Integrated Production and Distribution Planning: An Improved Formulation and the Value of Integrated Planning

Bütünleşik Üretim ve Dağıtım Planlaması: İyileştirilmiş Bir Model ve Bütünleşik Planlamanın Değeri

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Abstract

This study considers a supply chain management problem in which a plant produces and distributes a product to multiple retailers using a homogeneous fleet of vehicles over a finite time horizon. The aim is to decide on the production quantities at the plant, delivery quantities to retailers, the set of vehicles to use and the assignment of retailers to vehicles in each period such that the system-wide costs are minimized. A mixed integer linear programming formulation of the problem outperforming the existing ones in the literature is proposed. This study also compares integrated production and distribution planning with the sequential planning in which retailers place their own orders and the plant makes its plan based on these orders, and assesses the value of integrated planning. The computational results indicate that average cost savings of 8.9% and maximum cost savings of 28% can be obtained with the integrated planning over the sequential planning.

Keywords: *Supply Chain Management, Production Planning, Distribution Planning, Mixed Integer Linear Programming*

JEL Codes: *C44, C61, M11*

Öz

Bu çalışma, sonlu bir planlama ufku boyunca bir üreticinin bir ürünü üretip birçok perakendeciye türdeş bir araç filosu ile dağıttığı bir tedarik zinciri problemini ele almaktadır. Amaç, her bir dönemde üreticideki üretim miktarlarına, perakendecilere dağıtılacak ürün miktarlarına, kullanılacak araçlara ve ziyaret edilecek perakendecilerin hangi araçlara atanacağına, sistem maliyetini enazlayacak şekilde karar vermektir. Bu problem için literatürde varolanlardan daha iyi sonuçlar veren bir karışık tam sayılı doğrusal programlama modeli önerilmiştir. Ayrıca, üretim ve dağıtım planlamasının bütünleşik ele alınması perakendecilerin kendi siparişlerini verdikleri ve üreticinin planlamasını bu siparişlere göre yaptığı ardışık planlama ile karşılaştırılmış ve bütünleşik planlamanın değeri değerlendirilmiştir. Sayısal deney sonuçları bütünleşik planlamanın ardışık planlamaya göre ortalama %8.9 ve en çok %28 maliyet tasarrufu sağladığını göstermektedir.

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JEL Kodları: *C44, C61, M11*

Introduction

This study considers the supply chain management problem introduced by Senoussi et al. (2016). In this problem, a plant produces and distributes a product to multiple retailers using a homogeneous fleet of vehicles over a finite time horizon in a vendor-managed inventory setting in a way that the plant manages its own inventories as well as those of the retailers.

The problem considered in this study, referred to as Production-Inventory-Distribution Problem (PIDP) in Senoussi et al. (2016), integrates production management at the plant, inventory management at the plant and retailers, and distribution management of products from the plant to retailers. The aim in PIDP is to decide on the production quantities at the plant, delivery quantities to retailers, the set of vehicles to use and the assignment of retailers to vehicles in each period such that the sum of the production setup costs at plant, inventory holding costs at plant and retailers, vehicle usage costs and delivery costs to retailers is minimized. It is assumed in PIDP that the plant is far from the retailers to be served but retailers are clustered (i.e., close to each other). Thus, the traveling cost of a vehicle from the plant to retailers is considered to incur a major fixed cost and visiting a retailer by a vehicle incurs a minor fixed cost instead of considering the traveling cost based on the order of retailers visited.

Integration of production, inventory and distribution management has significant cost savings potential and arises in industries where vendor-managed inventory policy is adopted or when both the plant and retailers are owned by the same firm. Examples of such industries are petrochemical industry, suppliers for supermarkets, department store chains, home products, clothing industry, and parts distribution of automotive industry (see e.g., Campbell and Savelsbergh 2004:489). In particular, the PIDP arises in bottled water production and distribution industry, maritime transportation industry and distribution to service roads (Senoussi et al. 2016:972; 2018:109).

There are two studies on the PIDP. The first one, Senoussi et al. (2016), propose two mixed integer linear programming (MILP) formulations of the PIDP which are strengthened by several valid inequalities. They solve these two formulations using a commercial MILP solver. The second study, Senoussi et al. (2018), develop five heuristics based on genetic algorithms to solve the PIDP to near optimality. Kang and Kim (2010) consider the same problem as the PIDP except that replenishment decisions to the plant and inventory carrying at the plant are completely ignored. The PIDP is also closely related to the one-warehouse multi-retailer (OWMR) problem and the production routing problem (PRP).

The OWMR problem is a two-level lot sizing problem in which replenishment quantities to the supplier (i.e. warehouse) and to retailers are decided. Because there is no vehicle assignment consideration in the OWMR problem, the PIDP is an extension of the OWMR problem. Different variants of the basic OWMR problem are considered in the literature. For example, Federgruen and Tzur (1999) study a multi-item variant of the OWMR problem. Chan et al. (2002) consider different ordering cost structures. Levi et al. (2008) study different inventory holding cost structures. Jin and Muriel (2009) consider capacitated replenishments to both supplier and retailers. Melo and Wolsey (2012) consider multiple products and multiple suppliers. Solyalı and Süral (2012) and Cunha and Melo (2016) present and compare several strong MILP reformulations for the basic OWMR problem in which there are no production or distribution capacity constraints.

The PIDP can be considered as a special case of the PRP. Unlike the PIDP, the PRP considers detailed routing of vehicles for distribution of products to retailers and their associated cost (i.e., routing cost of vehicles based on the order of retailers visited). The PRP is studied under different conditions: the single product case (e.g., Absi et al. 2015), the multi-product case (e.g., Fumero and Vercellis 1999), with the time windows for deliveries to retailers (e.g., Qui et al. 2018), under demand uncertainty (e.g., Adulyasak et al. 2015b). Because of its complexity, there are just a few exact algorithms for the PRP (Ruokokoski et al. 2010, Archetti et al. 2011, Adulyasak et al. 2014). The most successful heuristic algorithms for the PRP are mathematical programming based heuristics and are those of Absi et al. (2015), Solyalı and Süral (2017), Russell (2017), and Chitsaz et al. (2018). For a review on formulations and solution algorithms proposed for the PRP, one can refer to Adulyasak et al. (2015a).

The contribution of this paper is twofold. First, an improved formulation of the problem is proposed. The computational results clearly reveal that the proposed formulation outperforms the existing ones in the literature. Second, integrated production and distribution planning is compared with the sequential planning in which retailers place their own orders and the plant makes its plan based on these orders in order to assess the value of integrated planning. While there is no study comparing integrated approaches with sequential approaches in the PIDP literature, a few studies are available in the PRP literature (see e.g., Chandra and Fisher 1994, Fumero and Vercellis 1999, Ruokokoski et al. 2010, and Absi et al. 2018). Note that there are only a few studies in the literature because of the complexity of the integrated planning problem and the difficulty of solving it to optimality or near-optimality. The major difference between this study and those comparing integrated approaches with sequential approaches in the PRP literature is that all studies in the PRP literature consider heuristics in solving the integrated planning problem except Ruokokoski et al. (2010) whereas this study attempts to exactly solve the integrated planning problem. Note that unlike others, Ruokokoski et al. (2010) consider an uncapacitated single vehicle for the distribution of products which makes the problem easier.

The rest of the paper is composed of the following sections. Section 1 describes the problem in detail and presents the mathematical programming formulation of the problem proposed in the literature. Section 2 includes the proposed mathematical programming formulation of the problem. The sequential planning approach is formulated and described in Section 3. The computational results are presented in Section 4. Finally, last section concludes the paper.

1. Problem Description and Formulation

A production distribution system in which a plant produces and distributes a single product to multiple retailers using a fleet of vehicles is considered. The system operates under a vendor-managed inventory system where the plant manages the inventories of itself and multiple retailers. Retailers face with known demands in discrete time periods which have to be satisfied without backlogging. In order to meet these demands, the plant makes production and distributes to retailers. The plant incurs a fixed setup cost every time it makes a production. The plant cannot produce more than its production capacity in a period. However, it can produce more than it distributes in a time period, and thus it can hold the excess amount as inventory which incurs an inventory holding cost for each unit stocked in each time period. The production cost per unit produced is ignored as this cost is constant. The distribution from the plant to retailers is made using a fleet of homogenous vehicles. Each retailer is replenished by at most one vehicle in a period (i.e., split deliveries are not allowed) while a vehicle can replenish multiple retailers in a period provided that its capacity is not exceeded. The usage of a vehicle in a period incurs a fixed vehicle usage cost. As it is assumed that retailers are close to each other, every delivery to a retailer incurs a fixed delivery cost rather than a sequence-dependent routing cost (i.e., there are no vehicle routing decisions). Like the plant, retailers can hold inventory which incurs an inventory holding cost for each unit carried in a period. However, the inventory level of a retailer cannot exceed its storage capacity. Thus, the integrated production, inventory, and distribution problem (PIDP) is to determine the production quantities at plant, distribution quantities to retailers as well as the vehicles to deliver to retailers such that the total cost comprised of setup costs and inventory holding costs at the plant, inventory holding costs at retailers, vehicle usage costs, and delivery costs to retailers is minimized.

The strictest assumptions of the problem described above are the adoption of vendor-managed inventory policy, known demands, and retailers being close to each other. Without assuming a vendor-managed inventory policy, the plant cannot manage the inventories of retailers as well as itself which means integrated planning would not be possible. If demands were uncertain, the model formulations presented in this paper would not be used and different model formulations incorporating demand uncertainty would be needed. Lastly, if the order of visits to retailers had a significant impact on the delivery cost, one would formulate the problem as a PRP which includes vehicle routing decisions instead of the model formulations given in this paper. The other assumptions of the problem like having a single plant, a single product, a fleet of homogeneous vehicles, not allowing backlogging at retailers, not allowing split deliveries and the cost terms are not rigid as one can easily modify the model formulations given in this paper to address multiple plants and products, a fleet of heterogeneous vehicles, and allowing backlogging and split deliveries.

In the following, parameters and decision variables to be used in the model formulation are defined.

Parameters:

c_j: Delivery cost to retailer *j*.

h_0: Inventory holding cost at plant.

h_j: Inventory holding cost at retailer *j*.

S: Fixed setup cost at plant.

R: Fixed cost of using a vehicle.

J: Number of retailers.

K: Number of vehicles.

T: Number of time periods in the planning horizon.

Q: Production capacity at plant.

W: Capacity of a vehicle.

U_j: Storage capacity at retailer *j*.

d_{jt}: Demand at retailer *j* in period *t*.

Decision Variables:

I_{0t}: Inventory level of plant at the end of period *t*.

I_{jt}: Inventory level of retailer *j* at the end of period *t*.

p_t: Production quantity at plant in period *t*.

q_{jkt}: Delivery quantity to retailer *j* by vehicle *k* in period *t*.

$x_{jkt} = \begin{cases} 1 \text{ if vehicle } k \text{ delivers to retailer } j \text{ in period } t. \\ 0 \text{ otherwise.} \end{cases}$

$y_t = \begin{cases} 1 \text{ if production occurs in period } t. \\ 0 \text{ otherwise.} \end{cases}$

$v_{kt} = \begin{cases} 1 \text{ if vehicle } k \text{ is used in period } t. \\ 0 \text{ otherwise.} \end{cases}$

Using the above defined parameters and decision variables, Senoussi et al. (2016) formulated the PIDP as the following MILP formulation:

$$AF\text{: Min } \sum_{t=1}^{T}\left(Sy_t + \sum_{j=0}^{J} h_j I_{jt} + \sum_{k=1}^{K} Rv_{kt} + \sum_{j=1}^{J}\sum_{k=1}^{K} c_j x_{jkt}\right) \qquad (1)$$

s.t.

$$I_{0t} = I_{0,t-1} + p_t - \sum_{j=1}^{J}\sum_{k=1}^{K} q_{jkt} \quad 1 \le t \le T \qquad (2)$$

$$I_{jt} = I_{j,t-1} + \sum_{k=1}^{K} q_{jkt} - d_{jt} \quad 1 \le j \le J, 1 \le t \le T \qquad (3)$$

$$I_{jt} \le U_j \quad 1 \le j \le J, 1 \le t \le T \qquad (4)$$

$$p_t \le Qy_t \quad 1 \le t \le T \qquad (5)$$

$$\sum_{j=1}^{J} q_{jkt} \le Wv_{kt} \quad 1 \le k \le K, 1 \le t \le T \qquad (6)$$

$$q_{jkt} \le Wx_{jkt} \quad 1 \le j \le J, 1 \le k \le K, 1 \le t \le T \qquad (7)$$

$$\sum_{k=1}^{K} x_{jkt} \le 1 \quad 1 \le j \le J, 1 \le t \le T \qquad (8)$$

$$I_{0t}, I_{jt}, p_t, q_{jkt} \ge 0 \quad 1 \le j \le J, 1 \le k \le K, 1 \le t \le T \qquad (9)$$

$$x_{jkt}, y_t, v_{kt} \in \{0,1\} \quad 1 \le j \le J, 1 \le k \le K, 1 \le t \le T. \qquad (10)$$

The objective function (1) minimizes the sum of fixed setup cost, inventory holding costs at the plant and retailers, fixed vehicle usage cost and delivery costs to retailers. Equations (2) and (3) ensure the inventory balance at the plant and at retailers, respectively. Constraints (4) stipulate that the inventory level of retailers does not exceed their storage capacities. Constraints (5) guarantee that the amount produced at the plant in a period does not exceed the production capacity of the plant. Constraints (5) also ensure that if a production occurs in a period, fixed setup cost is incurred in that period. Constraints (6) stipulate that total amount shipped to retailers by a vehicle is no greater than the vehicle capacity. Constraints (6) also ensure that if any quantity is shipped by a vehicle in a period, a fixed usage cost is incurred for that vehicle. Constraints (7) guarantee that if any delivery occurs to a retailer in a period, the fixed delivery cost to that retailer is incurred. Constraints (8) ensure that a retailer can be replenished by at most one vehicle in a period. Constraints (9) stipulate that those variables cannot be negative whereas constraints (10) ensure the integrality of those variables.

In order to solve the A*F* formulation presented above more efficiently, this formulation is strengthened with the following inequalities by Senoussi et al. (2016):

$$v_{k+1,t} \leq v_{kt} \qquad 1 \leq k < K, 1 \leq t \leq T \qquad (11)$$

$$\sum_{k=1}^{K} v_{kt} \leq \left\lceil \sum_{j=1}^{J} (U_j + d_{jt}) / W \right\rceil \qquad 1 \leq t \leq T \qquad (12)$$

$$\sum_{i=1}^{J} q_{ikt} \geq W\left(x_{jkt} + z_{j,t-1} - 1\right) \qquad 1 \leq j \leq J, 1 \leq k \leq K, 1 \leq t \leq T \qquad (13)$$

$$I_{jt} \leq U_j z_{jt} \qquad 1 \leq j \leq J, 1 \leq t \leq T \qquad (14)$$

$$z_{jt} \in \{0,1\} \qquad 1 \leq j \leq J, 1 \leq t \leq T \qquad (15)$$

$$I_{0,u-1} + \sum_{j=1}^{J} I_{j,u-1} + Q \sum_{l=u}^{t} y_l + W \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{l=t+1}^{s} x_{jkl} \geq \sum_{j=1}^{J} D_{jus} \qquad 1 \leq u \leq t \leq s \leq T \qquad (16)$$

$$I_{j,u-1} + W \sum_{t=u}^{s} \sum_{k=1}^{K} x_{jkt} \geq D_{jus} \qquad 1 \leq j \leq J, 1 \leq u \leq s \leq T \qquad (17)$$

$$\sum_{l=2}^{K} x_{jlt} \leq \sum_{i=1}^{j-1} x_{i1t} \qquad 1 \leq j \leq J, 1 \leq t \leq T, \qquad (18)$$

where $D_{jus} = \sum_{t=u}^{s} d_{jt}$ for $1 \leq j \leq J, 1 \leq u \leq s \leq T$.

Constraints (11) are symmetry-breaking constraints for *v* variables. Constraints (12) limit the number of vehicles to use in a period. Constraints (13) together with (14) and (15) are the full-truckload inequalities ensuring that a full vehicle is sent to retailers in a period if that vehicle in that period visits a retailer which has a positive inventory level at the end of the previous period. Note that z_{jt} variable is equal to 1 if the inventory level of retailer *j* is positive at the end of period *t*, and 0 otherwise. Constraints (16) and (17) are the extensions of well-known (*l*,*s*) inequalities for the single-item uncapacitated lot sizing problem (Barany et al. 1984). Constraints (18) reduce the number of variables.

Senoussi et al. (2016) called the formulation (1)–(10) as Aggregate Formulation (*AF*) and proposed another MILP formulation, referred to as the Echelon Stock Formulation (*ESF*), which is based on the echelon stock idea. In order to derive ESF, echelon stock variable E_t in period *t* is defined as follows:

$$E_t = I_{0t} + \sum_{j=1}^{J} I_{jt} \qquad 1 \le t \le T. \qquad (19)$$

Using (19), Senoussi et al. (2016) replaced I_{0t} variables in *AF* with $E_t - \sum_{j=1}^{J} I_{jt}$ and obtained *ESF*. Because *AF* yielded slightly better results than *ESF* in the computational experiments of Senoussi et al. (2016), *ESF* will not be considered further in this study.

It should be noted that constraints (12) are not valid for the PIDP because split deliveries to retailers are not allowed (i.e., every retailer can be visited by at most one vehicle in a period) and constraints (12) may eliminate some feasible solutions. For instance, consider the following example: T=1, J=3, $I_{10} = I_{20} = I_{30} = 0, d_{11} = d_{21} = d_{31} = 10, U_1 = U_2 = U_3 = 2$, W = 19. For this example, the right-hand side of (12) is two but three vehicles are needed to satisfy the demand of retailers. Thus, constraints (12) eliminate the feasible solution in this example. Because constraints (12) are invalid for the PIDP, they will be removed from the formulation proposed by Senoussi et al. (2016).

2. Proposed Formulation

Because the *AF* formulation presented in Section 1 could not be solved to optimality in many test problem sets, an improved mixed integer linear programming formulation is proposed. In this improved formulation, tighter (*l*,*s*) inequalities are proposed, some of the inequalities in *F* are made stronger, and more effective symmetry-breaking inequalities are added. Using the echelon stock idea discussed in Section 1, the proposed formulation (*PF*) is as follows:

$$PF\text{: Min} \sum_{t=1}^{T}\Big(Sy_t + h_0 E_t + \sum_{j=1}^{J}(h_j - h_0) I_{jt} + \sum_{k=1}^{K} R v_{kt} + \sum_{j=1}^{J}\sum_{k=1}^{K} c_j x_{jkt}\Big) \qquad (20)$$

s.t. (3), (4), (6), (8)–(11), (18),

$$E_t = E_{t-1} + p_t - d_{0t} \qquad 1 \le t \le T \qquad (21)$$

$$E_t \ge \sum_{j=1}^{J} I_{jt} \qquad 1 \le t \le T \qquad (22)$$

$$E_{u-1} + \sum_{l=u}^{t} \min(Q, D_{0lt}) y_l \ge D_{0ut} \qquad 1 \le u \le t \le T \qquad (23)$$

$$I_{j,u-1} + \sum_{t=u}^{s} \min(W, U_j + d_{jt}, D_{jts}) \sum_{k=1}^{K} x_{jkt} \ge D_{jus} \qquad 1 \le j \le J, 1 \le u \le s \le T \qquad (24)$$

$$p_t \le \min(Q, D_{0tT}) y_t \qquad 1 \le t \le T \qquad (25)$$

$$q_{jkt} \le \min\big(W, U_j + d_{jt}, D_{jtT}\big) x_{jkt} \qquad 1 \le j \le J, 1 \le k \le K, 1 \le t \le T \qquad (26)$$

$$\sum_{i=1}^{j} 2^{(j-i)} x_{i,k+1,t} \leq \sum_{i=1}^{j} 2^{(j-i)} x_{ikt} \qquad 1 \leq j \leq J, 1 \leq k < K, \quad 1 \leq t \leq T, \tag{27}$$

where $D_{0ut} = \sum_{j=1}^{J} D_{jut}$ for $1 \leq u \leq t \leq T$.

The objective function (20) is equivalent to (1). Equations (21) are the inventory balance equations of the whole system. Constraints (22) ensure that the inventory level of the plant is nonnegative (i.e., $I_{0t} \geq 0$ for $1 \leq t \leq T$). Constraints (23) and (24) are the extended (*l*,*s*) inequalities with Wagner-Whitin cost structure for the PIDP. Note that these constraints are new to the PIDP and they are tighter (i.e., stronger) than constraints (16) and (17) as shown below.

Using (19), $D_{0ut} = \sum_{j=1}^{J} D_{jut}$, and $s = t$, constraints (16) can be recast as:

$$E_{u-1} + Q \sum_{l=u}^{t} y_l + W \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{l=t+1}^{t} x_{jkl} \geq D_{0ut} \qquad 1 \leq u \leq t \leq T.$$

As the last term of the right-hand side of the above inequality is equal to zero, constraints (16) for $s = t$ can be rewritten as

$$E_{u-1} + Q \sum_{l=u}^{t} y_l + \geq D_{0ut} \qquad 1 \leq u \leq t \leq T.$$

Because $\sum_{l=u}^{t} \min(Q, D_{0lt}) y_l \leq \sum_{l=u}^{t} Q y_l$, constraints (23) are tighter than constraints (16) when $s = t$. Note that constraints (23) are a subset of constraints (16) and there is no relationship between (16) and (23) when $t > s$ in (16).

It is easy to observe that constraints (24) are tighter than constraints (17) because $\sum_{t=u}^{s} \sum_{k=1}^{K} \min(W, U_j + d_{jt}, D_{jts}) x_{jkt} \leq \sum_{t=u}^{s} \sum_{k=1}^{K} W x_{jkt}$.

As the right-hand sides of (25) and (26) are tighter than constraints (5) and (7), respectively, constraints (25) and (26) are stronger inequalities. Constraints (27) which are effective symmetry-breaking constraints for x variables are adapted from Adulyasak et al. (2014).

Note that constraints (11) and (18) from *AF* are used in *PF* as well because they help solve *PF* faster. On the other hand, constraints (13)–(15) are not used in *PF* as they do not improve the solution of *PF* according to preliminary computational experiments.

3. Sequential Planning Approach

In this section, the sequential planning approach is presented. In this approach, retailers independently place orders to the plant in order to minimize their own costs whereas the plant decides on its production and distribution plan based on the orders of retailers.

The total cost of sequential planning approach can be found using a two-phase method. In the first phase, the following formulation is solved to determine the time and quantity of orders for each retailer:

$$S1\text{: Min } \sum_{t=1}^{T} \sum_{j=1}^{J} \left(h_j I_{jt} + c_j \bar{x}_{jt} \right) \quad (28)$$

s.t. (4),

$$I_{jt} = I_{j,t-1} + \bar{q}_{jt} - d_{jt} \qquad 1 \le j \le J, 1 \le t \le T \quad (29)$$

$$\bar{q}_{jt} \le W \bar{x}_{jt} \qquad 1 \le j \le J, 1 \le t \le T \quad (30)$$

$$I_{jt}, \bar{q}_{jt} \ge 0 \qquad 1 \le j \le J, 1 \le t \le T \quad (31)$$

$$\bar{x}_{jt} \in \{0,1\} \qquad 1 \le j \le J, 1 \le t \le T, \quad (32)$$

where $\bar{q}_{jt}$ denotes the quantity ordered by retailer *j* in period *t* and $\bar{x}_{jt}$ is equal to 1 if retailer *j* is visited in period *t*, and 0 otherwise.

Note that *S*1 formulation decomposes for each retailer and the objective function (28) is minimizing the inventory holding cost as well as delivery cost at each retailer. Constraints (29) and (30) are equivalent to constraints (3) and (7), respectively, with the only difference being the elimination of the index that specifies the assigned vehicle in (29) and (30). Constraints (31) are for nonnegativity of variables and constraints (32) are for the integrality of variables.

Once the optimal time and quantity of orders for each retailer (i.e., $\bar{q}_{jt}$) are determined, the following formulation is solved in the second phase in order to determine the production schedule at the plant and the assignment of order quantities of retailers to vehicles. Note that unlike the original PIDP, there is no restriction on the production capacity of plant and the number of vehicles in sequential planning approach in order to ensure the feasibility of solutions found.

$$S2\text{: Min } \sum_{t=1}^{T} \left(S y_t + h_0 I_{0t} + \sum_{k=1}^{K} R v_{kt} \right) \quad (33)$$

s.t. (6) – (8), (10),

$$I_{0t} = I_{0,t-1} + p_t - \sum_{j=1}^{J} \bar{q}_{jt} \qquad 1 \le t \le T \quad (34)$$

$$p_t \le \left(\sum_{u=t}^{T} \sum_{j=1}^{J} \bar{q}_{ju} \right) y_t \qquad 1 \le t \le T \quad (35)$$

$$\bar{q}_{jt} = \sum_{k=1}^{K} q_{jkt} \qquad 1 \le j \le J, 1 \le t \le T \quad (36)$$

$$I_{0t}, p_t, q_{jkt} \ge 0 \qquad 1 \le j \le J, 1 \le k \le K, 1 \le t \le T, \quad (37)$$

where *K* is a sufficiently large number (e.g., *K*=*J*).

The objective function (33) of *S*2 formulation is the sum of fixed setup cost and inventory holding cost at plant and the fixed vehicle usage costs. Constraints (34) and (35) are equivalent to constraints (2) and (5), respectively, with the only difference being known order quantities to retailers in (34) and (35). Equations (36) assign order quantities of retailers to vehicles. Constraints (37) ensure nonnegativity of variables. The sum of objective functions (28) and (33) gives the total cost of sequential planning approach.

4. Computational Results

Computational experiments have been performed on benchmark instances in order to compare the formulation proposed by Senoussi et al. (2016) (i.e., *AF* with (11) and (13)–(18)) and the formulation proposed by this study (i.e., *PF*). The value of integration in PIDP compared to a sequential planning approach has also been assessed through computational experiments.

The computational experiments have been carried out on a Workstation with a 2.4 GHz CPU, 12 cores, and 48 GB RAM that runs on Windows 7. As an MILP solver, CPLEX, which is one of the state-of-the-art commercial solvers available in the market, is used. All formulations presented in this study have been solved using the default settings of CPLEX 12.7.1 with a single thread. One hour of time limit has been implemented for CPLEX 12.7.1 to solve the formulations.

The tests have been performed on test instances generated using the generation scheme of Senoussi et al. (2016). The generated instances involve different number of time periods, retailers and vehicles as well as different levels of production and vehicle capacities. Table 1 shows important parameters of instances and their values.

Table 1. Values of important parameters in instances*

Parameter	Values
T	3, 6, 9
J	5, 10, 15, 20
K	2, 3, 4
Q	$2\alpha, 1.5\alpha$ where $\alpha = \sum_{j=1}^{J}\sum_{t=1}^{T} d_{jt}/T$
W	$2\beta, 1.5\beta$ where $\beta = max_{1\le t\le T}(\sum_{j=1}^{J} d_{jt})/K$

* Table from Senoussi et al. (2016)

All combinations of the parameters in Table 1 except those involving $Q = 1.5\alpha$ and $W = 1.5\beta$ have been considered. The instances with combinations involving $Q = 1.5\alpha$ and $W = 1.5\beta$ have not been generated as these instances are generally infeasible. For each combination, five instances have been randomly generated using uniform distribution from the intervals [5,25] for demand, [1,5] for unit inventory holding cost at retailers, [100,500] for the delivery cost to retailers, and [2,6] multiplied with average demand (i.e., $\sum_{j=1}^{J}\sum_{t=1}^{T} d_{jt}/(T \text{ x } J)$) for the storage capacity at retailers. The unit inventory holding cost at plant, fixed setup cost at plant, fixed cost of using a vehicle, and initial inventory levels of both plant and retailers have been set to 1, 2000, 1000, and 0, respectively. Thus, 108 combinations multiplied with five instances for each combination results in the generation of 540 instances in total.

Summary of computational results obtained with the formulation proposed in this study and that of Senoussi et al. (2016) are presented in Table 2 where first and second columns show parameters and their values, third and six columns the computational time needed by *AF* with (11) and (13)–(18) and *PF*, respectively, fourth and seventh columns remaining integrality gap obtained by *AF* with (11) and (13)–(18) and *PF*, respectively, and fifth and eighth columns the number of instances that could not be solved to optimality by *AF* with (11) and (13)–(18) and *PF*, respectively.

Table 2. Summary of computational results with formulations

		***AF* with (11) & (13)–(18)**			***PF***		
Parameter	**Value**	**Time**	**Gap%**	**#NO**	**Time**	**Gap%**	**#NO**
T	**3**	16.88	0.00	0	6.55	0.00	0
	6	441.21	0.02	13	275.70	0.01	9
	9	1596.46	0.22	68	1347.89	0.15	56
J	**5**	80.54	0.00	0	30.74	0.00	0
	10	650.85	0.13	20	487.06	0.07	14
	15	860.29	0.12	27	687.35	0.08	21
	20	1147.71	0.08	34	968.37	0.07	30
K	**2**	76.74	0.00	0	40.59	0.00	0
	3	727.04	0.05	26	512.38	0.02	17
	4	1250.77	0.20	55	1077.17	0.14	48
Q	**2**	741.42	0.10	59	595.69	0.07	48
	1.5	571.70	0.05	22	438.76	0.03	17
W	**2**	522.71	0.04	39	394.58	0.03	31
	1.5	1009.05	0.16	42	840.98	0.11	34
Average		684.85	0.08	81	543.38	0.05	65

Key results of Table 2 are as follows:

- The proposed formulation outperforms the existing one with regard to the average solution time and the remaining gap%. Furthermore, the new formulation managed to solve 80 more instances to optimality.

- As the length of the planning horizon increases, the difficulty of solving PIDP to optimality becomes harder for both formulations.
- Similar to the length of the planning horizon, when the number of retailers or the number of vehicles increases, instances become harder to solve to optimality.
- All instances with $= 3$, $J = 5$, or $K = 2$ have been solved to optimality by both formulations.
- The largest remaining gap% values are obtained when $T = 9$.

In order to assess the value of integrated planning over the sequential planning, the benchmark instances presented in this section are adapted by making both the production capacity of plant and the number of vehicles unlimited. In these instances, the value of *K* is set to three when determining the value of *W*. Because *Q* and *K* are unlimited, there are 120 instances in total for assessing the value of integrated planning. Note that *S*1 and *S*2 formulations are solved for the sequential planning approach whereas *PF* formulation with sufficiently large values of *Q* (i.e., $Q = \sum_{j=1}^{J} \sum_{t=1}^{T} d_{jt}$) and *K* (i.e., $K = J$) is solved for the integrated planning approach.

Summary of computational results to assess the value of integrated planning are presented in Table 3 where first and second columns indicate parameters and their values, and third column shows the average percent improvement obtained by integrated planning over sequential planning.

Table 3. Summary of computational results for the value of integrated planning

Parameter	**Value**	**Ave-Imp%**
T	**3**	5.74
	6	9.64
	9	11.21
J	**5**	10.92
	10	9.72
	15	7.48
	20	7.34
W	**2**	8.48
	1.5	9.25
Average		8.87
Maximum		28.03

Key results of Table 3 are as follows:

- On average, 8.87% improvement is obtained by solving the integrated problem over the sequential planning approach.

- The maximum percent improvement by solving the integrated problem over the sequential planning approach reaches to 28.03%.
- As the length of planning horizon increases, the value of integrated planning increases.
- The value of integrated planning decreases as the number of products becomes larger.
- When the vehicle capacity is tighter, the value of integrated planning increases.

It should be noted that the solution time needed by the sequential planning approach is negligible whereas both the remaining integrality gap and the solution time for the integrated planning approach are similar to those of *PF* in Table 2.

In order to understand why integrated planning approach yields better results than sequential planning approach, average percentages of different cost components with respect to total cost are obtained for both integrated and sequential planning approaches and presented in Table 4. In Table 4, the first column shows the approach and the rest indicates average percentages of each cost component.

Table 4. Average percentages of cost components of integrated and sequential planning approaches*

Approach	**PlantInv Cost**	**Setup Cost**	**RetInv Cost**	**Deliv Cost**	**VehUse Cost**
Integrated	3.21	12.16	13.75	29.75	41.14
Sequential	3.75	11.08	11.56	27.57	46.04

* PlantInv Cost: Percent Inventory Holding Cost at Plant, Setup Cost: Percent Setup Cost at Plant, RetInv Cost: Percent Inventory Holding Cost at Retailers, Deliv Cost: Percent Delivery Cost to Retailers, VehUse Cost: Percent Vehicle Usage Cost.

As shown in Table 4, the main reason of obtaining an important improvement by integrated planning approach over sequential planning approach is the ability of the integrated planning to better plan usage of vehicles than the sequential planning. Although the sequential planning approach minimizes inventory holding cost and fixed delivery cost of each retailer, the integrated planning approach yields lower fixed vehicle usage costs than the sequential planning approach which predominates the gain from inventory holding cost and delivery cost of retailers. Unlike the sequential planning approach, the integrated planning approach considers vehicle usage costs together with the costs at plant and retailers. This enables the integrated planning approach to more efficiently use vehicles for distribution which in turn results in smaller total costs than the sequential planning approach.

Conclusions

In this study, a supply chain management problem that involves integrated planning of production of a product at a plant and distribution to multiple retailers with a homogeneous fleet of capacitated vehicles over a finite planning horizon is considered. The plant manages the inventories of itself and those of the retailers in a vendor-managed setting. An improved mixed integer linear programming formulation is proposed for the problem. The computational results show that the proposed formulation is superior to the existing formulations. As another contribution, this study assesses the value of integrated planning over the sequential planning where retailers place their orders and the plant makes production and distribution decisions based on the orders of retailers. The computational experiments indicate that average cost savings of 8.9% and maximum cost savings of 28% is achieved by the integrated planning over sequential planning.

As a future research avenue, more complicated integrated production and distribution planning problems (e.g., problem with production and distribution of multiple products, problems with uncertain demands, and problems with capacitated production at plant) can be considered. Another future research study is to develop tailored algorithms to solve larger instances of problem to optimality.

References

Absi, N., Archetti, C., Dauzere-Peres, S., Feillet, D. (2015). "A two-phase iterative heuristic approach for the production routing problem". *Transportation Science*, 49: 784–795.

Absi, N., Archetti, C., Dauzere-Peres, S., Feillet, D., Speranza, M.G. (2018). "Comparing sequential and integrated approaches for the production routing problem". *European Journal of Operational Research*, 269: 633–646.

Adulyasak, Y., Cordeau, J.-F., Jans, R. (2014). "Formulations and branch-and-cut algorithms for multivehicle production and inventory routing problems". *INFORMS Journal on Computing*, 26: 103–120.

Adulyasak, Y., Cordeau, J.-F., Jans, R. (2015a). "The production routing problem: a review of formulations and solution algorithms". *Computers & Operations Research*, 55: 141–152.

Adulyasak, Y., Cordeau, J.-F., Jans, R. (2015b). "Benders decomposition for production routing under demand uncertainty". *Operations Research*, 63: 851–867.

Archetti, C., Bertazzi, L., Paletta, G., Speranza, M.G. (2011). "Analysis of maximum level policy in a production-distribution system". *Computers & Operations Research*, 38: 1731–1746.

Barany, I., Van Roy T.J., Wolsey, L.A. (1984). "Uncapacitated lot sizing: the convex hull of solutions". *Mathematical Programming Studies*, 22: 32–43.

Campbell, A.M., Savelsbergh, M.W.P. (2004). "A decomposition approach for the inventory-routing problem". *Transportation Science*, 38: 488–502.

Chan, L.M.A., Muriel, A., Shen, Z.J.M., Simchi-Levi, D., Teo, C.P. (2002). "Effective zero-inventory ordering policies for the single-warehouse multiretailer problem with piecewise linear cost structures". *Management Science*, *48*: 1446–1460.

Chandra, P., Fisher, M.L., (1994). "Coordination of production and distribution planning". *European Journal of Operational Research*, 72: 503–517.

Chitsaz, M., Cordeau, J.-F., Jans, R. (2019). "A unified decomposition matheuristic for assembly, production, and inventory routing". *INFORMS Journal on Computing*, 31: 134–152.

Cunha, J.O., Melo, R.A. (2016). "On reformulations for one-warehouse multi-retailer problem". *Annals of Operations Research*, 238: 99–122.

Federgruen, A., Tzur, M. (1999). "Time-partitioning heuristics: Application to one warehouse, multiitem, multiretailer lot-sizing problems". *Naval Research Logistics*, 46: 463–486.

Fumero, F., Vercellis C. (1999). "Synchronized development of production, inventory, and distribution schedules". *Transportation Science*, 33: 330–340.

Jin, Y., Muriel, A. (2009). "Single-warehouse multi-retailer inventory systems with full truckload shipments". *Naval Research Logistics*, 56: 450–464.

Kang, J.H., Kim, Y.D. (2010). "Coordination of inventory and transportation managements in a two-level supply chain". *International Journal of Production Economics*, 123: 137–145.

Levi, R., Roundy, R., Shmoys, D.B., Sviridenko, M. (2008). "A constant approximation algorithm for the one-warehouse multiretailer problem". *Management Science*, 54: 763–776.

Melo, R.A., Wolsey, L.A. (2012). "MIP formulations and heuristics for two-level production-transportation problems". *Computers & Operations Research*, 39: 2776–2786.

Qui, Y., Wang, L., Fang, X., Pardalos, P.M., Goldengorin B. (2018). "Formulations and branch-and-cut algorithms for production routing problems with time windows". *Transportmetrica A: Transport Science*, 14: 669–690.

Ruokokoski, M., Solyalı, O., Cordeau, J.-F., Jans, R., Süral, H. (2010). "Efficient formulations and a branch-and-cut algorithm for a production-routing problem". GERAD Technical Report, G-2010-66, HEC Montreal, Canada, 1–43.

Russell, R.A. (2017). "Mathematical programming heuristics for the production routing problem". *International Journal of Production Economics*, 193: 40–49.

Senoussi, A., Mouss, N.K., Penz, B., Brahimi, N., Dauzere-Peres, S. (2016). "Modeling and solving a one-supplier multi-vehicle production-inventory-distribution problem with clustered retailers". *International Journal of Advanced Manufacturing Technologies*, 85: 971–989.

Senoussi, A., Dauzere-Peres, S., Brahimi, N., Penz, B., Mouss, N.K. (2018). "Heuristics based on genetic algorithms for the capacitated multi vehicle production distribution problem". *Computers & Operations Research*, 96: 108–119.

Solyalı, O., Süral, H. (2012). "The one-warehouse multi-retailer problem: Reformulation, classification, and computational results". *Annals of Operations Research*, 196: 517–541.

Solyalı, O., Süral, H. (2017). "A multi-phase heuristic for the production routing problem". *Computers & Operations Research*, 87: 114–124.