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INVESTIGATION ON THE EFFECTS OF NUMBER OF COMMON POINTS IN 2D TRANSFORMATION PROBLEM

Taylan Ocalan [1]*

[1]Yildiz Technical University, Civil Engineering Faculty, Department of Geomatic Engineering, İstanbul, Turkey
(tocalan@yildiz.edu.tr); **ORCID 0000-0003-0861-013X**

*Corresponding Author, Received: 23/07/2018, Accepted: 13/08/2018

ABSTRACT: Coordinate transformation from one datum to another is the basic problem in geodesy. Generally, the problem may be expressed by converting coordinates in a cartesian coordinate system with defined origin provided by the intersection of two or three axes into another system using mathematical equations. To compute the transformation parameters, a sufficient number of coordinates of the common points should be known in two systems. The problem involves either 2D or 3D coordinate systems. Traditionally the commonly used model for the estimation of the transformation parameters is the Least Squares (LS) method refers as to Helmert Transformation. This study aims to compare the performance of the spatial distribution and quantity of the common points in LS method for coordinate transformation problems. For this purpose, a geodetic network with 25 points, whose coordinates are commonly known in two datum are used to compute the performance of the transformation problem under the different scenarios. To compare the cases, the sum of the absolute coordinate differences is provided by subtracting the original coordinates of test points from computed coordinates by using estimated transformation parameters. The results show that increasing control points one by one to estimate the transformation parameters improve the results of the transformation parameters and reliable transformation parameters have been estimated when a homogeneously distributed 8 points are taken as common points for about a region as 1500 km^2.

Keywords: *Least Square (LS) Method, Coordinate Transformation, Transformation Parameter, Accuracy*

1. INTRODUCTION

Coordinate transformation is one of the most common issues in geodesy phenomena. It is used to transform coordinates from one datum to other by using parameters as translation terms, scale and rotation angle. The increase of application areas in engineering surveys and integration of layouts with different datum has been increased the necessity of accurate datum transformation. The problem in datum transformation is to compute the transformation parameters using common points with known coordinates into two different datum.

Researchers use a number of strategies in order to estimate transformation parameters. For 2D networks, Helmert Transformation is the most commonly used method. Helmert Transformation employs a linear transformation between two systems (Chen and Hill, 2005). Its parameters are two translation terms along the two axes, scale and rotation angle between the axes of two coordinate systems (Akyilmaz, 2007). In Helmert Transformation, at least two common points that their coordinates are well known both datum are required. The numbers of common points given above are the minimum numbers required for the solution and Helmert Transformation uses Least-Squares (LS) method using these common points' coordinates. Moreover, distribution of common points in both datum provides different datum parameters (Kutoglu and Ayan, 2006), which is significant in estimating transformation parameters (Tan et al., 2013) by LS method (Kutoglu and Vaníček, 2006). Here, the number of the common points to be used in transformation problem is also important. Even though the minimum three common points in both datum should be known for adjustment, in this case, it is seen that the reliability of the results obtained from estimation is low. If the number of the common points are increased for estimation of the transformation parameters, the differences between original and converted coordinates decrease. For this purpose, in this paper, first, the mathematical expressions of 2D coordinate transformations using LS method is introduced. Then, the latter section presents the case study depending on a real network and the results of them. The last section concludes with the analyses of the comparative performances of the LS method. In this study, when the number of the common points reaches 8, which the total number of common points is 11 with a homogeneous geometrical distribution, improved results computed by the differences of the test points have been obtained.

2. *THE MATHEMATICAL MODEL OF 2-D TRANSFORMATION PROBLEM (HELMERT TRANSFORMATION)*

2D Coordinate transformation, so-called Helmert Transformation today, has been formulated by F.R. Helmert, and considers only one system contains error in the stochastic model. The transformation parameters are estimated by the LS method. 2D Helmert transformation problem includes four transformation parameters; two translation terms, one rotation component and one scale factor. To estimate the parameters, common points in two different systems are used. The equations of Helmert Transformation problem are mentioned in Eq. (1) and (2).

$$\boldsymbol{x = t_\chi + k.cos\varepsilon.\chi - k.sin\varepsilon.\gamma} \quad (1)$$

$$\boldsymbol{y = t_\gamma + k.sin\varepsilon.\chi + k.cos\varepsilon.\gamma} \quad (2)$$

Where t_χ and t_γ are translation terms, k is scale factor, ε is rotation component. The sub-matrices of design matrix **A** are written as follow,

$$\boldsymbol{A_i} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \chi & -\gamma \\ \mathbf{0} & \mathbf{1} & \gamma & \chi \end{bmatrix} \text{ i=1,.....,n} \quad (3)$$

Here, n is the number of common points in both systems. $\boldsymbol{l_i} = [\boldsymbol{x_i} \quad \boldsymbol{y_i}]^T, \boldsymbol{i} = \mathbf{1}, \ldots, \boldsymbol{n}$ is the observation vector of the transformation problem. To estimate the transformation parameters $\widehat{\boldsymbol{\beta}} = [\boldsymbol{t_\chi} \quad \boldsymbol{t_\gamma} \quad \boldsymbol{k.cos\varepsilon} \quad \boldsymbol{k.sin\varepsilon}\ ]^T$ the linear observation equation can be formed as follows (Koch, 1999):

$$\boldsymbol{v_{LS} = A\widehat{\beta} - l} \quad (4)$$

$$\begin{bmatrix} v_\chi \\ v_\gamma \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \chi & -\gamma \\ \mathbf{0} & \mathbf{1} & \gamma & \chi \end{bmatrix} \begin{bmatrix} t_\chi \\ t_\gamma \\ k.cos\varepsilon \\ k.sin\varepsilon \end{bmatrix} - \begin{bmatrix} x \\ y \end{bmatrix} \quad (5)$$

Once transformation parameters between two systems are estimated by common points, χ and γ coordinates are converted to second system namely x and y coordinates (as seen Fig.1).

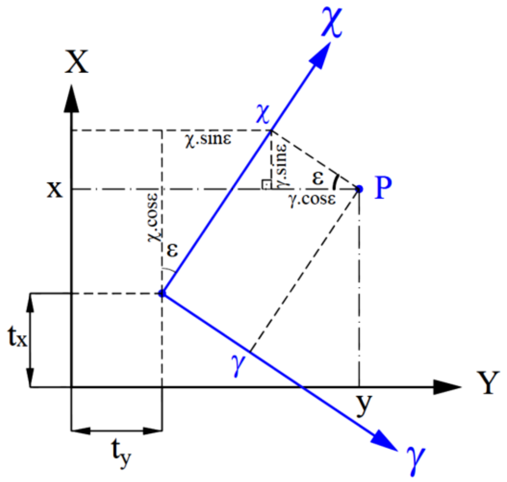

Figure 1. 2D Coordinate Transformation

3. CASE STUDY

For the comparative analysis of computing transformation parameters, a 25-points network established in the Asian side of the İstanbul, Turkey with the known coordinates in both ITRF96 and ED50 has been used. The location of the study area is represented in Figure 2. The total area is about 1450 km^2.

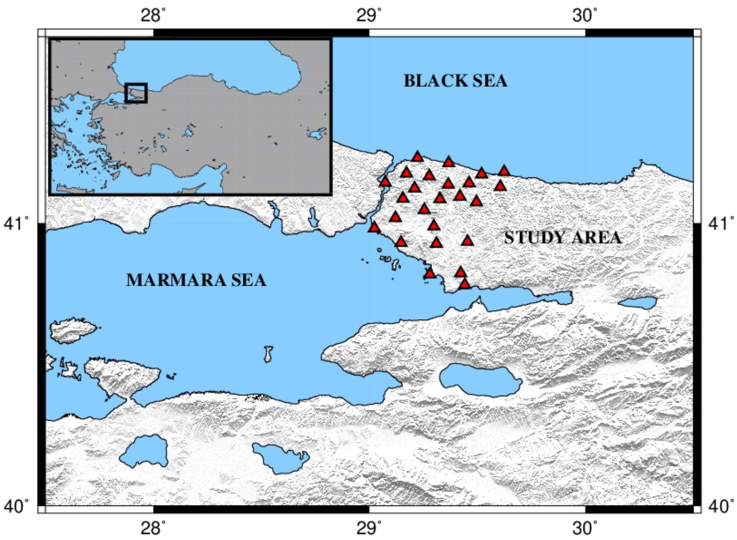

Figure 2. The area of interest (the red colored triangles represent the points in ITRF96 datum)

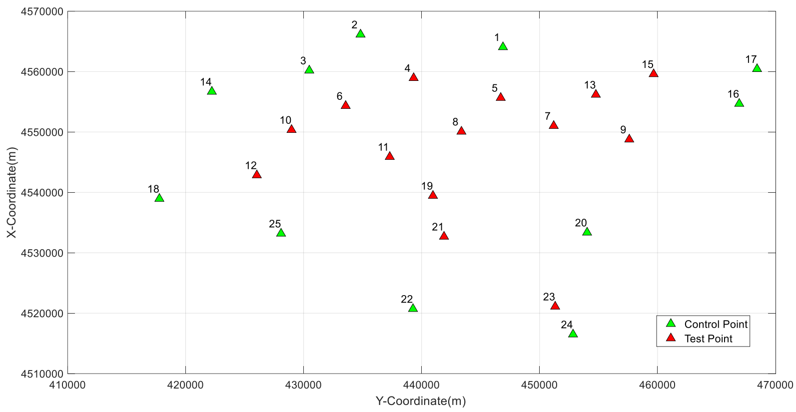

Figure 3. Distribution of the control (green colored triangle) and test (red colored triangle) points in area of interest

The distribution of control and test points is represented in Figure 3, where the green colored triangles show the control points, a total number of 11 and red colored triangles indicate the locations of the test points, a total number of 14. The representative figure has been drawn using ITRF96 datum coordinates of the points. The control points have been used to estimate the transformation parameters by changing the locations and numbers of them in each case. While increasing the number of the points in each case, the locations of the points are considered to cover the study area homogeneously. The test points have been used to check the validation of the estimated parameters that they are whether close to the original coordinates or not. The 14-test points have been converted to the second datum by using the transformation parameters estimated in each case and the obtained coordinates are compared with the original coordinates of the test points.

In the transformation problem, it is important that the common points in both datum should surround the area of interest to represent the region as a frame in terms of estimating the accurate transformation parameters. For this reason, this effect was considered in this study when the number of common points (here, control points) has been increased one by one, the distribution of the control points have been applied as surrounding the test points homogeneously. As mentioned, the 2D transformation parameters have been determined by increasing the number of control points that starts from 3 points, which is the minimum requirement for adjustment and up to 11 points. The name of the control points used in each case can be seen in Table 1.

Table 1 Definition of the cases with control points

Case	# Control Points	Control Points
1	3	17-14-24
2	4	17-2-18-24
3	5	17-2-18-24-25
4	6	17-2-18-24-25-1
5	7	17-2-18-24-25-1-14
6	8	17-2-18-24-25-1-14-20
7	9	17-2-18-24-25-1-14-20-3
8	10	17-2-18-24-25-1-14-20-3-22
9	11	17-2-18-24-25-1-14-20-3-22-16

In each parameter estimation case, the test points have been transferred to the other datum by using the estimated parameters and the coordinates of the test points transferred have been compared with the original datum coordinates. The four transformation parameters (two translation terms, one rotation component and one scale factor) are denoted as tx, ty, ε and k, respectively in 2D network (seen in Tables 2). Table 2 shows the 2D network solutions consisting of several scenarios. The cases are formed to detect the spatial and quantity relations between translation terms and rotation component, also to obtain the reliability of the results of the LS method. In Table 2, the transformation parameters are estimated for transferring coordinates from ITRF96 to ED50 datum.

Table 2 Estimated transformation parameters

# Case	t_x (m)	t_y (m)	k	ε (°)
1	174.0549509	44.5675821	1.0000024	359.9998570
2	175.5153203	45.1053020	1.0000020	359.9998526
3	175.7280878	46.2908093	1.0000020	359.9998382
4	174.9094627	46.5868773	1.0000021	359.9998335
5	174.2245164	47.0875161	1.0000023	359.9998263
6	176.8750128	47.0830701	1.0000017	359.9998294
7	175.6652566	47.8693917	1.0000020	359.9998180
8	177.5116814	45.3846153	1.0000016	359.9998511
9	177.2992547	45.9525747	1.0000016	359.9998438

Table 3 Coordinate differences between estimated and original test points in ED50 datum

	Case 1		Case 2		Case 3	
# Test Point	Δx (mm)	Δy (mm)	Δx (mm)	Δy (mm)	Δx (mm)	Δy (mm)
4	-28.1075	-32.4222	-26.9873	11.9954	-34.7223	17.0079
5	-38.4552	-26.7569	-35.6952	15.4949	-41.3323	20.7975
6	-9.4101	-29.5503	-7.2204	17.1075	-16.0715	23.7010
7	-41.2389	0.4561	-36.6152	41.5932	-40.7839	47.7386
8	-43.0124	-18.6888	-38.6761	25.0825	-44.7461	32.0361
10	0.7316	-16.5026	3.8706	31.9622	-5.8478	39.8899
11	-28.0566	-17.6561	-22.8189	28.4243	-30.1158	36.8710
12	2.6158	-30.7370	7.9874	19.2663	-1.9268	29.2988
13	-34.0988	-22.9626	-30.8915	16.6088	-34.5350	21.1942
19	-41.1820	2.0933	-33.5535	47.4689	-39.4601	57.2730
21	-36.8647	-2.8020	-26.9486	42.7850	-32.1274	54.2256
9	-72.5805	16.2343	-66.7297	55.4463	-69.1214	61.6929
15	-36.8233	-8.8609	-34.3552	28.8465	-37.0126	32.2189
23	-25.7702	23.3955	-11.3413	66.7847	-13.3056	80.4574
	Case 4		Case 5		Case 6	
4	-26.3673	14.7037	-26.2198	7.7581	-22.6288	-4.5261
5	-32.9398	20.0433	-32.3256	14.5271	-27.2439	-2.1807
6	-8.9904	20.7765	-10.1990	13.6096	-3.6433	4.3923
7	-32.8274	48.1446	-32.2950	43.8276	-24.7846	24.2868
8	-37.5976	31.1636	-38.1719	25.8840	-29.6930	10.7925
10	0.1671	36.4959	-2.1628	29.1899	6.9255	22.3993
11	-24.1907	35.2878	-26.0994	29.6888	-14.8919	17.8681
12	2.5443	26.0113	-1.1900	19.2344	12.3755	13.7329
13	-25.3895	21.7959	-23.6980	17.3296	-19.3497	-3.9899
19	-34.3520	56.8543	-36.6953	52.5648	-21.9766	38.2914
21	-28.1160	54.5255	-31.2778	51.2092	-12.7164	36.0325
9	-61.0287	63.3947	-60.0087	60.2435	-51.5488	36.8964
15	-26.8745	33.3903	-24.1022	29.1765	-21.9751	5.2229
23	-10.5283	83.3428	-14.1162	82.7720	10.6000	61.5524
	Case 7		Case 8		Case 9	
4	-17.0197	-17.3251	-13.9011	-22.0265	-10.9672	-24.0004
5	-20.9764	-12.5004	-20.9778	-21.6699	-17.2106	-22.9654
6	-0.3229	-8.9130	7.7319	-14.2711	9.7654	-15.8535
7	-18.7709	16.0019	-19.7481	2.5866	-15.5662	2.0434
8	-25.4725	0.7623	-21.5998	-10.4763	-18.4553	-11.1699
10	8.3494	8.7499	20.4395	2.6969	21.7455	1.4646
11	-12.9137	7.1665	-4.0771	-4.3656	-1.8579	-4.7350
12	11.3567	0.8505	27.7628	-8.5126	28.4295	-8.8839
13	-11.3476	-12.4187	-16.1882	-24.0984	-11.3677	-25.1796
19	-20.8637	29.7786	-11.8948	13.2482	-9.4303	13.8336
21	-13.0905	29.0990	-2.3043	8.3337	0.0446	9.8203
9	-44.8160	30.6438	-48.7086	13.6968	-43.7830	13.6656
15	-12.1562	-2.6723	-21.0110	-14.0933	-15.4434	-15.4407
23	9.2379	59.2548	18.6188	28.5184	21.7735	31.8196

To investigate the effect of the number of the control points used in the transformation problem, 9 different cases have been realized. In each case, the number of the control points has been increased one by one starting from 3 to 11. Table 3 shows the coordinate differences of the test points, which are obtained by subtracting the original coordinates from transformed coordinates. In addition to these results, to figure out the importance of the number of the control points to be used in transformation problem, the sum of the absolute differences of the coordinate components has been taken into consideration. When the number of the control points has been increased, a significant and clear improvement has been detected in the results. Table 4 shows this situation case by case in terms of the sum of absolute coordinate differences subtracted from original test points coordinates in ED50 datum and estimated coordinates of test points in ED50 datum. According to the Table 4, since from 3 to 7 reference points (Cases 1-5) provide similar results, the similar trend in results also is provided for the number of points from 8 to 11. However, a significant difference has been detected when the number of reference points is increased to 8. As can be seen from Table 4, although Case 1 provides a lower value than Case 2, the increase in Case 2 is not seen as a meaningful improvement. Contrary to this, after Case 6, a continuous decrease has been tracked.

Table 4. The sum of the absolute coordinate differences of test points between original and estimated ED50 datum coordinates.

The sum of the absolute coordinate differences $\|\Delta x\|+\|\Delta y\|$ (m)		
Case 1	Case 2	Case 3
0.6881	0.8326	0.9955
Case 4	Case 5	Case 6
0.8978	0.8356	0.5625
Case 7	Case 8	Case 9
0.4628	0.4436	0.4267

4. CONCLUSION

In this study, the basic geodetic application, so-called coordinate transformation problem has been examined with Helmert Transformation by increasing the quantity of the control points to achieve reliable accuracy for solving the datum transformation. The problem is tested by a network that includes 25 points with known coordinates in two different datum. 11 points have been used as control points to estimate the transformation parameters in defined cases in which the number of control points starts with 3 points and ends when reached to 11 points. The 14 points in this network have been used as test points, where the coordinates of them have been compared with transformed coordinates computed from the estimated parameters.

As mentioned before, for 2D coordinate transformation problem, minimum 3 common points should be known to estimate the transformation parameters as adjusted. However, the distribution and number of these common points should also be taken into account to obtain accurate transformation parameters. In this study, it is seen that the number of the common points affects the reliability of the results of the transformation problems, meanwhile the accuracy of the coordinates calculated by using these parameters. For this study area, the homogeneously distributed 8 common points provide reliable and accurate results to solve the transformation problem.

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