Konuralp Journal of Mathematics, *7 (1) (2019) 233-234*

Konuralp Journal of Mathematics

Journal Homepage: www.dergipark.gov.tr/konuralpjournalmath
e-ISSN: 2147-625X

Corrigendum to Translation Surfaces in the 3-Dimensional Simply Isotropic $\mathbb{I}_3^1$ Satisfying $\Delta^{III}x_i = \lambda_i x_i$

Konuralp Journal of Mathematics, 4(1) (2016), 275-281

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Abstract

In [1], there is a mistake in **Theorem 4.2** that appered in the paper. We here provide a correct theorem.

Keywords: *Simply isotropic space, translation surfaces, laplace operator.*
2010 Mathematics Subject Classification: *53A35,53B30*

In previous paper [1], the coefficients and the Laplacian operator $\Delta^{\mathbf{III}}$ of the third fundamental form are not correct. The third fundamental form and the Laplacian operator $\Delta^{\mathbf{III}}$ with respect to non-degenerate third fundamental form **III** on **M** in simply isotropic 3-space are defined by

$$\mathbf{III} = Rdu^2 + 2T\,dudv + Sdv^2 \tag{4.1}$$

and

$$\Delta^{\mathbf{III}}\mathbf{x} = -\frac{1}{\sqrt{RS-T^2}}\left(\partial_u\left(\frac{S\mathbf{x}_u - T\mathbf{x}_v}{\sqrt{RS-T^2}}\right) - \partial_v\left(\frac{T\mathbf{x}_u - R\mathbf{x}_v}{\sqrt{RS-T^2}}\right)\right),$$

where

$$R = L^2 + M^2,\ T = M(L+N),\ S = N^2 + M^2,$$

respectively [2, 3]. In fact, the third fundamental form **III** is expressed in terms of the first fundamental form **I** and the second fundamental form **II** in simply isotropic 3-space, that is,

$$\mathbf{K}(\mathbf{I}) + \mathbf{III} - 2\mathbf{H}(\mathbf{II}) = 0,$$

where **K** and **H** are the Gaussian curvature and the mean curvature, respectively [2 ,3]. Now following the similar type of steps as in section 4 and section 5, we can easily find out:

$$\Delta^{\mathbf{III}}\mathbf{x} = \left(\Delta^{\mathbf{III}}\mathbf{x}_1, \Delta^{\mathbf{III}}\mathbf{x}_2, \Delta^{\mathbf{III}}\mathbf{x}_3\right) = \left(\frac{f'''}{f''^3}, \frac{g'''}{g''^3}, -\frac{1}{f''} - \frac{1}{g''} + \frac{f'f'''}{f''^3} + \frac{g'g'''}{g''^3}\right) \tag{4.2}$$

The equation in (4.2) gives rise to the following system of differential equations

$$\frac{f'''}{f''^3} = \lambda_1 u, \tag{4.3}$$

$$\frac{g'''}{g''^3} = \lambda_2 v, \tag{4.4}$$

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$$-\frac{1}{f''}-\frac{1}{g''}+\frac{f'f'''}{f''^3}+\frac{g'g'''}{g''^3}=\lambda_3\left(f+g\right), \tag{4.5}$$

where $\lambda_1,\lambda_2,\lambda_3\in\mathbb{R}$. Combining equations (4.3), (4.4) and (4.5), we get

$$-f''-g''+\left(\lambda_1uf'+\lambda_2vg'-\lambda_3\left(f+g\right)\right)f''g''=0. \tag{4.6}$$

Since the differential equation (4.6) cannot be solved analytically, therefore apart from the harmonic case, the cases with respect to non-zero constants λ_1,λ_2 and λ_3 does not exists. Supposing $f''=0$ and $g''=0$. This implies that the third fundamental form in this case is degenerate, which is a contradiction. Now Suppose that **M** is **III**-harmonic, i.e., $\lambda_1=\lambda_2=\lambda_3$, then from (4.6), we get

$$-f''=g''. \tag{4.7}$$

Therefore for some $p\in\mathbb{R}\backslash\{0\}$, we obtain

$$-f''=g''=p,$$

and

$$f(u)=-\frac{pu^2}{2}+c_1u+c_2,\ g(v)=\frac{pv^2}{2}+c_3v+c_4,$$

where $c_i\in\mathbb{R}$. In this case **M** is parameterized by

$$\mathbf{x}(u,v)=\left(u,v,\left(-\frac{pu^2}{2}+c_1u+c_2\right)+\left(\frac{pv^2}{2}+c_3v+c_4\right)\right). \tag{4.8}$$

Therefore, we have the following:

Theorem 4.2. Let **M** be a translation surface with non-degenerate the third fundamental form in the three dimensional simply isotropic space $\mathbb{I}_3^1$. Then, the surface **M** satisfying the condition $\Delta^{\mathbf{III}}\mathbf{x}_i=\lambda_i\mathbf{x}_i$ is only **III**-harmonic and is parameterized by (4.8)

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