Süleyman Demirel Üniversitesi
Fen Bilimleri Enstitüsü Dergisi
Cilt 23, Sayı 1, 1-5, 2019

Süleyman Demirel University
Journal of Natural and Applied Sciences
Volume 23, Issue 1, 1-5, 2019

DOI: 10.19113/sdufenbed.439699

Common Fixed Point Theorems via Implicit Contractions in Soft Quasi Metric Spaces

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(Alınış / Received: 02.07.2018, Kabul / Accepted: 29.01.2019, Online Yayınlanma / Published Online: 10.04.2019)

Keywords
Common fixed point,
Implicit contractions,
Well posedness,
Soft quasi metric space

Abstract:
Some common fixed point results involving implicit contractions on soft quasi metric spaces are presented in this research article. Also, the well posedness property of the common fixed point problem of mappings is defined and a theorem is given about it. Finally, some fixed point results on soft G-metric spaces are indicated to be urgent outcomes of main theorems are given in this article .

Esnek Simetrik Olmayan Metrik Uzaylarda Kapalı Büzülmeler yoluyla Ortak Sabit Nokta Teoremleri

Anahtar Kelimeler
Ortak sabit nokta,
Kapalı büzülmeler,
İyi belirlenme,
Esnek simetrik olmayan metrik uzay

Özet:
Bu araştırma makalesinde, esnek simetrik olmayan metrik uzaylarda kapalı büzülmeler içeren bazı ortak sabit nokta sonuçları sunulmuştur. Ayrıca dönüşümlerin ortak sabit nokta probleminin well-posedness özelliği tanımlanmış ve bununla ilgili bir teorem verilmiştir. Son olarak, esnek G-metrik uzaylardaki bazı sabit nokta sonuçlarının, bu makalede verilen ana teoremlerin ivedi sonuçları olduğu gösterilmiştir.

1. Introduction

Throughout this paper, we follow the notations and definitions, used in [2], [3] and [4]. For the sake of completeness, we recall some basic definitions, notations and results.

Definition 1.1. ([2]) A mapping $d : SE(\widetilde{X}) \times SE(\widetilde{X}) \to \mathbb{R}(E)^*$ is said to be a soft metric on $\widetilde{X}$ if d satisfies the following conditions:

(M1) $d(\widetilde{x},\widetilde{y}) \widetilde{\geq} \bar{0}$, for all $\widetilde{x},\widetilde{y} \widetilde{\in} \widetilde{X}$.

(M2) $d(\widetilde{x},\widetilde{y}) = \bar{0}$ if and only if $\widetilde{x} = \widetilde{y}$,

(M3) $d(\widetilde{x},\widetilde{y}) = d(\widetilde{y},\widetilde{x})$, for all $\widetilde{x},\widetilde{y} \widetilde{\in} X$,

(M4) $d(\widetilde{x},\widetilde{y}) \widetilde{\leq} d(\widetilde{x},\widetilde{z}) + d(\widetilde{z},\widetilde{y})$, for all $\widetilde{x},\widetilde{y},\widetilde{z} \widetilde{\in} X$.
The soft set $\widetilde{X}$ with a soft metric d on $\widetilde{X}$ is said to be a soft metric space and is denoted by $(\widetilde{X},d)$.

Definition 1.2. ([2]) Let $(\widetilde{x_n})$ be a sequence of soft elements in $(\widetilde{X},d)$.The sequences $(\widetilde{x_n})$ is said to be convergent in $(\widetilde{X},d)$, if there is a soft element $\widetilde{x} \widetilde{\in} \widetilde{X}$ such that $d(\widetilde{x_n},\widetilde{x}) \to \bar{0}$ as $n \to \infty$.
A sequence $(\widetilde{x_n})$ of soft elements in $(\widetilde{X},d)$ is said to be Cauchy sequence in $\widetilde{X}$, if for every $\widetilde{\varepsilon} \widetilde{\geq} \bar{0}$, there is a natural number m such that $d(\widetilde{x_i},\widetilde{x_j}) \widetilde{\leq} \widetilde{\varepsilon}$, whenever $i, j \geq m$.

Definition 1.3. ([2]) A soft metric space $(\widetilde{X},d)$ is said to be complete if every Cauchy sequence in $\widetilde{X}$ converges to some soft element of $\widetilde{X}$.

Definition 1.4. ([3]) Let X be a nonempty set and E be the nonempty set of parameters. A mapping $\widetilde{G} : SE(\widetilde{X}) \times SE(\widetilde{X}) \times SE(\widetilde{X}) \to \mathbb{R}(E)^*$ is said to be a soft generalized metric or soft G-metric on $\widetilde{X}$, if $\widetilde{G}$ satisfies the following conditions:

$(\widetilde{G}_1)$ $\widetilde{G}(\widetilde{x},\widetilde{y},\widetilde{z}) = \bar{0}$, if $\widetilde{x} = \widetilde{y} = \widetilde{z}$,

$(\widetilde{G}_2)$ $\widetilde{G}(\widetilde{x},\widetilde{x},\widetilde{y}) \widetilde{>} \bar{0}$, for all $\widetilde{x},\widetilde{y} \in SE(\widetilde{X})$ with $\widetilde{x} \neq \widetilde{y}$

$(\widetilde{G}_3)$ $\widetilde{G}(\widetilde{x},\widetilde{x},\widetilde{y}) \widetilde{\leq} \widetilde{G}(\widetilde{x},\widetilde{y},\widetilde{z})$, for all $\widetilde{x},\widetilde{y},\widetilde{z} \in SE(\widetilde{X})$ with $\widetilde{y} \neq \widetilde{z}$,

$(\widetilde{G}_4)$ $\widetilde{G}(\widetilde{x},\widetilde{y},\widetilde{z}) = \widetilde{G}(\widetilde{x},\widetilde{z},\widetilde{y}) = \widetilde{G}(\widetilde{y},\widetilde{z},\widetilde{x}) = \ldots$

$(\widetilde{G}_5)$ $\widetilde{G}(\widetilde{x},\widetilde{y},\widetilde{z}) \widetilde{\leq} \widetilde{G}(\widetilde{x},\widetilde{a},\widetilde{a}) + \widetilde{G}(\widetilde{a},\widetilde{y},\widetilde{z})$, for all $x,y,z,a \in SE(\widetilde{X})$.
The soft set $\widetilde{X}$ with a soft G-metric $\widetilde{G}$ on $\widetilde{X}$ is said to be a soft G-metric space and is denoted by $(\widetilde{X},\widetilde{G},E)$.

Proposition 1.5. *([3]) For any soft metric d on $\widetilde{X}$, we can construct a soft G-metric by the following mappings $\widetilde{G}_s$ and $\widetilde{G}_m$:*

(1) $\widetilde{G}_s(d)(\widetilde{x},\widetilde{y},\widetilde{z}) = \frac{1}{3}(d(\widetilde{x},\widetilde{y}) + d(\widetilde{y},\widetilde{z}) + d(\widetilde{x},\widetilde{z}))$,

(2) $\widetilde{G}_m(d)(\widetilde{x},\widetilde{y},\widetilde{z}) = \max\{d(\widetilde{x},\widetilde{y}) + d(\widetilde{y},\widetilde{z}) + d(\widetilde{x},\widetilde{z})\}$,

Proposition 1.6. *([3]) For any soft G-metric $\widetilde{G}$ on $\widetilde{X}$, we can construct a soft metric $d_{\widetilde{G}}$ on $\widetilde{X}$ defined by*

$$d_{\widetilde{G}}(\widetilde{x},\widetilde{y}) = \widetilde{G}(\widetilde{x},\widetilde{y},\widetilde{y}) + \widetilde{G}(\widetilde{x},\widetilde{x},\widetilde{y}). \qquad (1)$$

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Definition 1.7. ([3]) $(\widetilde{X},\widetilde{G},E)$ be a soft G-metric space and $(\widetilde{x_n})$ be a sequence of soft elements in $\widetilde{X}$. The sequence $(\widetilde{x_n})$ is said to be soft G-convergent at $\widetilde{x}$ in $\widetilde{X}$, if for every $\widetilde{\varepsilon}\widetilde{\geq}\bar{0}$, chosen arbitrarily, there exists a natural number $N=N(\widetilde{\varepsilon})$ such that $\bar{0}\widetilde{\leq}\widetilde{G}(\widetilde{x_n},\widetilde{x_n},\widetilde{x})\widetilde{<}\widetilde{\varepsilon}$, whenever $n\geq N$,i,e., $n\geq N\Rightarrow(\widetilde{x_n})\in B_{\widetilde{G}}(\widetilde{x},\widetilde{\varepsilon})$.
We denote this by $(\widetilde{x_n})\to\widetilde{x}$ as $n\to\infty$ or by $\lim_{n\to\infty}(\widetilde{x_n})=\widetilde{x}$.

Proposition 1.8. *([3]) Let $(\widetilde{X},\widetilde{G},E)$ be a soft G-metric space, for a sequence $(\widetilde{x_n})$ in $\widetilde{X}$ and soft element $\widetilde{x}$, then the followings are equivalent:*

(1) $(\widetilde{x_n})$ is soft G-convergent to $\widetilde{x}$,

(2) $d_{\widetilde{G}}(\widetilde{x_n},\widetilde{x})\to\bar{0}$ as $n\to\infty$,

(3) $\widetilde{G}(\widetilde{x_n},\widetilde{x_n},\widetilde{x})\to\bar{0}$ as $n\to\infty$,

(4) $\widetilde{G}(\widetilde{x_n},\widetilde{x},\widetilde{x})\to\bar{0}$ as $n\to\infty$,

(5) $\widetilde{G}(\widetilde{x_n},\widetilde{x_m},\widetilde{x})\to\bar{0}$ as $n,m\to\infty$.

Definition 1.9. ([3]) A soft G-metric space $(\widetilde{X},\widetilde{G},E)$ is symmetric if

$(\widetilde{G}_6)$ $\quad\widetilde{G}(\widetilde{x},\widetilde{y},\widetilde{y})=\widetilde{G}(\widetilde{x},\widetilde{x},\widetilde{y})$ for all $x,y\in SE(\widetilde{X})$.

When $(\widetilde{X},\widetilde{G},E)$ is a symmetric , many fixed point theorems on this space are particular cases of existing fixed point theorems in soft metric spaces. But for treating the non-symmetric case, Bilgili Gungor ([1]) introduced soft quasi-metric space and showed that non-symmetric soft G-metric space have a soft quasi-metric form and then many results on non-symmetric soft G-metric spaces can be reproduced from fixed point on soft quasi-metric spaces.

Definition 1.10. ([1]) Let E be the nonempty set of parameters, X be a nonempty set. If $\widetilde{Q}:SE(\widetilde{X})\times SE(\widetilde{X})\to\mathbb{R}(E)^*$ is a mapping which satisfies conditions,

$(\widetilde{Q}_1)$ $\quad\widetilde{Q}(\widetilde{u},\widetilde{v})=\bar{0}\iff\widetilde{u}=\widetilde{v}$,

$(\widetilde{Q}_2)$ $\quad\widetilde{Q}(\widetilde{u},\widetilde{v})\widetilde{\leq}\widetilde{Q}(\widetilde{u},\widetilde{w})+\widetilde{Q}(\widetilde{w},\widetilde{v})$, for all $\widetilde{u},\widetilde{v},\widetilde{w}\in SE(\widetilde{X})$
then it is said to be soft quasi-metric on $\widetilde{X}$.

And $(\widetilde{X},\widetilde{Q},E)$ is said to be soft quasi metric space.

It is simple to see that any soft metric space is a soft quasi-metric space, but any soft quasi metric space is not soft metric space.
Taking to advantages of this definition Bilgili Gungor presented important results between soft G-metric spaces and soft quasi-metric spaces (See [1]).

2. Material and Method

In the following we will define an implicit contraction mapping via soft real numbers inspired from the article of Popa and Patriciu ([5]).

Definition 2.1. Let Δ be the set of all continuous functions $L(\widetilde{s_1},...,\widetilde{s_6}):\mathbb{R}^6(E)^*\to\mathbb{R}(E)^*$ such that
$(L1)$: L is nonincreasing in variable $\widetilde{s}_5$,
$(L2)$: There exists $l_1\in\Sigma$ so that for all $\widetilde{x},\widetilde{y}\widetilde{\geq}\bar{0},L(\widetilde{x},\widetilde{y},\widetilde{y},\widetilde{x},\widetilde{x+y},\bar{0})\widetilde{\leq}\bar{0}$ means $\widetilde{x}\leq l_1(\widetilde{y})$,
$(L3)$: There exists $l_2\in\Sigma$ so that for all $\widetilde{x},\widetilde{y}\widetilde{>}\bar{0},L(\widetilde{x},\widetilde{x},\bar{0},\bar{0},\widetilde{x},\widetilde{y})\widetilde{<}\bar{0}$ means $\widetilde{x}\widetilde{\leq}l_2(\widetilde{y})$,
$\Sigma=\{\sigma:\sigma:\mathbb{R}(E)^*\to\mathbb{R}(E)^*,\sigma$ is nondecreasing functions, $\lim_{n\to\infty}\sigma^n(\widetilde{s})\widetilde{=}\bar{0}$ for each $\widetilde{s}\widetilde{\geq}\bar{0}\}$.

Example 2.2. $L(\widetilde{s}_1,...,\widetilde{s}_6)\widetilde{=}\widetilde{s}_1-\bar{a}\widetilde{s}_2-\bar{b}\widetilde{s}_3-\bar{c}\widetilde{s}_4-\bar{d}\widetilde{s}_5-\bar{e}\widetilde{s}_6$, where $\bar{a},\bar{b},\bar{c},\bar{d},\bar{e}\widetilde{\geq}\bar{0},\bar{a}+\bar{b}+\bar{c}+2\bar{d}+\bar{e}\widetilde{<}\bar{1}$.
$(L1)$: Obviously.
$(L2)$: Let $\widetilde{x},\widetilde{y}\widetilde{\geq}\bar{0}$ be and $L(\widetilde{x},\widetilde{y},\widetilde{y},\widetilde{x+y},\bar{0})\widetilde{=}\widetilde{x}-\bar{a}\widetilde{y}-\bar{b}\widetilde{y}-\bar{c}\widetilde{x}-\bar{d}\widetilde{(x+y)}\widetilde{\leq}\bar{0}$ which implies $\widetilde{x}\widetilde{\leq}\frac{\bar{a}+\bar{b}+\bar{d}}{\bar{1}-\bar{c}-\bar{d}}\widetilde{y}$ and $(L2)$ is satisfied for $l_1(\widetilde{s})\widetilde{=}\frac{\bar{a}+\bar{b}+\bar{d}}{\bar{1}-(\bar{c}+\bar{d})}\widetilde{s}$.
$(L3)$: Let $\widetilde{x},\widetilde{y}\widetilde{>}\bar{0}$ be and $L(\widetilde{x},\widetilde{x},\bar{0},\bar{0},\widetilde{x},\widetilde{y})\widetilde{=}\widetilde{x}-\bar{a}\widetilde{x}-\bar{d}\widetilde{x}-\bar{e}\widetilde{y}\widetilde{\leq}\bar{0}$ which implies $\widetilde{x}\widetilde{\leq}\frac{\bar{e}}{\bar{1}-(\bar{a}+\bar{d})}\widetilde{y}$ and $(L3)$ is satisfied for $l_2(\widetilde{s})\widetilde{=}\frac{\bar{e}}{\bar{1}-(\bar{a}+\bar{d})}\widetilde{s}$.

Example 2.3. $L(\widetilde{s}_1,...,\widetilde{s}_6)\widetilde{=}\widetilde{s}_1-\bar{k}\max\{\widetilde{s}_2,...,\widetilde{s}_6\}$, where $\bar{k}\in[\bar{0},\frac{\bar{1}}{2})$.
$(L1)$: Obviously.
$(L2)$: Let $\widetilde{x},\widetilde{y}\widetilde{\geq}\bar{0}$ be and $L(\widetilde{x},\widetilde{y},\widetilde{y},\widetilde{x},\widetilde{x+y},\bar{0})\widetilde{=}\widetilde{x}-\bar{k}\max\{\widetilde{x},\widetilde{y},\widetilde{x+y}\}\widetilde{\leq}\bar{0}$.Thus, $\widetilde{x}\widetilde{\leq}\frac{\bar{k}}{\bar{1}-\bar{k}}\widetilde{y}$ and $(L2)$ is satisfied for $l_1(\widetilde{s})\widetilde{=}\frac{\bar{k}}{\bar{1}-\bar{k}}\widetilde{s}$.
$(L3)$: Let $\widetilde{x},\widetilde{y}\widetilde{>}\bar{0}$ be and $L(\widetilde{x},\widetilde{x},\bar{0},\bar{0},\widetilde{x},\widetilde{y})\widetilde{=}\widetilde{x}-\bar{k}\max\{\widetilde{x},\widetilde{y}\}\widetilde{\leq}\bar{0}$. If $\widetilde{x}\widetilde{>}\widetilde{y}$, then $\widetilde{x}(\bar{1}-\bar{k})\widetilde{\leq}\bar{0}$, a contradiction. Hence $\widetilde{x}\widetilde{\leq}\widetilde{y}$ which implies $\widetilde{x}\widetilde{\leq}\bar{k}\widetilde{y}$ and $(L3)$ is satisfied for $l_2(\widetilde{s})\widetilde{=}\bar{k}\widetilde{s}$.

3. Results

In the following, we prove the existence and uniqueness of a common fixed point of operators that provide specific inequalities via implicit contractions.

Lemma 3.1. *Let $(\widetilde{X},\widetilde{Q},E)$ be a soft quasi-metric space and $h,\rho:(\widetilde{X},\widetilde{Q},E)\to(\widetilde{X},\widetilde{Q},E)$ satisfying*

$$L(\widetilde{Q}(h\widetilde{u},h\widetilde{v}),\widetilde{Q}(\rho\widetilde{u},\rho\widetilde{v}),\widetilde{Q}(\rho\widetilde{u},h\widetilde{u}),\widetilde{Q}(\rho\widetilde{v},h\widetilde{v}),\widetilde{Q}(\rho\widetilde{u},\widetilde{v}),\widetilde{Q}(\rho\widetilde{v},h\widetilde{u}))\widetilde{\leq}\bar{0},\forall\widetilde{u},\widetilde{v}\in\widetilde{X} \tag{2}$$

and L satisfying property $(L3)$. In this case, h and ρ have at most one point of coincidence.

Proof. We assume that h and ρ have two distinct point of coincidence $\widetilde{x}$ and $\widetilde{y}$. In this case, there exist $p,r\in\widetilde{X}$ so that $\widetilde{x}\widetilde{=}h\widetilde{p}\widetilde{=}\rho\widetilde{p}$ and $\widetilde{y}\widetilde{=}h\widetilde{r}\widetilde{=}\rho\widetilde{r}$. Then by using (2) we get

$$L(\widetilde{Q}(h\widetilde{p},h\widetilde{r}),\widetilde{Q}(\rho\widetilde{p},\rho\widetilde{r}),\widetilde{Q}(\rho\widetilde{p},h\widetilde{p}),\widetilde{Q}(\rho\widetilde{r},h\widetilde{r}),\widetilde{Q}(\rho\widetilde{p},h\widetilde{r}),\widetilde{Q}(\rho\widetilde{r},\rho\widetilde{p}))\widetilde{\leq}\bar{0}, \tag{3}$$

so

$$L(\widetilde{Q}(\rho\widetilde{p},\rho\widetilde{r}),\widetilde{Q}(\rho\widetilde{p},\rho\widetilde{r}),0,0,\widetilde{Q}(\rho\widetilde{p},\rho\widetilde{r}),\widetilde{Q}(\rho\widetilde{r},\rho\widetilde{p}))\widetilde{\leq}\bar{0}. \tag{4}$$

By the property (L_3) of L, we get

$$\widetilde{Q}(\rho\widetilde{p},\rho\widetilde{r})\widetilde{\leq}l_2(\widetilde{Q}(\rho\widetilde{r},\rho\widetilde{p})). \tag{5}$$

Similarly, we get

$$\widetilde{Q}(\rho\widetilde{r},\rho\widetilde{p})\widetilde{\leq}l_2(\widetilde{Q}(\rho\widetilde{p},\rho\widetilde{r})). \tag{6}$$

With using (5),(6), l is nondecreasing and $l(t)<t$ for $t>0$,

$$0\widetilde{<}\widetilde{Q}(\rho\widetilde{p},\rho\widetilde{r})\widetilde{\leq}l_2(\widetilde{Q}(\rho\widetilde{r},\rho\widetilde{p}))\widetilde{\leq}l_2^2(\widetilde{Q}(\rho\widetilde{p},\rho\widetilde{r}))\widetilde{<}\widetilde{Q}(\rho\widetilde{p},\rho\widetilde{r}). \tag{7}$$

This is a contradiction and $\rho\widetilde{p}\widetilde{=}\rho\widetilde{r}$. Thus, $\widetilde{x}\widetilde{=}h\widetilde{p}\widetilde{=}\rho\widetilde{p}\widetilde{=}\rho\widetilde{r}\widetilde{=}h\widetilde{r}\widetilde{=}\widetilde{y}$.

□

Theorem 3.2. *Let $(\widetilde{X},\widetilde{Q},E)$ be a soft quasi-metric space and $h,\rho:(\widetilde{X},\widetilde{Q},E)\to(\widetilde{X},\widetilde{Q},E)$ satisfying*

$$L(\widetilde{Q}(h\widetilde{u},h\widetilde{v}),\widetilde{Q}(\rho\widetilde{u},\rho\widetilde{v}),\widetilde{Q}(\rho\widetilde{u},h\widetilde{u}),\widetilde{Q}(\rho\widetilde{v},h\widetilde{v}),\widetilde{Q}(\rho\widetilde{u},h\widetilde{v}),\widetilde{Q}(\rho\widetilde{v},h\widetilde{u}))\widetilde{\leq}\bar{0},\forall\widetilde{u},\widetilde{v}\in\widetilde{X} \tag{8}$$

and

$$L(\widetilde{Q}(h\widetilde{u},h\widetilde{v}),\widetilde{Q}(\rho\widetilde{u},\rho\widetilde{v}),\widetilde{Q}(\rho\widetilde{u},\rho\widetilde{v}),\widetilde{Q}(h\widetilde{u},\rho\widetilde{u}),\widetilde{Q}(h\widetilde{u},\rho\widetilde{v}),\widetilde{Q}(\rho\widetilde{u},h\widetilde{v}))\widetilde{\leq}\bar{0},\forall\widetilde{u},\widetilde{v}\in\widetilde{X} \tag{9}$$

for $L\in\Delta$. Accept that $\rho(\widetilde{X})$ is a soft complete quasi metric subspace of $(\widetilde{X},\widetilde{Q},E)$ and $h(\widetilde{X})\subset\rho(\widetilde{X})$. So h and ρ have a unique point of coincidence. Besides, if h and ρ are weakly compatible, in this case h and ρ have a unique common fixed point.

Proof. Let $\widetilde{u_0}\in SE(\widetilde{X})$ be an arbitrary soft element and because of $h(\widetilde{X})\subset\rho(\widetilde{X})$, there is $\widetilde{u_1}\in SE(\widetilde{X})$ such that $h\widetilde{u_0}\widetilde{=}\rho\widetilde{u_1}$. In this way, we get $\widetilde{u_{n+1}}\in SE(\widetilde{X})$ and $h\widetilde{u_n}\widetilde{=}\rho\widetilde{u_{n+1}}$. Thus, by using (8) we obtain

$$L(\widetilde{Q}(h\widetilde{u}_{n-1},h\widetilde{u}_n),\widetilde{Q}(\rho\widetilde{u}_{n-1},\rho\widetilde{u}_n),\widetilde{Q}(\rho\widetilde{u}_{n-1},h\widetilde{u}_{n-1}),\widetilde{Q}(\rho\widetilde{u}_n,h\widetilde{u}_n),\widetilde{Q}(\rho\widetilde{u}_{n-1},h\widetilde{u}_n),\widetilde{Q}(\rho\widetilde{u}_n,h\widetilde{u}_{n-1}))\widetilde{\leq}\bar{0}, \tag{10}$$

that is,

$$L(\widetilde{Q}(\rho\widetilde{u}_n,\rho\widetilde{u}_{n+1}),\widetilde{Q}(\rho\widetilde{u}_{n-1},\rho\widetilde{u}_n),\widetilde{Q}(S\widetilde{u}_{n-1},\rho\widetilde{u}_n),\widetilde{Q}(\rho\widetilde{u}_n,h\widetilde{u}_{n+1}),\widetilde{Q}(\rho\widetilde{u}_{n-1},\rho\widetilde{u}_{n+1}),\bar{0})\widetilde{\leq}\bar{0}. \tag{11}$$

From $(L1)$ and $(\widetilde{Q}_2)$, we obtain

$$L(\widetilde{Q}(\rho\widetilde{u}_n,\rho\widetilde{u}_{n+1}),\widetilde{Q}(\rho\widetilde{u}_{n-1},\rho\widetilde{u}_n),\widetilde{Q}(S\widetilde{u}_{n-1},\rho\widetilde{u}_n),\widetilde{Q}(\rho\widetilde{u}_n,h\widetilde{u}_{n+1}),\widetilde{Q}(\rho\widetilde{u}_{n-1},\rho\widetilde{u}_n)+\widetilde{Q}(\rho\widetilde{u}_n,\rho\widetilde{u}_{n+1}),\bar{0})\widetilde{\leq}\bar{0}. \tag{12}$$

By $(L2)$, we have

$$\widetilde{Q}(\rho\widetilde{u}_n,\rho\widetilde{u}_{n+1})\leq l_1(\widetilde{Q}(\rho\widetilde{u}_{n-1},\rho\widetilde{u}_n)). \tag{13}$$

If we continue similar way, we get

$$\widetilde{Q}(\rho\widetilde{u}_n,\rho\widetilde{u}_{n+1})\widetilde{\leq}l_1^n(\widetilde{Q}(\rho\widetilde{u}_0,\rho\widetilde{u}_1)). \tag{14}$$

So with using $(\widetilde{Q}_2)$, for $m>n$,

$$\begin{aligned}\widetilde{Q}(\rho\widetilde{u}_n,\rho\widetilde{u}_m) &\widetilde{\leq}\widetilde{Q}(\rho\widetilde{u}_n,\rho\widetilde{u}_{n+1})+\widetilde{Q}(\rho\widetilde{u}_{n+1},\rho\widetilde{u}_{n+2})+\\ &\quad ...+\widetilde{Q}(\rho\widetilde{u}_{m-1},\rho\widetilde{u}_m)\\ &\widetilde{\leq}(l_1^n+l_1^{n+1}+...+l_1^{m-1})(\widetilde{Q}(\rho\widetilde{u}_0,\rho\widetilde{u}_1))\\ &\widetilde{\leq}\frac{l_1^n}{1-l_1}(\widetilde{Q}(\rho\widetilde{u}_0,\rho\widetilde{u}_1)),\end{aligned} \tag{15}$$

which means that $\widetilde{Q}(\rho\widetilde{u}_n,\rho\widetilde{u}_m)\to\bar{0}$. So $\{\rho\widetilde{u}_n\}$ is a right-Cauchy sequence.
Now, by using (9) we get

$$L(\widetilde{Q}(h\widetilde{u}_n,h\widetilde{u}_{n-1}),\widetilde{Q}(\rho\widetilde{u}_n,\rho\widetilde{x}_{n-1}),\widetilde{Q}(\rho\widetilde{u}_n,\rho\widetilde{u}_{n-1}),\widetilde{Q}(h\widetilde{u}_n,\rho\widetilde{u}_n),\widetilde{Q}(h\widetilde{u}_n,\rho\widetilde{u}_{n-1}),\widetilde{Q}(\rho\widetilde{u}_n,h\widetilde{u}_{n-1}))\widetilde{\leq}\bar{0}, \tag{16}$$

that is,

$$L(\widetilde{Q}(\rho\widetilde{u}_{n+1},\rho\widetilde{u}_n),\widetilde{Q}(\rho\widetilde{u}_n,\rho\widetilde{u}_{n-1}),\widetilde{Q}(\rho\widetilde{u}_n,\rho\widetilde{u}_{n-1}),\widetilde{Q}(\rho\widetilde{u}_{n+1},\rho\widetilde{u}_n),\widetilde{Q}(\rho\widetilde{u}_{n+1},\rho\widetilde{u}_{n-1}),\bar{0})\widetilde{\leq}\bar{0}. \tag{17}$$

From $(L1)$ and $(\widetilde{Q}_2)$, we obtain

$$L(\widetilde{Q}(\rho\widetilde{u}_{n+1},\rho\widetilde{u}_n),\widetilde{Q}(\rho\widetilde{u}_n,\rho\widetilde{u}_{n-1}),\widetilde{Q}(\rho\widetilde{u}_n,\rho\widetilde{u}_{n-1}),\widetilde{Q}(\rho\widetilde{u}_{n+1},\rho\widetilde{u}_n),\widetilde{Q}(\rho\widetilde{u}_{n+1},\rho\widetilde{u}_n)+\widetilde{Q}(\rho\widetilde{u}_n,\rho\widetilde{u}_{n-1}),\bar{0})\widetilde{\leq}\bar{0}. \tag{18}$$

By $(L2)$, we have

$$\widetilde{Q}(\rho\widetilde{u}_{n+1},\rho\widetilde{u}_n)\leq l_1(\widetilde{Q}(\rho\widetilde{u}_n,\rho\widetilde{u}_{n-1})). \tag{19}$$

If we continue similar way, we get

$$\widetilde{Q}(\rho\widetilde{u}_{n+1},\rho\widetilde{u}_n)\widetilde{\leq}l_1^n(\widetilde{Q}(\rho\widetilde{u}_1,\rho\widetilde{u}_0)). \tag{20}$$

So with using $(\widetilde{Q}_2)$, for $n>m$,

$$\begin{aligned}\widetilde{Q}(\rho\widetilde{u}_n,\rho\widetilde{u}_m) &\widetilde{\leq}\widetilde{Q}(\rho\widetilde{u}_n,\rho\widetilde{u}_{n-1})+\widetilde{Q}(\rho\widetilde{u}_{n-1},\rho\widetilde{u}_{n-2})+\\ &\quad ...+\widetilde{Q}(\rho\widetilde{u}_{m+1},\rho\widetilde{u}_m)\\ &\widetilde{\leq}(l_1^{n-1}+l_1^{n-2}+...+l_1^m)(\widetilde{Q}(\rho\widetilde{u}_1,\rho\widetilde{x}_0))\\ &\widetilde{\leq}\frac{l_1^m}{1-l_1}(\widetilde{Q}(\rho\widetilde{u}_1,\rho\widetilde{u}_0)),\end{aligned} \tag{21}$$

which means that $\widetilde{Q}(\rho\widetilde{u}_n,\rho\widetilde{u}_m)\to\bar{0}$. So $\{\rho\widetilde{u}_n\}$ is a left-Cauchy sequence.
Therefore, $\{\rho u_n\}$ is a Cauchy sequence. Because of $\rho(\widetilde{X})$ is completeness, there is a point $\widetilde{x}\widetilde{=}\rho\widetilde{p}$ in $\rho(\widetilde{X})$ so that $\rho\widetilde{u}_n\to\widetilde{x}\widetilde{=}\rho\widetilde{p}$ as $n\to\infty$. Now, we shall see $h\widetilde{p}\widetilde{=}\rho\widetilde{p}$. Indeed, if we choose $\widetilde{u}=\widetilde{u}_n$ and $\widetilde{v}=\widetilde{p}$ in (8),

$$L(\widetilde{Q}(h\widetilde{u}_{n-1},h\widetilde{p}),\widetilde{Q}(\rho\widetilde{u}_{n-1},\rho\widetilde{p}),\widetilde{Q}(\rho\widetilde{u}_{n-1},h\widetilde{u}_{n-1}),\widetilde{Q}(\rho\widetilde{p},h\widetilde{p}),\widetilde{Q}(\rho\widetilde{u}_{n-1},h\widetilde{p}),\widetilde{Q}(\rho\widetilde{p},h\widetilde{u}_{n-1}))\widetilde{\leq}\bar{0}, \tag{22}$$

this is

$$L(\widetilde{Q}(\rho\widetilde{u}_n,h\widetilde{p}),\widetilde{Q}(\rho\widetilde{u}_{n-1},\rho\widetilde{p}),\widetilde{Q}(\rho\widetilde{u}_{n-1},\rho\widetilde{u}_n),\widetilde{Q}(\rho\widetilde{p},h\widetilde{p}),\widetilde{Q}(\rho\widetilde{u}_{n-1},h\widetilde{p}),\widetilde{Q}(\rho\widetilde{p},\rho\widetilde{u}_n))\widetilde{\leq}\bar{0}. \tag{23}$$

In case of the limit $n\to\infty$, we get

$$L(\widetilde{Q}(\rho\widetilde{p},h\widetilde{p}),0,0,\widetilde{Q}(\rho\widetilde{p},h\widetilde{p}),\widetilde{Q}(\rho\widetilde{p},h\widetilde{p}),0)\widetilde{\leq}\bar{0}. \tag{24}$$

From the property $(L2)$ of L, we get $\widetilde{Q}(\rho\widetilde{p},h\widetilde{p})\widetilde{=}\bar{0}$ which means $\rho\widetilde{p}\widetilde{=}h\widetilde{p}$. So $\widetilde{k}\widetilde{=}h\widetilde{p}\widetilde{=}\rho\widetilde{p}$ is a point of coincidence. If we use Lemma 3.1, $\widetilde{k}$ is the unique point of coincidence. Furthermore, assume that h anf ρ are weakly compatible. From Lemma 3.1, h and ρ have a unique common fixed point. □

If ρ replace with the identity function in Theorem 3.2, the following corollary is obtained.

Corollary 3.3. *Let $(\widetilde{X},\widetilde{Q},E)$ be a complete soft quasi-metric space and $h:(\widetilde{X},\widetilde{Q},E)\to(\widetilde{X},\widetilde{Q},E)$ satisfying*

$$L(\widetilde{Q}(h\widetilde{u},h\widetilde{v}),\widetilde{Q}(\widetilde{u},\widetilde{v}),\widetilde{Q}(\widetilde{u},h\widetilde{u}),\widetilde{Q}(\widetilde{v},h\widetilde{v}),\widetilde{Q}(\widetilde{u},h\widetilde{v}),\widetilde{Q}(\widetilde{v},h\widetilde{u}))\widetilde{\leq}\bar{0},\forall\widetilde{u},\widetilde{v}\in\widetilde{X} \tag{25}$$

and

$$L(\widetilde{Q}(h\widetilde{u},h\widetilde{v}),\widetilde{Q}(\widetilde{u},\widetilde{v}),\widetilde{Q}(\widetilde{u},\widetilde{v}),\widetilde{Q}(h\widetilde{u},\widetilde{u}),\widetilde{Q}(h\widetilde{u},\widetilde{v}),\widetilde{Q}(\widetilde{u},h\widetilde{v}))\widetilde{\leq}\bar{0},\forall\widetilde{u},\widetilde{v}\in\widetilde{X} \tag{26}$$

where $L\in\Delta$. Then h has a unique fixed point.

We shall give the following corollary related to Ciric contraction type in [6] with choosing L as given in Example 2.3 and applying Theorem 3.2.

Corollary 3.4. *Let $(\widetilde{X},\widetilde{Q},E)$ be a soft quasi-metric space and $h,\rho:(\widetilde{X},\widetilde{Q},E)\to(\widetilde{X},\widetilde{Q},E)$ satisfying*

$$\widetilde{Q}(h\widetilde{u},h\widetilde{v})\widetilde{\leq}k\max\{\widetilde{Q}(\rho\widetilde{u},\rho\widetilde{v}),\widetilde{Q}(\rho\widetilde{u},h\widetilde{u}),\widetilde{Q}(\rho\widetilde{v},h\widetilde{v}),\widetilde{Q}(\rho\widetilde{u},h\widetilde{v}),\widetilde{Q}(\rho\widetilde{v},h\widetilde{u})\},\forall\widetilde{u},\widetilde{v}\in\widetilde{X} \tag{27}$$

and

$$\widetilde{Q}(h\widetilde{u},h\widetilde{v})\widetilde{\leq}k\max\{\widetilde{Q}(\rho\widetilde{u},\rho\widetilde{v}),\widetilde{Q}(\rho\widetilde{u},\rho\widetilde{v}),\widetilde{Q}(h\widetilde{u},\rho\widetilde{u}),\widetilde{Q}(h\widetilde{u},\rho\widetilde{v}),\widetilde{Q}(\rho\widetilde{u},h\widetilde{v})\},\forall\widetilde{u},\widetilde{v}\in\widetilde{X} \tag{28}$$

for $k\in[0,\frac{1}{2})$. Accept that $\rho(\widetilde{X})$ is a soft complete quasi metric subspace of $(\widetilde{X},\widetilde{Q},E)$ and $h(\widetilde{X})\subset\rho(\widetilde{X})$. So h and ρ have a unique point of coincidence. Furthermore, if h and ρ are weakly compatible, in this case h and ρ have a unique common fixed point.

Example 3.5. Let $\widetilde{\mathbb{R}}$ be the soft real set equipped with the soft quasi metric

$$\widetilde{Q}(\widetilde{x},\widetilde{y})=\begin{cases}\widetilde{y}-\widetilde{x} & \text{if } \widetilde{x}\leq\widetilde{y},\\ \bar{1} & \text{if } \widetilde{x}>\widetilde{y}.\end{cases} \tag{29}$$

$(\widetilde{\mathbb{R}},\widetilde{Q},E)$ is a complete soft quasi metric space and if the mapping $h:(\widetilde{\mathbb{R}},\widetilde{Q},E)\to(\widetilde{\mathbb{R}},\widetilde{Q},E)$ is chosen as $h:SE(\widetilde{\mathbb{R}})\to SE(\widetilde{\mathbb{R}}),h(\widetilde{x})=\bar{c}$, for any constant $c\in\mathbb{R}$. If ρ replace with the identity function in Theorem 3.2 and $L\in\Delta$ is chosen in Example 2.3, then the mapping h satisfies the inequalities 8 and 9. Thus, all conditions of Corollary 3.3 is satisfied and so h has a unique fixed point. Indeed, $\bar{c}$ is a soft real number which is unique fixed point of h.

3.1. Well posedness in soft quasi-metric spaces

In the following, the subject of the well-posedness in soft quasi-metric spaces are presented taking into account other work done in this area (see [[7]], [[8]] and [[9]]).

Definition 3.6. Let $(\widetilde{X},\widetilde{Q},E)$ be a soft quasi-metric space and $h:(\widetilde{X},\widetilde{Q},E)\to(\widetilde{X},\widetilde{Q},E)$ be a given mapping. When the following conditions satisfy, we say the fixed point problem of f is said to be well posed.

(1) $\widetilde{u_0}\in\widetilde{X}$ is the unique fixed point of h,

(2) $(\widetilde{u}_n)\subseteq\widetilde{X}$ is any sequence with $\lim_{n\to\infty}\widetilde{Q}(h\widetilde{u}_n,\widetilde{u}_n)\widetilde{=}\lim_{n\to\infty}\widetilde{Q}(\widetilde{u}_n,\widetilde{h}u_n)\widetilde{=}\bar{0}$, then we get $\lim_{n\to\infty}\widetilde{Q}(\widetilde{u_0},\widetilde{u}_n)\widetilde{=}\lim_{n\to\infty}\widetilde{Q}(\widetilde{u}_n,\widetilde{u_0})\widetilde{=}\bar{0}$.

Definition 3.7. Let $(\widetilde{X},\widetilde{Q},E)$ be a soft quasi-metric space and $h,\rho:(\widetilde{X},\widetilde{Q},E)\to(\widetilde{X},\widetilde{Q},E)$ be given mappings. When the following conditions satisfy, we say the common fixed point problem of h and ρ is said to be well posed:

(1) $\widetilde{u_0}\in\widetilde{X}$ is the unique common fixed point of h and ρ,

(2) $(\widetilde{u}_n)\subseteq\widetilde{X}$ is any sequence with

$$\begin{gathered}\lim_{n\to\infty}\widetilde{Q}(h\widetilde{u}_n,\widetilde{u}_n)\widetilde{=}\lim_{n\to\infty}\widetilde{Q}(\widetilde{u}_n,\widetilde{h}u_n)\widetilde{=}\bar{0}\\ \text{and}\\ \lim_{n\to\infty}\widetilde{Q}(\rho\widetilde{u}_n,\widetilde{u}_n)\widetilde{=}\lim_{n\to\infty}\widetilde{Q}(\widetilde{u}_n,\widetilde{\rho}u_n)\widetilde{=}\bar{0},\end{gathered} \tag{30}$$

then we get $\lim_{n\to\infty}\widetilde{Q}(\widetilde{u_0},\widetilde{u}_n)\widetilde{=}\lim_{n\to\infty}\widetilde{Q}(\widetilde{u}_n,\widetilde{u_0})\widetilde{=}\bar{0}$.

Definition 3.8. Let $L(\widetilde{s}_1,\ldots,\widetilde{s}_6):\mathbb{R}^6(E)^*\to\mathbb{R}(E)^*$ be a mapping. It is said to L has property (L_k) if $L(\widetilde{x},\widetilde{y},\widetilde{y},\widetilde{z},\widetilde{y},\widetilde{x})\widetilde{\leq}\bar{0}$ and $L(\widetilde{x},\widetilde{y},\widetilde{y},\widetilde{z},\widetilde{x},\widetilde{y})\widetilde{\leq}\bar{0}$ for all $\widetilde{x},\widetilde{y},\widetilde{z}\geq\bar{0}$, then there exists $k\in(0,1)$ such that $\widetilde{x}\widetilde{\leq}k\max\{\widetilde{y},\widetilde{z}\}$.

Theorem 3.9. *Let $(\widetilde{X},\widetilde{Q},E)$ be a soft quasi-metric space and $h,\rho:(\widetilde{X},\widetilde{Q},E)\to(\widetilde{X},\widetilde{Q},E)$ satisfy hypotheses of Theorem 3.2 and L has property (L_k). In this case, the common fixed point problem h and ρ is well posed.*

Proof. Using Theorem 3.2, h and ρ have a unique common fixed point $\widetilde{u}$. Let $\{\widetilde{u}_n\}\in\widetilde{X}$ be a sequence such that

$$\begin{gathered}\lim_{n\to\infty}\widetilde{Q}(h\widetilde{u}_n,\widetilde{u}_n)\widetilde{=}\lim_{n\to\infty}\widetilde{Q}(\widetilde{u}_n,\widetilde{h}u_n)\widetilde{=}\bar{0}\\ \text{and}\\ \lim_{n\to\infty}\widetilde{Q}(\rho\widetilde{u}_n,\widetilde{u}_n)\widetilde{=}\lim_{n\to\infty}\widetilde{Q}(\widetilde{u}_n,\widetilde{\rho}u_n)\widetilde{=}\bar{0}.\end{gathered} \tag{31}$$

Now, by using (9) we get that

$$L(\widetilde{Q}(h\widetilde{u},h\widetilde{u}_n),\widetilde{Q}(\rho\widetilde{u},\rho\widetilde{u}_n),\widetilde{Q}(\rho\widetilde{u},\rho\widetilde{u}_n),\widetilde{Q}(h\widetilde{u},\rho\widetilde{u}),\widetilde{Q}(h\widetilde{u},\rho\widetilde{u}_n),\widetilde{Q}(\rho\widetilde{u},h\widetilde{u}_n))\widetilde{\leq}\bar{0}, \tag{32}$$

so

$$L(\widetilde{Q}(\widetilde{u},h\widetilde{u}_n),\widetilde{Q}(\widetilde{u},\rho\widetilde{u}_n),\widetilde{Q}(\widetilde{u},\rho\widetilde{u}_n),\bar{0},\widetilde{Q}(\widetilde{u},\rho\widetilde{u}_n),\widetilde{Q}(\widetilde{u},h\widetilde{u}_n))\widetilde{\leq}\bar{0}. \tag{33}$$

And by using (L_k) property of L, we have

$$\begin{aligned}\widetilde{Q}(\widetilde{u},h\widetilde{u}_n)&\widetilde{\leq}k\max\{\widetilde{Q}(\widetilde{u},\rho\widetilde{u}_n),\bar{0}\}\\&\widetilde{\leq}k\widetilde{Q}(\widetilde{u},\rho\widetilde{u}_n)\end{aligned} \tag{34}$$

From $(\widetilde{Q}_2)$ and (34), we get

$$\begin{aligned}\widetilde{Q}(\widetilde{u},\widetilde{u}_n)&\widetilde{\leq}\widetilde{Q}(\widetilde{u},h\widetilde{u}_n)+\widetilde{Q}(h\widetilde{u}_n,\widetilde{u}_n)\\&\widetilde{\leq}k\widetilde{Q}(\widetilde{u},\rho\widetilde{u}_n)+\widetilde{Q}(h\widetilde{u}_n,\widetilde{u}_n)\\&\widetilde{\leq}k[\widetilde{Q}(\widetilde{u},\widetilde{u}_n)+\widetilde{Q}(\widetilde{u}_n,\rho\widetilde{u}_n)]+\widetilde{Q}(h\widetilde{u}_n,\widetilde{u}_n).\end{aligned} \tag{35}$$

Hence,

$$\widetilde{Q}(\widetilde{u},\widetilde{u}_n)\widetilde{\leq}\frac{k}{1-k}\widetilde{Q}(\widetilde{u}_n,\rho\widetilde{u}_n)]+\frac{1}{1-k}\widetilde{Q}(h\widetilde{u}_n,\widetilde{u}_n). \tag{36}$$

If we take limit as $n\to\infty$, we get $\lim_{n\to\infty}\widetilde{Q}(\widetilde{u},\widetilde{u}_n)\widetilde{=}\bar{0}$. Similarly, if we use (9) again, we get that

$$L(\widetilde{Q}(h\widetilde{u}_n,h\widetilde{u}),\widetilde{Q}(\rho\widetilde{u}_n,\rho\widetilde{u}),\widetilde{Q}(\rho\widetilde{u}_n,\rho\widetilde{u}),\widetilde{Q}(h\widetilde{u}_n,\rho\widetilde{x}_n),\widetilde{Q}(h\widetilde{u}_n,\rho\widetilde{u}),\widetilde{Q}(\rho\widetilde{u}_n,h\widetilde{u}))\widetilde{\leq}\bar{0}, \tag{37}$$

so

$$L(\widetilde{Q}(h\widetilde{u}_n,\widetilde{u}),\widetilde{Q}(\rho\widetilde{u}_n,\widetilde{u}),\widetilde{Q}(\rho\widetilde{u}_n,\widetilde{u}),\widetilde{Q}(h\widetilde{u}_n,\rho\widetilde{u}_n),\widetilde{Q}(h\widetilde{u}_n,\widetilde{u}),\widetilde{Q}(\rho\widetilde{u}_n,\widetilde{u}))\widetilde{\leq}\bar{0}, \tag{38}$$

And by using (L_k) property of L, we have

$$\begin{aligned}\widetilde{Q}(h\widetilde{u}_n,\widetilde{u})&\widetilde{\leq}k\max\{\widetilde{Q}(\rho\widetilde{u}_n,\widetilde{u}),\widetilde{Q}(h\widetilde{u}_n,\rho\widetilde{u}_n)\}\\&\widetilde{\leq}k[\widetilde{Q}(\rho\widetilde{u}_n,\widetilde{u})+\widetilde{Q}(h\widetilde{u}_n,\rho\widetilde{u}_n)]\end{aligned} \tag{39}$$

From $(\widetilde{Q}_2)$ and (39), we get

$$\begin{aligned}\widetilde{Q}(\widetilde{u}_n,\widetilde{u})&\widetilde{\leq}\widetilde{Q}(\widetilde{u}_n,h\widetilde{u}_n)+\widetilde{Q}(h\widetilde{u}_n,\widetilde{u})\\&\widetilde{\leq}\widetilde{Q}(\widetilde{u}_n,h\widetilde{u}_n)+k[\widetilde{Q}(\rho\widetilde{u}_n,\widetilde{u})+\widetilde{Q}(h\widetilde{u}_n,\rho\widetilde{u}_n)]\\&\widetilde{\leq}\widetilde{Q}(\widetilde{u}_n,h\widetilde{u}_n)+k[\widetilde{Q}(\rho\widetilde{u}_n,\widetilde{u}_n)+\widetilde{Q}(\widetilde{u}_n,\widetilde{u})\\&+\widetilde{Q}(h\widetilde{u}_n,\widetilde{u}_n)+\widetilde{Q}(\widetilde{u}_n,\rho\widetilde{u}_n)].\end{aligned} \tag{40}$$

Hence,

$$\widetilde{Q}(\widetilde{u}_n,\widetilde{u})\widetilde{\leq}\frac{1}{1-k}\widetilde{Q}(\widetilde{u}_n,h\widetilde{u}_n) + \frac{k}{1-k}[\widetilde{Q}(\rho\widetilde{u}_n,\widetilde{u}_n)+\widetilde{Q}(h\widetilde{u}_n,\widetilde{u}_n)+\widetilde{Q}(\widetilde{u}_n,\rho\widetilde{u}_n)]. \tag{41}$$

If we take limit as $n\to\infty$, we get $\lim_{n\to\infty}\widetilde{Q}(\widetilde{u}_n,\widetilde{u})\widetilde{=}\bar{0}$. Thus, proof is completed. □

3.2. Results on soft G-metric spaces

In this section, we obtain some consequences of our main theorems.

Corollary 3.10. *Let $(\widetilde{X},\widetilde{G},E)$ be a soft G-metric space and $h,\rho:(\widetilde{X},\widetilde{G},E)\to(\widetilde{X},\widetilde{G},E)$ satisfying*

$$L(\widetilde{G}(h\widetilde{u},h\widetilde{v},h\widetilde{v}),\widetilde{G}(\rho\widetilde{u},\rho\widetilde{v},\rho\widetilde{v}),\widetilde{G}(\rho\widetilde{u},h\widetilde{u},h\widetilde{u}), \widetilde{G}(\rho\widetilde{v},h\widetilde{v},h\widetilde{v}),\widetilde{G}(\rho\widetilde{u},h\widetilde{v},h\widetilde{v}),\widetilde{G}(\rho\widetilde{v},h\widetilde{u},h\widetilde{u}))\widetilde{\leq}\bar{0}, \tag{42}$$

and

$$L(\widetilde{G}(h\widetilde{u},h\widetilde{v},h\widetilde{v}),\widetilde{G}(\rho\widetilde{u},\rho\widetilde{v},\rho\widetilde{v}),\widetilde{G}(\rho\widetilde{u},\rho\widetilde{v},\rho\widetilde{v}), \widetilde{G}(h\widetilde{u},\rho\widetilde{u},\rho\widetilde{u}),\widetilde{G}(h\widetilde{u},\rho\widetilde{v},\rho\widetilde{v}),\widetilde{G}(\rho\widetilde{u},h\widetilde{v},h\widetilde{v}))\widetilde{\leq}\bar{0}, \tag{43}$$

for all $\widetilde{u},\widetilde{v}\in\widetilde{X}$ where $L\in\Delta$. Accept that $\rho(\widetilde{X})$ is a soft G-complete metric subspace of $(\widetilde{X},\widetilde{G},E)$ and $h(\widetilde{X})\subset\rho(\widetilde{X})$. Thus h and ρ have a unique point of coincidence. Besides, if h and ρ are weakly compatible, then h and ρ have a unique common fixed point.

Proof. If we choose $\widetilde{Q}(\widetilde{u},\widetilde{v})=\widetilde{G}(\widetilde{u},\widetilde{v},\widetilde{v})$, $\widetilde{Q}$ is a soft quasi-metric and also $(\rho(\widetilde{X}),\widetilde{Q},E)$ is a soft complete quasi-metric subspace of $(\widetilde{X},\widetilde{Q},E)$. Then we obtain the hypothesis of Theorem 3.2 and the result gets from Theorem 3.2. □

Now, we define the notion of the well posedness on soft G-metric spaces.

Definition 3.11. Let $(\widetilde{X},\widetilde{G},E)$ be a soft G-metric space and $h,\rho:(\widetilde{X},\widetilde{G},E)\to(\widetilde{X},\widetilde{G},E)$ be given mappings. When the following conditions satisfy, we say the common fixed point problem of h and ρ is said to be well posed:

(1) $\widetilde{u}_0\in\widetilde{X}$ is the unique common fixed point of h and ρ,

(2) $(\widetilde{u}_n)\subseteq\widetilde{X}$ is any sequence with

$$\lim_{n\to\infty}\widetilde{G}(\widetilde{u}_n,\widetilde{f}u_n,\widetilde{h}u_n)\widetilde{=}\bar{0} \text{ and } \lim_{n\to\infty}\widetilde{G}(\widetilde{u}_n,\widetilde{g}u_n,\widetilde{\rho}u_n)\widetilde{=}\bar{0}, \tag{44}$$

then we get $\lim_{n\to\infty}\widetilde{G}(\widetilde{u}_n,\widetilde{u}_0,\widetilde{u}_0)\widetilde{=}\bar{0}$.

Corollary 3.12. *Let $(\widetilde{X},\widetilde{G},E)$ be a soft G-metric space and $h,\rho:(\widetilde{X},\widetilde{G},E)\to(\widetilde{X},\widetilde{G},E)$ satisfy hypotheses of Corollary 3.3 and L has property (L_k). Then, the common fixed point problem h and ρ is well posed.*

Proof. If we choose $\widetilde{Q}(\widetilde{u},\widetilde{v})=\widetilde{G}(\widetilde{u},\widetilde{v},\widetilde{v})$, $\widetilde{Q}$ is a soft quasi-metric.

Then we obtain the hypothesis of Theorem 3.9 and the result gets from Theorem 3.9. Here we should attend to $\widetilde{G}(\widetilde{u},\widetilde{v},\widetilde{v})\widetilde{\leq}2\widetilde{G}(\widetilde{v},\widetilde{u},\widetilde{u})$ for all $\widetilde{u},\widetilde{v}$. □

4. Discussion and Conclusion

In this work, firstly the implicit contraction mapping via soft real numbers is defined. In this way, common fixed point results via implicit contractions on soft quasi metric spaces are given. Then, the well posedness property of the common fixed point problem of mappings are defined and a theorem is given about it.Finally, some fixed point results on soft G-metric spaces are indicated to be immediate consequences of our main theorems on soft quasi metric spaces .

Acknowledgment

This research article was assistanced by Amasya University Research Fund Project (FMB-BAP 17-0272).

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