, the results for different parameters and their different levels are discussed in Section 3. Finally, a summary concludes the paper in Section 4.

2. A New Power Allocation Method

Stratified random sampling is frequently used in surveys. Providing that target population is separated into L homogenous strata; for stratum h, stratum size, and weight as well as sample size selected in the strata are defined by N_h, $W_h=N_h/N$, and n_h (h=1,2,...,L), respectively. Population mean $\bar{Y} = \sum_h W_h \bar{Y}_h$ is estimated via weighted sample mean $\bar{y}_{st} = \sum_h W_h \bar{y}_h$. In stratified random sampling, when the strata sizes are considerably different, standard allocation methods may lead to some problems. For instance, Neyman allocation minimizes the variance of $\bar{y}_{st}$ under the constraint of $n = \sum_h n_h$. However, it may cause some strata estimators to have large coefficient of variations ($CV(\bar{y}_h) = C_h / \sqrt{n_h}$). On the other hand, equal allocation ($n_h=n/L$) is efficient for the estimation of strata means. Nevertheless, its CV is larger than Neyman allocation for $\bar{y}_{st}$ estimator.

Bankier proposed a power allocation method utilizing Neyman and equal allocation methods [1]. Let $C_h = S_h / \bar{Y}_h$ be the coefficient of variation of h^{th} stratum. The loss function,

$$F = \sum_h \left\{ X_h^q CV(\bar{y}_h) \right\}^2 \tag{3}$$

is minimized subject to the constraint $n = \sum_{h=1}^{L} n_h$ and Bankier's power allocation method is given below:

$$n_h^B = \frac{C_h X_h^q}{\sum_h C_h X_h^q} n, \quad h = 1, 2, \ldots, L \tag{4}$$

Here, q is a constant in the range of $0 \le q \le 1$ as determined by the researcher, X_h is some measure of size or importance of stratum h. As seen in Eq.(4), Bankier's power allocation does not take the cost into account. Therefore, the new power allocation method is obtained by minimizing the loss function in Eq.(3) with $t = t_0 + \sum_{i=1}^{L} t_h n_h^{\alpha}$ constraint.

Theorem: In stratified random sampling with the non-linear cost function $t = t_0 + \sum_{i=1}^{L} t_h n_h^{\alpha}$, $\alpha \geq 0$, the loss function given in Eq.(4) is minimum for a specified cost t when

$$n_h = \frac{\left(X_h^{2q} C_h^2 / t_h\right)^{\frac{1}{\alpha+1}}}{\sum_h \left(X_h^{2q} C_h^2 / t_h\right)^{\frac{1}{\alpha+1}}} n \quad (5)$$

Proof:

$$\min\{F\} = \min\left\{\sum_h \left\{X_h^q CV(\bar{y}_h)\right\}^2\right\}$$

with respect to F subject to

$$t = t_0 + \sum_{i=1}^{L} t_h n_h^{\alpha}$$

Take λ as the Lagrangian multiplier and minimize

$$L = \left\{\sum_h \left\{X_h^q \frac{C_h}{\sqrt{n_h}}\right\}^2\right\} + \lambda\left(\sum_h t_h n_h^{\alpha} - T\right)$$

for chosen n_h.

Hence, to minimize the loss function F for fixed t, we have

$$\lambda \alpha t_h n_h^{\alpha+1} = X_h^{2q} C_h^2$$

and

$$n_h = \left(\frac{X_h^{2q} C_h^2}{\lambda \alpha t_h}\right)^{\frac{1}{\alpha+1}}$$

$$n = \sum_{h=1}^{L} n_h = \sum_{h=1}^{L} \left(\frac{X_h^{2q} C_h^2}{\lambda \alpha t_h}\right)^{\frac{1}{\alpha+1}}$$

$$n = \frac{1}{\lambda^{\frac{1}{\alpha+1}} \alpha^{\frac{1}{\alpha+1}}} \sum_{h=1}^{L} \left(X_h^{2q} C_h^2 / t_h\right)^{\frac{1}{\alpha+1}}$$

dividing n_h by n, we obtain Eq.(5).

The new power allocation model in Eq.(5) is a modification of Bankier's power allocation method since it utilizes a non-linear cost constraint. When $\alpha = 1$, the cost function in Eq.(2) would be in linear form. If $q = 1$, $\alpha = 1$, and $X_h = N_h \bar{Y}_h$, then Eq.(5) turns into the optimum allocation method.

$$n_h = \frac{N_h S_h / \sqrt{t_h}}{\sum_h N_h S_h / \sqrt{t_h}} n \quad (6)$$

If $q = 0$, $\alpha = 1$, and $C_h = C$ for each stratum, then Eq.(5) turns into the square root allocation method as shown below:

$$n_h = \frac{1 / \sqrt{t_h}}{\sum_h 1 / \sqrt{t_h}} n \quad (7)$$

3. Real Data Example

In this section, we discuss how the proposed allocation model works for sample size allocation. Moreover, the performance of the model was analyzed for different cases. For this purpose, a subset of 2012 SBS data were used. For analyzing the performance of the proposed model, pre-defined strata and population values of some parameters were needed. Therefore, enterprises in manufacturing sector with more than 20 employees from SBS were included as enumeration. By this way, we assumed this part as population and defined the size groups as strata. Turnover of these enterprises specified as target variable (Y) and used in the analysis. Strata sizes (N_h), strata population means ($\bar{Y}_h$), strata standard deviations (S_h), and strata CV_s (C_h) are given in Table 1 for five size groups. Besides, in Table 1, each size group indicates each stratum. For the proposed model implementation, cost function t_h values were needed. However, they are not included in 2012 SBS. Therefore, t_h values were produced hypothetically.

Table 1. Population values for SBS

Size Groups	N_h	$\bar{Y}_h$	S_h	C_h	t_h
20-49	17427	5,510,399	10,200,503	1.85	0.96
50-99	4752	14,836,096	48,390,547	3.26	0.30
100-249	3315	36,550,161	55,473,303	1.52	1.21
250-499	986	99,548,400	116,525,154	1.17	0.98
500-999	371	253,470,468	359,408,319	1.42	0.94
Turkey	26851				

The proposed model in Eq.(5) allocates the pre-determined sample size n to strata. By the help of this model, it would be possible to estimate average turnover for both Turkey and specified strata levels approaching to target $CV(\bar{y}_{st})$ and $CV(\bar{y}_h)$ values. Since t_h values are taken into consideration, we can make efficient estimations as far as cost function permitted. The Non-Linear Programming (NLP) model, proposed by Choudry et al. [7], was used to decide the sample size. This model is given in Eq.(8).

$$f : \min\left\{\sum_{h=1}^{L} n_h\right\} \quad (8)$$

with respect to *f* subject to

$$CV(\bar{y}_h) \le CV_{oh}$$

$$CV(\bar{y}_{st}) = \frac{\sqrt{V(\bar{y}_{st})}}{\bar{Y}} \le CV_o \quad 1 \le n_h \le N_h, \quad h = 1, 2, \ldots, L.$$

Using Eq.(8) and 2012 SBS parameter values in Table 1, we obtain the model in Eq.(9), which determines the sample size. For the proposed model allocation, our target values for *CVs* were specified as $CV(\bar{y}_h) \le 0.15$ for the strata means $\bar{y}_h$ and $CV(\bar{y}_{st}) \le 0.06$ for the weighted sample mean $\bar{y}_{st}$. As a result of Eq.(8), the following was obtained:

$$\min\{n\} = \min\{n_1 + n_2 + n_3 + n_4 + n_5\} \qquad (9)$$

$$n_1 \ge 150.96, n_2 \ge 430.72, n_3 \ge 99.31,$$

$$n_4 \ge 57.32, \; n_5 \ge 72.07$$

$$\frac{0.1372}{n_1} + \frac{0.2296}{n_2} + \frac{0.1468}{n_3} + \frac{0.0573}{n_4} + \frac{0.0772}{n_5} \le 0.00397$$

$$1 \le n_1 \le 17427, \; 1 \le n_2 \le 4752, \; 1 \le n_3 \le 3315,$$
$$1 \le n_4 \le 986, \; 1 \le n_5 \le 371.$$

Minimum sample size *n*=902 was obtained using the NLP allocation satisfying the specified *CV* values. This sample size calculated by iterative methods using MATLAB 2017a. Using the proposed allocation model, overall sample size *n*=902 was allocated to strata, and then $CV(\bar{y}_h)$, $CV(\bar{y}_{st})$ values were evaluated. The new allocation method was analyzed whether $CV(\bar{y}_h)$ and $CV(\bar{y}_{st})$ indicators provide the target values. For the analysis, some combination of α and q variables were used as follows:

q=0, 0.2, 0.4, 0.5, 0.6, 0.8, 1 and $\alpha = 0$, 0.25, 0.5, 0.75, 1, 1.25, 1.5, 2
where $0 \le q \le 1$ and $\alpha \ge 0$. Results are given in Tables 2-8.

Tables 2-8 in Appendix A.

When $q = 0$, strata size (N_h, $h = 1, 2, \ldots, 5$), and strata mean ($\bar{Y}_h$, $h = 1, 2, \ldots, 5$) have no effect on the determination of the sample size. For this reason, the coefficient of variation (C_h, $h = 1, 2, \ldots, 5$) and costs of strata (t_h, $h = 1, 2, \ldots, 5$) are important factors on sample size calculation. However, the coefficient of variation (C_h) is more effective than strata costs(t_h). As seen in Table 2, when $\alpha = 0$, the maximum sample size is obtained from the 2nd stratum with the largest C_h, and the minimum sample size appears in the 4th stratum with the smallest C_h. However, as α increases, the sample size of the 2nd stratum decreases and the sizes of other strata increase. In other words, the sample size differences between strata reduce. As α increases, the target value for the population, which is $CV(\bar{y}_{st}) \le 0.06$, would be ensured. Besides that, when $\alpha \le 0.75$, target values for strata $CV(\bar{y}_h) \le 0.15$ are not ensured for the strata with C_h values closer with each other. For the 2nd stratum, all cases provide the target values except $\alpha = 2$. As seen in Table 2, all target values are ensured for $1 \le \alpha \le 1.5$

As q increases, strata sizes and strata means also affect the sample sizes. As q increases, a more proper allocation giving more weight to the size of strata and mean of strata would be attained. For the same α value, all $CV(\bar{y}_h)$ values of the strata decrease, except the 2nd stratum, and approach the target value 0.15 as q increases. For example, in Table 3, while $q = 0.2$ and $\alpha = 0.25$, $CV(\bar{y}_h)$ is 0.18, and, in Table 6, while $q = 0.6$ and $\alpha = 0.25$, $CV(\bar{y}_h)$ is 0.15 for the 3rd stratum. Thus, for the same α value, $CV(\bar{y}_h)$ values reduce as q increases in all strata other than the 2nd stratum, which belongs to the largest C_h Furthermore, as q increases, $CV(\bar{y}_{st})$ values reduce for the same α value. Besides, as q increases, the range between strata sample sizes decrease for the same α value.

For fixed q, as α increases, $CV(\bar{y}_h)$ values reduce in all strata except for the 2nd stratum. For example, in Table 3.5, while $q = 0.5$ and $\alpha = 0$, the target value is $CV(\bar{y}_h) = 0.19$, and while $q = 0.5$ and $\alpha = 0.75$, the target value is $CV(\bar{y}_h) = 0.12$ for the 4th stratum. Moreover, as α increases, $CV(\bar{y}_{st})$ values decrease and provide the target value.

As seen in the results provided in Tables 2-8, for $0.75 \le \alpha \le 1$, all values of q provide the target value $CV(\bar{y}_{st}) \le 0.06$. Moreover, the target value for strata $CV(\bar{y}_h) \le 0.15$ is also provided for all cases except for some cases of the 2nd stratum. As the coefficient of variation for the 2nd stratum is greater than other strata, we obtain different results for this stratum compared to the others.

4. Discussion and Conclusion

Coefficients of variations (*CV*) for target variables are used as quality indicators regarding accuracy and reliability in most of the EU commission regulations about quality of statistics. For this reason, satisfying the specified CV values becomes an important issue. Traditional allocation methods in stratified random sampling have some difficulties to cover the needs related to CV, especially for official statistics. Bankier [1], Costa et al. [5], and Longford [6] have proposed some compromise models that can be used to overcome these difficulties. Choudry et al. [7] utilized non-linear programming in satisfying specified reliability requirements. However, none of these models used the cost function. Therefore, a new allocation model is proposed in the present study satisfying the specified CV values which takes into account the non-linear cost function.

In this newly proposed model, as q value increases for the same α value, $CV(\bar{y}_h)$ and $CV(\bar{y}_{st})$ decrease and then approach to target values when strata coefficient of variation values are close to each other. For fixed q, as α value increases, $CV(\bar{y}_h)$ and $CV(\bar{y}_{st})$ indicators decrease and then approach to target values. For fixed α, as q value increases, the survey cost decreases. For the same q value, the survey cost increases as α increases.

According to the results of the application data, when a C_h of stratum is substantially larger than others, for the same α value, $CV(\bar{y}_h)$ increases as q increases. For fixed q, $CV(\bar{y}_h)$ also increases as α increases. As seen in the results, the most important advantage of this model is the flexibility of the researchers in assigning the α and q values based on their needs. Besides, when C_h values are closer with each other, the proposed model is effective in ensuring the target values. Proposed model make more productive the survey studies compared to classical allocation methods not using the cost function, since it handles the allocation issue in more realistic way by using the cost. This model can also be improved for multivariate stratified random sampling.

References

[1] Bankier M. 1988. Power Allocation: Determining Sample Sizes for Sub-national Areas. The American Statistician, 42, 174-177.

[2] Cochran W. G. 1977. Sampling Techniques. John Wiley and Sons, 428s.

[3] Carroll J. 1970. Allocation of a Sample Between States. Unpublished memorandum, Australian Bureau of Census and Statistics.

[4] Felligi I. P. 1981. Should be Census Counts be Adjusted for Allocation Purposes? -Equity Considerations. In Current Topics in Survey Sampling, eds. D. Krewski, R. Platek, and J.N.K. Rao, New York: Academic Press, pp. 47-76.

[5] Costa A, Satorra A. & Venture E. 2004. Improving both Domain and Total Area Estimation by Composition. SORT, 28, 69-8.

[6] Longford N. T. 2006. Sample Size Calculation for Small-area Estimation. Survey Methodology, 32, 87-966.

[7] Choudry G. H., Rao J.N.K. & Hidiroglou M. A. 2012. On Sample Allocation for Efficient Domain Estimation. Survey Methodology, 38(1):23-29.

[8] Şahin Tekin T. S., Özdemir Y. A. & Metin C. B. 2017. A New Compromise Allocation Method in Stratified Random Sampling. Gazi University Journal of Science, 30(3): 181-194.

[9] Bretthauer K. M., Ross A. & Shetty B. 1999. Nonlinear İnteger Programming for Optimal Allocation in Stratified Sampling. European Journal of Operational Research, 116:667-680.

[10] Chernyak A. 2001. Optimal Allocation in Stratified and Double Random Sampling with a Nonlinear Cost Function. Journal of Mathematical Sciences, 103(4): 525-528.

Appendices

Appendix A. Tables 2-8.

Table 2. n_h, $CV(\bar{y}_h)$, and, $CV(\bar{y}_{st})$ values for $q = 0$

$q = 0$	$\alpha = 0$		$\alpha = 0.25$		$\alpha = 0.5$		$\alpha = 0.75$		$\alpha = 1$		$\alpha = 1.25$		$\alpha = 1.5$		$\alpha = 2$	
Stratum	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$
1	72	0.21	100	0.18	120	0.16	134	0.15	144	0.15	151	0.15	156	0.14	163	0.14
2	719	0.11	628	0.12	554	0.13	497	0.13	453	0.14	418	0.15	391	0.15	350	0.16
3	39	0.24	60	0.19	79	0.16	93	0.15	105	0.14	114	0.13	121	0.13	132	0.12
4	28	021	47	0.16	64	0.14	78	0.12	90	0.11	99	0.11	107	0.10	119	0.10
5	43	0.20	66	0.15	85	0.13	100	0.12	111	0.11	120	0.10	127	0.10	137	0.09
$CV(\bar{y}_{st})$	901	0.09	901	0.09	902	0.07	902	0.06	903	0.06	902	0.06	902	0.058	901	0.057

Table 3. n_h, $CV(\bar{y}_h)$, and, $CV(\bar{y}_{st})$ values for $q = 0.2$

$q = 0.2$	$\alpha = 0$		$\alpha = 0.25$		$\alpha = 0.5$		$\alpha = 0.75$		$\alpha = 1$		$\alpha = 1.25$		$\alpha = 1.5$		$\alpha = 2$	
Stratum	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$
1	79	0.20	106	0.17	125	0.16	138	0.15	147	0.15	154	0.14	158	0.14	165	0.14
2	697	0.11	605	0.12	533	0.13	478	0.14	436	0.14	404	0.15	378	0.16	340	0.17
3	46	0.22	69	0.18	88	0.15	102	0.14	113	0.14	121	0.13	128	0.13	138	0.12
4	31	0.20	51	0.15	67	0.13	81	0.12	93	0.11	102	0.10	109	0.10	121	0.09
5	47	0.19	70	0.15	89	0.13	103	0.11	113	0.11	122	0.10	129	0.10	139	0.09
$CV(\bar{y}_{st})$	900	0.091	901	0.075	902	0.067	902	0.06	902	0.060	903	0.059	902	0.058	903	0.056

Table 4. n_h, $CV(\bar{y}_h)$, and , $CV(\bar{y}_{st})$ values for q = 0.4

q = 0.4	α = 0		α = 0.25		α = 0.5		α = 0.75		α = 1		α = 1.25		α = 1.5		α = 2	
Stratum	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$
1	87	0.19	113	0.17	131	0.16	142	0.15	151	0.15	157	0.14	161	0.14	167	0.14
2	674	0.11	582	0.12	512	0.13	459	0.14	420	0.15	389	0.15	365	0.16	330	0.17
3	56	0.20	79	0.16	97	0.15	111	0.14	121	0.13	129	0.13	135	0.12	144	0.12
4	35	0.19	54	0.15	71	0.13	84	0.12	95	0.11	104	0.10	111	0.10	123	0.09
5	51	0.18	74	0.14	92	0.12	105	0.11	116	0.10	124	0.10	130	0.10	140	0.09
$CV(\bar{y}_{st})$	903	0.085	902	0.072	903	0.065	901	0.061	903	0.059	903	0.058	902	0.057	904	0.056

Table 5. n_h, $CV(\bar{y}_h)$, and , $CV(\bar{y}_{st})$ values for q = 0.5

q = 0.5	α = 0		α = 0.25		α = 0.5		α = 0.75		α = 1		α = 1.25		α = 1.5		α = 2	
Stratum	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$
1	91	0.19	116	0.17	133	0.15	145	0.15	152	0.14	158	0.14	162	0.14	167	0.14
2	661	0.11	569	0.12	501	0.13	450	0.14	412	0.15	382	0.16	359	0.16	325	0.17
3	61	0.19	85	0.16	102	0.14	115	0.13	125	0.13	132	0.12	138	0.12	147	0.12
4	36	0.19	56	0.15	72	0.13	86	0.12	96	0.11	105	0.10	112	0.10	123	0.09
5	53	0.18	76	0.14	93	0.12	107	0.11	117	0.10	125	0.10	131	0.09	140	0.09
$CV(\bar{y}_{st})$	902	0.083	902	0.070	901	0.064	903	0.061	902	0.059	902	0.058	902	0.057	902	0.056

Table 6. n_h, $CV(\bar{y}_h)$, and , $CV(\bar{y}_{st})$ values for q = 0.6

q = 0.6	α = 0		α = 0.25		α = 0.5		α = 0.75		α = 1		α = 1.25		α = 1.5		α = 2	
Stratum	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$
1	94	0.19	119	0.16	136	0.15	147	0.15	154	0.14	159	0.14	163	0.14	168	0.14
2	648	0.11	557	0.12	490	0.13	440	0.14	403	0.15	375	0.16	352	0.16	319	0.17
3	67	0.18	90	0.15	108	0.14	120	0.13	129	0.13	136	0.12	142	0.12	150	0.12
4	38	0.18	58	0.14	74	0.13	87	0.11	98	0.11	106	0.10	113	0.10	124	0.09
5	55	0.17	78	0.14	95	0.12	108	0.11	118	0.10	125	0.10	132	0.09	141	0.09
$CV(\bar{y}_{st})$	902	0.08	902	0.069	903	0.064	902	0.06	902	0.058	901	0.057	902	0.057	902	0.056

Table 7. n_h, $CV(\bar{y}_h)$, and , $CV(\bar{y}_{st})$ values for q = 0.8

q = 0.8	α = 0		α = 0.25		α = 0.5		α = 0.75		α = 1		α = 1.25		α = 1.5		α = 2	
Stratum	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$
1	102	0.18	126	0.16	141	0.15	151	0.15	157	0.14	162	0.14	165	0.14	170	0.14
2	620	0.12	531	0.13	468	0.14	421	0.15	387	0.15	360	0.16	340	0.17	309	0.17
3	79	0.16	102	0.14	119	0.13	130	0.13	138	0.12	144	0.12	149	0.12	156	0.11
4	41	0.17	61	0.14	77	0.12	90	0.11	100	0.11	108	0.10	115	0.10	126	0.09
5	59	0.16	81	0.13	98	0.12	110	0.11	120	0.10	127	0.10	133	0.09	141	0.09
$CV(\bar{y}_{st})$	901	0.076	901	0.066	903	0.062	902	0.059	902	0.058	901	0.057	902	0.056	902	0.055

Table 8. n_h , $CV(\bar{y}_h)$, and , $CV(\bar{y}_{st})$ values for q = 1

q = 1	α = 0		α = 0.25		α = 0.5		α = 0.75		α = 1		α = 1.25		α = 1.5		α = 2	
Stratum	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$	n_h	$CV(\bar{y}_h)$
1	110	0.17	132	0.16	145	0.15	154	0.14	160	0.14	164	0.14	167	0.14	171	0.14
2	590	0.12	505	0.13	445	0.14	402	0.15	371	0.16	346	0.16	327	0.17	299	0.18
3	94	0.15	116	0.13	130	0.13	140	0.12	148	0.12	153	0.11	157	0.11	162	0.11
4	45	0.17	65	0.14	80	0.12	93	0.11	102	0.10	110	0.10	117	0.10	127	0.09
5	63	0.16	85	0.13	101	0.12	112	0.11	121	0.10	128	0.10	134	0.09	142	0.09
$CV(\bar{y}_{st})$	902	0.073	903	0.064	901	0.060	901	0.058	902	0.057	901	0.056	902	0.056	901	0.055