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A New Type of Soft Covering Based Rough Sets Applied to Multicriteria Group Decision Making for Medical Diagnosis

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Abstract
In this work we define a new type of soft covering upper approximation operator and study its basic and topological properties. Comparing with other type of soft covering operation, our soft covering upper approximation is more accurate and have more properties. Based on the new type of soft covering upper approximation operator, we give a new kind of soft covering based rough sets. Also we present an example in medicine which determines the risk of prostate cancer. Our aim is to gain results more reasonable by using upper and lower approximations of a new kind of soft covering based rough sets and to help the doctor to determine that the patient needs biopsy or not.

Keywords: *Soft set; rough set; soft covering approximation space; soft covering based rough set; prostate cancer.*

AMS Subject Classification (2010): *Primary: 03E72 ; Secondary: 90B50; 06D72.*

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1. Introduction

The classical mathematical methods are inadequate to solve many complex problems in economics, engineering, environmental science, sociology, medical science and many other fields, since these kinds of problems have their own uncertainties. To avoid difficulties in dealing with uncertainties, researches proposed many mathematical methods such as fuzzy set theory, rough set theory and soft set theory. The most successful theoretical approach to the vagueness is undoubtedly fuzzy set theory [4] proposed by Zadeh in 1965. The fuzzy set theory is based on fuzzy membership function. Fuzzy membership function determines the belongingness of an element to a set to a degree. The rough set theory [1] proposed by Pawlak in 1982 is another mathematical tool for dealing with uncertainty of imprecise data and vagueness. The fundamental concepts of classical rough sets are lower and upper approximations based on equivalence relations. But equivalence relations are too restrictive to deal with many applications in real world problems. To handle such type of circumstances, it has been extended to covering based rough sets [5, 6]. In 1999, Molodtsov [2] proposed the concept of a soft set, which is completely new mathematical approach to vagueness. The absence of any restrictions on the approximate description in soft set theory makes this theory very convenient and easily applicable in practice. Maji et al. [7] carried out Molodtsov's idea by introducing several operations in soft set theory. Ali et al. [8] introduced some operations over soft sets. It has been found that fuzzy sets, rough sets and soft sets are closely related concepts [9]. Feng et al. investigated the concept of soft rough set in 2010 [10] which is a combination of soft set and rough set. In [10, 11] basic properties of soft rough approximations were presented and supported by some illustrative examples. In fact, a soft set instead of an equivalence relation was used to granulate the universe of discourse. Feng [12] gave an application of soft rough approximations in multicriteria group decision making problems and his method enables us to select the optimal object in more reliable manner.

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Topology is a useful theoretical framework for the study of rough set theory and soft set theory. Many authors investigated topological structures of rough sets and soft sets [1, 3, 5, 6, 19, 23–28].

Yüksel et al. established a soft covering approximation space in 2014 [3] and investigated the concept of soft covering based rough set which is a combination of covering soft set and rough set. In this work we study a new kind of soft covering based rough sets and its basic and topological properties. Then we give counterexamples for unsatisfied properties. We use a new type of soft covering approximations at Feng's method in medicine and aim to help the doctors for diagnosing the prostate cancer risk.

Prostate cancer is the second most common cause of cancer death among men in most industrialized countries. It depends on various factors as family's cancer history, age, ethnic background and the level of prostate specific antigen (PSA) in the blood. Since PSA is a substance produced by the prostate, it is very important factor to an initial diagnosis of level PSA for patients [13–15]. As known when the prostate cancer can be diagnosed earlier, the patient can be completely treated. The definitive diagnose of the prostate cancer is possible with prostate biopsy. The results of PSA test, rectal examination and transrectal findings help to the doctor to decide biopsy is necessary or not [16–18]. If biopsy is applied for diagnosing canser disease, the cancer may spread to the other vital organs [16]. For this reason the biopsy method is undesirable.

In this study, we aim to reduce the number of patients who are applied biopsy. Therefore, we develop a prediction method which determines the necessity of biopsy and it gives to user a range of the risk of the cancer. For this process prostate specific antigen (PSA), free prostate specific antigen (fPSA), prostate volume (PV) and age of the patient are use as laboratory data . We determine the risk of prostate cancer by using upper and lower approximations of a new kind of soft covering based rough sets generated with the values PSA, fPSA, PV and age data of patients. We observed that this method is more fast, economical and without risk than the traditional diagnostic methods.

2. Preliminaries

In this section, we recall some basic concepts such as rough sets, soft sets, fuzzy sets, fuzzy soft sets and soft rough sets to be used in this paper. For more details on these topics, some references are mentioned for relevant readers [1, 2, 4, 7, 8, 10, 11, 19, 22]. Throughout this paper, the universe U is supposed to be a finite nonempty set. $\emptyset$ be the empty set, $-X$ the complement of X in U.

We first recall some fundamental facts about Pawlak's rough sets.

Definition 2.1. [1] Let U be a finite set and R be an equivalence relation on U. Then the pair (U, R) is called a Pawlak approximation space.

R generates a partition $U/R = \{Y_1, Y_2, ..., Y_m\}$ on U, where $Y_1, Y_2, ..., Y_m$ are the equivalence classes generated by the equivalence relation R. In the rough set theory, these are also called elementary sets of R.

For any $X \subseteq U$, we can describe X by the elementary sets of R and two sets:

$$\underline{R}(X) = \cup\{Y_i \in U/R : Y_i \subseteq X\},$$

$$\overline{R}(X) = \cup\{Y_i \in U/R : Y_i \cap X \neq \emptyset\}$$

which are called the lower and upper approximation of X, respectively. In addition,

$$Pos(X) = \underline{R}(X), Neg(X) = U - \overline{R}(X), Bnd(X) = \overline{R}(X) - \underline{R}(X)$$

are called the positive, negative and boundary regions of X, respectively. Now we ready to give the definition of rough sets:

Definition 2.2. [1] Let (U, R) be a Pawlak approximation space and $X \subseteq U$. If $Bnd(X) \neq \emptyset$, i.e., $\overline{R}(X) \neq \underline{R}(X)$, X is said to be rough (or inexact); in the opposite case, i.e., if the boundary region of X is empty, i.e., $\overline{R}(X) = \underline{R}(X)$, then X is called definable (or crisp).

Proposition 2.1. *[1] Let (U, R) be a Pawlak approximation space and $X, Y \subseteq U$. The properties of the Pawlak's rough sets:*

1) $\underline{R}(\emptyset) = \emptyset, \overline{R}(\emptyset) = \emptyset$

2) $\underline{R}(U) = U, \overline{R}(U) = U$

3) $\underline{R}(X) \subseteq X \subseteq \overline{R}(X)$

4) $X \subseteq Y \Rightarrow \underline{R}(X) \subseteq \underline{R}(Y)$

5) $X \subseteq Y \Rightarrow \overline{R}(X) \subseteq \overline{R}(Y)$

6) $\underline{R}(X \cap Y) = \underline{R}(X) \cap \underline{R}(Y)$
7) $\overline{R}(X \cup Y) = \overline{R}(X) \cup \overline{R}(Y)$
8) $\underline{R}(\underline{R}(X) = \underline{R}(X)$
9) $\overline{R}(\overline{R}(X) = \overline{R}(X)$
10) $\underline{R}(-X) = -\overline{R}(X)$
11) $\overline{R}(-X) = -\underline{R}(X)$
12) $\underline{R}(-\underline{R}(X)) = -\underline{R}(X)$
13) $\overline{R}(-\overline{R}(X)) = -\overline{R}(X)$
14) $\forall K \in U/R$, $\underline{R}(K) = K$
15) $\forall K \in U/R$, $\overline{R}(K) = K$

Pawlak [1] has obtained some important results from the topological point of view in the rough set theory. As you see below:

Proposition 2.2. *Let U be a finite set, the domain of discourse, and R an equivalence relation on U. The lower and upper approximations generated by R are the interior and closure operators, respectively.*

Let us recall now the soft set notion.

Definition 2.3. [2] Let U be an initial universe set and E be the set of parameters. Let $P(U)$ denote the power set of U. A pair $G = (F, A)$ is called a soft set over U where $A \subseteq E$ and $F : A \to P(U)$ is a set valued mapping.

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\forall \varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of $\varepsilon-$ approximate elements of the soft set $G = (F, A)$. It is worth noting that $F(\varepsilon)$ may be arbitrary. Some of them may be empty and some may have nonempty intersection.

Example 2.1. [19] Miss Zeynep and Mr. Ahmet are going to marry and they want to hire a wedding room. The soft set (F, E) describes the "capacity of the wedding room". Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ be the wedding rooms under consideration, and $E = \{e_1 = big, e_2 = central, e_3 = cheap, e_4 = quality, e_5 = elegant\}$ be the parameter set $F(e_1) = \{u_2, u_4\}, F(e_2) = \{u_1, u_3, u_4\}, F(e_3) = \emptyset, F(e_4) = \{u_1, u_3, u_5\}, F(e_5) = \{u_1, u_6\}$. The soft set (F, E) is as follows:
$(F, E) = \{e_1 = \{u_2, u_4\},\ e_2 = \{u_1, u_3, u_4\},\ e_3 = \emptyset,\ e_4 = \{u_1, u_3, u_5\}, e_5 = \{u_1, u_6\}\}$.

U	e_1	e_2	e_3	e_4	e_5
u_1	0	1	0	1	1
u_2	1	0	0	0	0
u_3	0	1	0	1	0
u_4	1	1	0	0	0
u_5	0	0	0	1	0
u_6	0	0	0	0	1

Table 1. Tabular presentation of the soft set

Following the definition in [4], the concept of fuzzy set which is a newly-emerging mathematical approach to vagueness.

Definition 2.4. [4] Let U be a universe set. A fuzzy set A in U is a set of ordered pairs:

$$A = \{(x, \mu_A(x)) : x \in U\},$$

where $\mu_A : U \longrightarrow [0, 1] = I$ is a mapping and $\mu_A(x)$ (or $A(x)$) states the grade of belongness of x in A. The family of all fuzzy sets in U is denoted by I^U.

Maji et al. [22] defined the following hybrid model fuzzy soft sets, combining soft sets with fuzzy sets.

Definition 2.5. [22] Let $A \subseteq E$. (f_A, E) is defined to be a fuzzy soft set on (U, E) if $f_A : E \longrightarrow I^U$ is mapping defined by $f_A(e) = \mu^e_{f_A}$ where $\mu^e_{f_A} = \overline{O}$ if $e \in E - A$ and $\mu^e_{f_A} \neq \overline{O}$ if $e \in A$, where $\overline{O}(u) = 0$ for each $u \in U$.

Feng et al. [10] investigated the following concept of soft rough set in which is a combination of soft and rough sets.

Definition 2.6. [10] Let $G = (F, A)$ be a soft set over U. Then the pair $S = (U, G)$ is called a soft approximation space. Based on the approximation space S, we define the following two operations

$$\begin{aligned} \underline{apr}_S(X) &= \{u \in U : \exists a \in A, [u \in F(a) \subseteq X]\}, \\ \overline{apr}^S(X) &= \{u \in U : \exists a \in A, [u \in F(a),\ F(a) \cap X \neq \emptyset]\}, \end{aligned}$$

assigning to every subset $X \subseteq U$ two sets $\underline{apr}_S(X)$ and $\overline{apr}^S(X)$, which are called the soft $S-$lower approximation and the soft $S-$upper approximation of X, respectively. In general, we refer to $\underline{apr}_S(X)$ and $\overline{apr}^S(X)$ as soft rough approximations of X with respect to S. If $\overline{apr}^S(X) = \underline{apr}_S(X)$, X is said to be soft $S-$definable; otherwise X is called a soft $S-$rough set.

3. A new type of soft covering based rough sets

Yüksel et al. investigated the concept of soft covering based rough sets in 2014 [3] which are a hybrid model combining covering soft set and rough set. In this section, we present a new kind of soft covering based rough sets and its basic properties.

Interior and closure operators are two core concepts in classical topology and for Pawlak's rough sets, the lower and upper approximation operations on a set are also the interior and closure operators on this set, respectively. In this paper, we use these topological tools to investigate a new type of soft covering based rough sets. We present the similarity and difference between the properties of this type of soft covering based rough sets and those of Pawlak's rough sets.

Definition 3.1. [10] A soft set $G = (F, A)$ over U is called a full soft set if $\underset{a\in A}{\cup} F(a) = U$.

Definition 3.2. [10] A full soft set $G = (F, A)$ over U is called a covering soft set if $F(a) \neq \emptyset, \forall a \in A$.

We indicate a covering soft set with C_G.

Definition 3.3. [3] Let $G = (F, A)$ be a covering soft set over U. We call the ordered pair $S = (U, C_G)$ a soft covering approximation space.

Definition 3.4. [3] Let $S = (U, C_G)$ be a soft covering approximation space, $x \in U$, the soft minimal description of x is defined as follows: $Md_S(x) = \{F(a) : a \in A \wedge x \in F(a) \wedge (\forall e \in A \wedge x \in F(e) \subseteq F(a) \Rightarrow F(a) = F(e))\}$.

In order to describe an object, we need only the essential characteristics related to this object, not all characteristics for this object. That is the purpose of minimal description concept.

Definition 3.5. [3] Let $S = (U, C_G)$ be a soft covering approximation space. For a set $X \subseteq U$, soft covering lower and upper approximation are, respectively, defined as

$$\begin{aligned} \underline{S}(X) &= \underset{a\in A}{\cup}\{F(a) : F(a) \subseteq X\} \\ \overline{S}(X) &= \cup\{Md_S(x) : x \in X\}. \end{aligned}$$

Definition 3.6. [3] Let $S = (U, C_G)$ be a soft covering approximation space. A subset $X \subseteq U$ is called soft covering based definable set if $\overline{S}(X) = \underline{S}(X)$; in opposite case, i.e., if $\overline{S}(X) \neq \underline{S}(X)$, X is said to be soft covering based rough set.

We can call the soft covering based rough set which is given in Definition 3.5 [3] as the first type of soft covering based rough set in a soft covering approximation space.

The new type of soft covering based rough set model as follows:

Definition 3.7. Let $S = (U, C_G)$ be a soft covering approximation space. For a set $X \subseteq U$, the second type of soft covering lower and upper approximation are respectively defined as

$$\underline{S}_*(X) = \underset{a\in A}{\cup}\{F(a) : F(a) \subseteq X\}$$
$$\overline{S}^*(X) = \underline{S}_*(X) \cup (\cup\{\underset{F(a)\in Md_S(x)}{\cap} F(a) : x \in X - \underline{S}_*(X)\}).$$

In addition,

$$Pos_S(X) = \underline{S}_*(X), Neg_S(X) = U - \overline{S}^*(X), Bnd_S(X) = \overline{S}^*(X) - \underline{S}_*(X)$$

are called the second type of soft covering positive, negative and boundary regions of X , respectively.

Definition 3.8. Let $S = (U, C_G)$ be a soft covering approximation space. A subset $X \subseteq U$ is called second type of soft covering based definable set if $\overline{S}^*(X) = \underline{S}_*(X)$; in opposite case, i.e., if $\overline{S}^*(X) \neq \underline{S}_*(X)$, X is said to be second type of soft covering based rough set.

Example 3.1. Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ be universe and $G = (F, E)$ a covering soft set over U, where $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$, $F(e_1) = \{u_1, u_2\}$, $F(e_2) = \{u_1, u_2, u_3, u_4\}$, $F(e_3) = \{u_3, u_4\}$, $F(e_4) = \{u_3, u_4, u_5, u_6\}$, $F(e_5) = \{u_1, u_2, u_5, u_6\}$, and $F(e_6) = \{u_3, u_5, u_6\}$. Then $S = (U, C_G)$ is a soft covering approximation space.

For $X_1 = \{u_1, u_2\} \subseteq U$, we have $\underline{S}_*(X_1) = \{u_1, u_2\}$, $\overline{S}^*(X_1) = \{u_1, u_2\}$. Thus, $\underline{S}_*(X_1) = \overline{S}^*(X_1)$ and X_1 is second type of soft covering based definable set.

For $X_2 = \{u_1, u_2, u_5\} \subseteq U$, we have $\underline{S}_*(X_2) = \{u_1, u_2\}$, $\overline{S}^*(X_2) = \{u_1, u_2, u_5, u_6\}$. Thus, $\underline{S}_*(X_2) \neq \overline{S}^*(X_2)$ and X_2 is second type of soft covering based rough set.

Remark 3.1. From the definitions of two types of soft covering upper approximation operations, we have for a set $X \subseteq U$, $\overline{S}^*(X) \subseteq \overline{S}(X)$. However, $\overline{S}(X) \subseteq \overline{S}^*(X)$ is not true in general as shown in the following example.

Example 3.2. Let $U = \{u_1, u_2, u_3, u_4, u_5\}$ be universe and $G = (F, E)$ a covering soft set over U, where $E = \{e_1, e_2, e_3, e_4\}$, $F(e_1) = \{u_1, u_2\}$, $F(e_2) = \{u_2, u_3\}$, $F(e_3) = \{u_3, u_4\}$, and $F(e_4) = \{u_4, u_5\}$. Then $S = (U, C_G)$ is a soft covering approximation space. For $X = \{u_1, u_2\} \subseteq U$, we have

$$\begin{aligned} \overline{S}(X) &= \{u_1, u_2, u_3\} \\ \overline{S}^*(X) &= \{u_1, u_2\}. \end{aligned}$$

Thus, we obtain $\overline{S}(X) \subsetneq \overline{S}^*(X)$.

Remark 3.2. It is easy to see from the definitions that the soft covering lower approximation is the same as that in the first type of soft covering based rough set model.

Comparing with the properties of classical rough sets, the soft covering lower approximation has the following properties:

Theorem 3.1. *[3, 11] Let $G = (F, A)$ be a covering soft set over U, $S = (U, C_G)$ be a soft covering approximation space and $X, Y \subseteq U$. Then the soft covering lower approximation has the following properties:*

1) $\underline{S}(U) = U$
2) $\underline{S}(\emptyset) = \emptyset$
3) $\underline{S}(X) \subseteq X$
4) $X \subseteq Y \Rightarrow \underline{S}(X) \subseteq \underline{S}(Y)$
5) $\underline{S}(\underline{S}((X)) = \underline{S}(X)$
6) $\forall a \in A$, $\underline{S}(F(a)) = F(a)$
7) $\underline{S}(X \cap Y) \subseteq \underline{S}(X) \cap \underline{S}(Y)$
8) $\underline{S}(X \cup Y) \supseteq \underline{S}(X) \cup \underline{S}(Y)$.

Now, we investigate the second type of soft covering upper approximation and present a theorem which is necessary to prove the properties of the second type of soft covering upper approximation.

Theorem 3.2. *Let $G = (F, A)$ be a covering soft set over U, $S = (U, C_G)$ be a soft covering approximation space and $X \subseteq U$. Then $\overline{S}^*(X) = \cup\{\underset{F(a)\in Md_S(x)}{\cap} F(a) : x \in X\}$.*

Proof. If $\underline{S}_*(X) = \emptyset$, from Definition 3.7, this is obvious. If $\underline{S}_*(X) \neq \emptyset$, then

$$\cup\{\underset{F(a)\in Md_S(x)}{\cap} F(a) : x \in X\} = (\cup\{\underset{F(a)\in Md_S(x)}{\cap} F(a) : x \in \underline{S}_*(X)\}) \cup (\cup\{\underset{F(a)\in Md_S(x)}{\cap} F(a) : x \in X - \underline{S}_*(X)\}).$$

So we only need to prove that

$$\cup\{\underset{F(a)\in Md_S(x)}{\cap} F(a) : x \in \underline{S}_*(X)\} = \underline{S}_*(X).$$

From Definition 3.4, for $\forall a \in A$, each $F(a) \subseteq X$, we have

$$\cup\{\underset{F'(a)\in Md_S(x)}{\cap} F'(a) : x \in F(a)\} = F(a),$$

so

$$\underline{S}_*(X) = \underset{a\in A}{\cup}\{F(a) : F(a) \subseteq X\} = \underset{a\in A}{\cup}(\cup\{\underset{F'(a)\in Md_S(x)}{\cap} F'(a) : x \in F(a) \subseteq X\}).$$

Since for $\forall a \in A$, $F(a) \subseteq X$, $F(a) \subseteq \underline{S}_*(X)$, we obtain

$$\underline{S}_*(X) = \cup\{\underset{F'(a)\in Md_S(x)}{\cap} F'(a) : x \in \underline{S}_*(X)\}.$$

This shows that $\overline{S}^*(X) = \cup\{\underset{F(a)\in Md_S(x)}{\cap} F(a) : x \in X\}$. □

Theorem 3.3. *Let $G = (F, A)$ be a covering soft set over U, $S = (U, C_G)$ be a soft covering approximation space and X, $Y \subseteq U$. Then the second type of soft covering upper approximation has the following properties:*

1) $\overline{S}^*(U) = U$
2) $\overline{S}^*(\emptyset) = \emptyset$
3) $X \subseteq \overline{S}^*(X)$
4) $\overline{S}^*(X \cup Y) = \overline{S}^*(X) \cup \overline{S}^*(Y)$
5) $\overline{S}^*(\overline{S}^*(X)) = \overline{S}^*(X)$
6) $X \subseteq Y \Rightarrow \overline{S}^*(X) \subseteq \overline{S}^*(Y)$
7) $\forall a \in A$, $\overline{S}^*(F(a)) = F(a)$

Proof. From Definition 3.7, we can easily prove that properties 1,2 and 3.

4) From Definition 3.7 and Theorem 3.2 we have

$$\overline{S}^*(X) = \cup\{\underset{F(a)\in Md_S(x)}{\cap} F(a) : x \in X\}$$

and

$$\overline{S}^*(Y) = \cup\{\underset{F(a)\in Md_S(x)}{\cap} F(a) : x \in Y\}$$

for all $X, Y \subseteq U$. So

$$\begin{aligned}\overline{S}^*(X) \cup \overline{S}^*(Y) &= (\cup\{\underset{F(a)\in Md_S(x)}{\cap} F(a) : x \in X\}) \cup (\cup\{\underset{F(a)\in Md_S(x)}{\cap} F(a) : x \in Y\}) \\ &= \cup\{\underset{F(a)\in Md_S(x)}{\cap} F(a) : x \in X \cup Y\} = \overline{S}^*(X \cup Y).\end{aligned}$$

5) From the property 3), we have $\overline{S}^*(X) \subseteq \overline{S}^*(\overline{S}^*(X))$. $\forall x \in \overline{S}^*(\overline{S}^*(X))$, from Definition 3.7 and Theorem 3.2, we have

$$\overline{S}^*(\overline{S}^*(X)) = \cup\{\underset{F'(a)\in Md_S(y)}{\cap} F'(a) : y \in \overline{S}^*(X)\}.$$

Thus exists

$$y_0 \in \overline{S}^*(X) = \cup\{\underset{F(a)\in Md_S(z)}{\cap} F(a) : z \in X\},$$

and

$$x \in \underset{F(a)'\in Md_S(y_0)}{\cap} F'(a);$$

Then exists $z_0 \in X$ and

$$y_0 \in \underset{F(a)\in Md_S(z_0)}{\cap} F(a).$$

So $\forall F(a) \in Md_S(z_0)$, we have $y_0 \in F(a)$. Therefore for every such $F(a)$ must exist $F'(a) \in Md_S(y_0)$ to satisfy $F'(a) \subseteq F(a)$. Thus

$$\underset{F(a)'\in Md_S(y_0)}{\cap} F'(a) \subseteq \underset{F(a)\in Md_S(z_0)}{\cap} F(a),$$

so

$$x \in \underset{F(a) \in Md_S(z_0)}{\cap} F(a).$$

Therefore, $\overline{S}^*(\overline{S}^*(X)) \subseteq \overline{S}^*(X)$.

6) If $X \subseteq Y$, from Theorem 3.2,

$$\overline{S}^*(X) = \cup\{\underset{F(a) \in Md_S(x)}{\cap} F(a) : x \in X\} \subseteq \cup\{\underset{F(a) \in Md_S(x)}{\cap} F(a) : x \in Y\} = \overline{S}^*(Y).$$

7) If $\forall a \in A$, then $\underline{S}_*(F(a)) = F(a)$. Thus from Definition 3.7, $\overline{S}^*(F(a)) = F(a)$.

Theorem 3.4. *Let $G = (F, A)$ be a covering soft set over U, $S = (U, C_G)$ be a soft covering approximation space and $X, Y \subseteq U$. Then the second type of soft covering lower and upper approximations do not have the following properties:*

1) $\underline{S}_*(X \cap Y) = \underline{S}_*(X) \cap \underline{S}_*(Y)$
2) $\underline{S}_*(-\underline{S}_*(X)) = -\underline{S}_*(X)$
3) $\overline{S}^*(-\overline{S}^*(X)) = -\overline{S}^*(X)$
4) $\underline{S}_*(X) = -\overline{S}^*(-X)$
5) $\overline{S}^*(X) = -\underline{S}_*(-X)$

The following examples show that the equalities mentioned above do not hold.

Example 3.3. Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ be universe and $G = (F, E)$ a covering soft set over U, where $E = \{e_1, e_2, e_3\}$, $F(e_1) = \{u_1, u_2\}$, $F(e_2) = \{u_1, u_2, u_3\}$ and $F(e_3) = \{u_3, u_4, u_5, u_6\}$. Then $S = (U, C_G)$ is a soft covering approximation space. Suppose that $X = \{u_1, u_2, u_3\} \subseteq U$ and $Y = \{u_3, u_4, u_5, u_6\} \subseteq U$.

1) $\underline{S}_*(X) = \{u_1, u_2, u_3\}$, $\underline{S}_*(Y) = \{u_3, u_4, u_5, u_6\}$, $\underline{S}_*(X) \cap \underline{S}_*(Y) = \{u_3\}$ and $\underline{S}_*(X \cap Y) = \emptyset$. This shows that $\underline{S}_*(X \cap Y) \neq \underline{S}_*(X) \cap \underline{S}_*(Y)$.

2) $\underline{S}_*(X) = \{u_1, u_2, u_3\}$, $-\underline{S}_*(X) = \{u_4, u_5, u_6\}$, $\underline{S}_*(-\underline{S}_*(X)) = \emptyset$. This shows that $\underline{S}_*(-\underline{S}_*(X)) \neq -\underline{S}_*(X)$.

3) $\overline{S}^*(X) = \{u_1, u_2, u_3\}$, $-\overline{S}^*(X) = \{u_4, u_5, u_6\}$, $\overline{S}^*(-\overline{S}^*(X)) = \{u_3, u_4, u_5, u_6\}$. This shows that $\overline{S}^*(-\overline{S}^*(X)) \neq -\overline{S}^*(X)$.

4) $\underline{S}_*(X) = \{u_1, u_2, u_3\}$, $\overline{S}^*(-X) = \{u_3, u_4, u_5, u_6\}$, $-\overline{S}^*(-X) = \{u_1, u_2\}$. This shows that $\underline{S}_*(X) \neq -\overline{S}^*(-X)$.

5) $\overline{S}^*(X) = \{u_1, u_2, u_3\}$, $\underline{S}_*(-X) = \emptyset$, $-\underline{S}_*(-X) = U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$. This shows that $\overline{S}^*(X) \neq -\underline{S}_*(-X)$.

4. An application of multicriteria group decision making by a new type of soft covering approximation operators

Feng [12] established the multi-criteria group decision-making based on soft rough sets. Feng's method refines the primary evaluation results of the whole expert group and allows to select the optimal object in a more reliable manner. In this work we use a new type of soft covering approximations instead of soft rough approximations at Feng's method and aim to obtain the optimal choice for apply biopsy to the patients with prostate cancer risk by using the PSA, fPSA, PV and age data of patients. We determine the risk of prostate cancer. Our aim is to help the doctor to determine that the patient needs biopsy or not.

We choose 78 patients from Selçuk University Medicine Faculty with prostate complaint as the data.

1. Step: Let $U = \{u_k : u_1 = 1, u_2 = 2, \ldots, u_{78} = 78, k = 1, \ldots, 78\}$ be the universe and $A = \{PSA, fPSA, PV, Age\}$ be the parameter set. Now we obtain parametrized subsets of the universe. We choose the patients whose PSA in blood is 50 and higher than 50, fPSA is 12 and bigger than 12, PV is 20 and bigger than 20, age is 54 and older than 54. We generate the soft set $G = (F, A)$ over U. Since $G = (F, A)$ be a covering soft set, $S = (U, C_G)$ is the soft covering approximation space.

$F(PSA) = \{1, 4, 6, 7, 9, 11, 13, 15, 16, 18, 19, 20, 22, 23, 25, 26, 28, 29, 31, 33, 34, 36, 37, 39, 40, 42, 43, 45, 46, 47, 48, 49, 52, 53, 55, 56, 58, 60, 62, 63, 64, 66, 68, 70, 71, 72, 73, 74, 75, 77\}$

$F(fPSA) = \{1, 4, 7, 8, 9, 10, 11, 13, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 28, 29, 31, 32, 33, 34, 35, 36, 37, 39, 40, 42, 43, 45, 46, 48, 49, 51, 52, 53, 55, 56, 58, 60, 62, 63, 64, 66, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78\}$

$F(Age) = \{1, ..., 29, 31, ..., 56, 58, 60, ..., 78\}$

$F(PV) = U$

U	PSA	$fPSA$	PV	Age
u_1	76	17	30	65
u_5	39	7	48	64
u_{21}	39	9	52	68
u_{30}	27	7	28	51
u_{46}	88	19	37	77
u_{51}	46	12	62	71
u_{54}	42	10	59	80
u_{71}	52	12	35	65
u_{74}	51	12	78	67
u_{78}	41	13	79	80

Table 2. The input PSA, fPSA, PV and Age values of several patients

U	PSA	$fPSA$	PV	Age
u_1	1	1	1	1
u_5	0	0	1	1
u_{21}	0	0	1	1
u_{30}	0	0	1	0
u_{46}	1	1	1	1
u_{51}	0	1	1	1
u_{54}	0	0	1	1
u_{71}	1	1	1	1
u_{74}	1	1	1	1
u_{78}	0	1	1	1

Table 3. Tabular presentation of the soft set $G = (F, A)$

2. Step: Let $T = \{T_{d_1}, T_{d_2}, T_{d_3}\}$ be the specialist doctors group who evaluate the patients with respect to the parameters PSA, fPSA, PV and Age. Now we generate the soft set $G_1 = (V, T)$ over U by using the first evaluation of the results of specialist doctors group T.

$X_{d_1} = V(T_{d_1}) = \{1, 4, 6, 7, 8, 9, 11, 13, 15, 16, 17, 18, 19, 20, 22, 23, 25, 26, 28, 29, 31, 33, 34, 36, 37, 39, 40, 42, 43, 45, 46, 47, 48, 49, 51, 52, 53, 55, 56, 58, 60, 62, 63, 64, 66, 68, 70, 71, 72, 73, 74, 75, 77\}$

$X_{d_2} = V(T_{d_2}) = \{1, 2, 3, 4, 6, 7, 9, 11, 13, 15, 16, 18, 20, 22, 23, 24, 25, 26, 28, 29, 31, 33, 34, 36, 37, 39, 40, 41, 42, 43, 45, 46, 47, 48, 49, 51, 52, 53, 55, 56, 58, 60, 62, 64, 66, 68, 70, 72, 73, 74, 75, 77, 78\}$

$X_{d_3} = V(T_{d_3}) = \{1, 2, 4, 6, 7, 9, 11, 13, 15, 16, 18, 19, 20, 22, 23, 25, 26, 28, 29, 31, 33, 34, 36, 37, 39, 40, 41, 42, 43, 45, 46, 47, 48, 49, 51, 52, 53, 54, 55, 56, 58, 60, 62, 64, 66, 67, 68, 70, 72, 73, 75, 76, 77, 78\}$

3. Step: In our work we use soft covering based rough sets instead of soft rough sets. Now, we show how to use second type of soft covering based rough sets to support this group decision making process. Let us choose $S = (U, C_G)$ as the soft covering approximation space. By using the soft covering approximations, we obtain two other soft sets $G_{1\_} = (\underline{V}, T)$ and $G_1^- = (\overline{V}, T)$ over U, where

$$\begin{aligned} \underline{V} : T \to P(U), \underline{V}(T_{d_i}) &= \underline{S}_*(X_{d_i}), i = 1, 2, 3 \\ \overline{V} : T \to P(U), \overline{V}(T_{d_i}) &= \overline{S}^*(X_{d_i}), i = 1, 2, 3 \end{aligned}$$

The soft set G_1^- can be seen as the evaluation result of the specialist doctor group T with low confidence while the soft set $G_{1\_}$ represents the evaluation result of the specialist doctor group T with high confidence.

Now we obtain the soft covering upper and lower approximations of three specialist doctors first eveluation results to get the soft sets G_1^- and $G_{1\_}$.

$$\begin{aligned}
\underline{V}(T_{d_1}) &= \underline{S}_*(X_{d_1}) = F(PSA)\\
\underline{V}(T_{d_2}) &= \underline{S}_*(X_{d_2}) = \emptyset\\
\underline{V}(T_{d_3}) &= \underline{S}_*(X_{d_3}) = \emptyset\\
\overline{V}(T_{d_1}) &= \overline{S}^*(X_{d_1}) = \{F(PSA) \cap F(fPSA)\} \cup F(PSA) \cup F(fPSA)\\
\overline{V}(T_{d_2}) &= \overline{S}^*(X_{d_2}) = F(PSA) \cup F(fPSA) \cup F(Age) = F(Age)\\
\overline{V}(T_{d_3}) &= \overline{S}^*(X_{d_3}) = F(PSA) \cup F(fPSA) \cup F(Age) = F(Age)
\end{aligned}$$

4. Step: The results of the specialist three doctors evaluation can be formulized in terms of fuzzy sets. For $X \subseteq U$, the characteristic function of X is denoted by χ_X. Based on the soft set $G_1 = (V, T)$ we can define fuzzy set μ_{G_1} in U by

$$\mu_{G_1} : U \to [0,1], u_k \to \mu_{G_1}(u_k) = \frac{1}{3}\sum_{i=1}^{3} \chi_{V(T_{d_i})}(u_k)$$

where $V(T_{d_i}) = X_{d_i}$ and $k = 1, \ldots, 78;\ i = 1, 2, 3$.

In a similar way, we can get the fuzzy sets $\mu_{G_{1\_}}$ and $\mu_{G_1^-}$ as follows:

$$\begin{aligned}
\mu_{G_{1_}} : U \to [0,1], u_k \to \mu_{G_{1_}}(u_k) &= \frac{1}{3}\sum_{i=1}^{3} \chi_{\underline{V}(T_{d_i})}(u_k),\\
\mu_{G_1^-} : U \to [0,1], u_k \to \mu_{G_1^-}(u_k) &= \frac{1}{3}\sum_{i=1}^{3} \chi_{\overline{V}(T_{d_i})}(u_k)
\end{aligned}$$

where $\underline{V}(T_{d_i}) = \underline{S}_*(X_{d_i}), \overline{V}(T_{d_i}) = \overline{S}^*(X_{d_i})$ and $k = 1, \ldots, 78;\ i = 1, 2, 3$.

From $G_{1\_} \subseteq G_1 \subseteq G_1^-$, it is easy to see that $\mu_{G_{1\_}} \subseteq \mu_{G_1} \subseteq \mu_{G_1^-}$. These fuzzy sets $\mu_{G_{1\_}}, \mu_{G_1}, \mu_{G_1^-}$ can be interpreted as some vague concepts like "the patients under high risk", "the patients under middle risk" and "the patients under low risk", respectively.

By this way we obtain the fuzzy sets $\mu_{G_1}, \mu_{G_{1\_}}, \mu_{G_1^-}$ by the memberships we get above. For example we obtain these fuzzy sets for the first patience,

$$\mu_{G_1}(1) = 1,\ \mu_{G_{1\_}}(1) = \frac{1}{3},\ \mu_{G_1^-}(1) = 1$$

5. Step: Let $C = \{L, M, H\}$ be a set of parameters, where L, M and H denote "under low risk", "under middle risk" and "under high risk" respectively. Now we can define a fuzzy soft set $G_F = (\alpha, C)$ over U, where $\alpha : C \to I^U$ is given by $\alpha(L) = \mu_{G_1^-}$, $\alpha(M) = \mu_{G_1}$ and $\alpha(H) = \mu_{G_{1\_}}$.

6. Step: Given a weighting vector $R = (r_L, r_M, r_H)$ such that $r_L + r_M + r_H = 1$,

$$v(u_k) = r_L.\alpha(L)(u_k) + r_M.\alpha(M)(u_k) + r_H.\alpha(H)(u_k)$$

is called the weighted evaluation value of the alternative $u_k \in U$, $k = 1, \ldots, 78$. Assume that, the weighting vector $R = (0.25, 0.5, 0.25)$. Finally we can select the object u_p such that $v(u_p) = max\{v(u_k) : k = 1, \ldots, 78\}$ as the patient with the highest cancer risk.

When we rank all the alternatives according to their weighted evaluation values we can select any of the objects with the largest weighted evaluation value as the highest cancer risk. The results are as follows:

$1 \approx 4 \approx 6 \approx 7 \approx 9 \approx 11 \approx 13 \approx 15 \approx 16 \approx 18 \approx 20 \approx 22 \approx 23 \approx 25 \approx 26 \approx 28 \approx 29 \approx 31 \approx 33 \approx 34 \approx 36 \approx 37 \approx 39 \approx 40 \approx 42 \approx 43 \approx 45 \approx 46 \approx 47 \approx 48 \approx 49 \approx 52 \approx 53 \approx 55 \approx 56 \approx 58 \approx 60 \approx 62 \approx 64 \approx 66 \approx 68 \approx 70 \approx 72 \approx 73 \approx 75 \approx 77 = 0.83 > 51 = 0.75 > 19 \approx 41 \approx 74 = 0.67 > 78 = 0.58 > 2 \approx 63 \approx 71 = 0.5 > 8 \approx 17 \approx 24 \approx 76 = 0.42 > 3 \approx 54 \approx 67 = 0.33 > 10 \approx 32 \approx 35 = 0.25 > 5 \approx 12 \approx 14 \approx 21 \approx 27 \approx 38 \approx 44 \approx 50 \approx 61 \approx 65 \approx 69 = 0.17 > 30 \approx 57 \approx 59 = 0$

Our results show that 0.83 is the highest value and 46 patients have this value and the patients with the membership 0.83 are potential cancer and they are under the highest risk. They need biopsy exactly. One patient with 0.75 value also under middle risk and they should be followed by the doctor. The other patients are under low risk and they do not need the biopsy.

U	$\mu_{G_{1\_}}$	μ_{G_1}	$\mu_{G_1^-}$	$v(u_k)$
u_1	$\frac{1}{3}$	1	1	0.83
u_5	0	0	$\frac{2}{3}$	0.17
u_{21}	0	0	$\frac{2}{3}$	0.17
u_{30}	0	0	0	0
u_{46}	$\frac{1}{3}$	1	1	0.83
u_{51}	0	1	1	0.75
u_{54}	0	$\frac{1}{3}$	$\frac{2}{3}$	0.33
u_{71}	$\frac{1}{3}$	$\frac{1}{3}$	1	0.5
u_{74}	$\frac{1}{3}$	$\frac{2}{3}$	1	0.67
u_{78}	0	$\frac{2}{3}$	1	0.58

Table 4. Tabular presentation of the fuzzy soft set $G_F = (\alpha, C)$ with weighted evaluation value of several patients

According to data from Selçuk University Medicine Faculty the biopsy is applied to all 78 patients, but only 44 patients were diagnosed with cancer. That is 34 patients do not need the biopsy. According to our study we obtained that the biopsy must be applied only to a group of 46 patients who are under high cancer risk. This group also contains 44 patients who were diagnosed with cancer. Hence, we reduce the number of patients who applied biopsy.

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