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On Right (σ, τ)-Jordan Ideals and One Sided Generalized Derivations

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Abstract

Let R be a prime ring with characteristic not 2 and σ, τ, α, β, λ, μ, γ automorphisms of R. Let h: R→R be a nonzero left (resp. right)-generalized (α,β)-derivation, b∈R and U, V nonzero right (σ,τ)-Jordan ideals of R. In this article we have investigated the following situations:

(1) bh(γ(U))=0, (2) h(γ(U))b=0, (3) h(γ(U))=0, (4) $U\subset C_{\lambda,\mu}(V)$, (5) $bh(I)\subset C_{\lambda,\mu}(U)$ or $h(I)b\subset C_{\lambda,\mu}(U)$, (6) $bV\subset C_{\lambda,\mu}(U)$ or $Vb\subset C_{\lambda,\mu}(U)$.

Keywords: Prime Ring, Generalized Derivation, (σ,τ)-Jordan Ideal.

Sağ (σ,τ)-Jordan İdealler ve Tek Yanlı Genelleştirilmiş Türevler Üzerine

Özet

R, karakteristiği 2 den farklı bir asal halka ve σ,τ,α,β,λ,μ,γ dönüşümleri R üzerinde otomorfizmler olsunlar. h:R→R sıfırdan farklı bir sol (sağ)-genelleştirilmiş (α,β)-türev, b∈R ve U ile V, R halkasının sıfırdan farklı sağ (σ,τ)-Jordan idealleri olsunlar. Bu makalede, aşağıdaki durumları araştırdık:

(1) bh(γ(U))=0, (2) h(γ(U))b=0, (3) h(γ(U))=0, (4) $U\subset C_{\lambda,\mu}(V)$, (5) $bh(I)\subset$

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$C_{\lambda,\mu}(U)$ or $h(I)b\subset C_{\lambda,\mu}(U)$, (6) $bV\subset C_{\lambda,\mu}(U)$ or $Vb\subset C_{\lambda,\mu}(U)$.

Anahtar Kelimeler: Asal Halka, Genelleştirilmiş Türev, (σ,τ)-Jordan Ideal.

1. Introduction

Let R be a ring and σ, τ two mappings of R. For each r, s∈R set $[r,s]_{\sigma,\tau}=r\sigma(s)-\tau(s)r$ and $(r,s)_{\sigma,\tau}=r\sigma(s)+\tau(s)r$. Let U be an additive subgroup of R. If (U, R)⊂U then U is called a Jordan ideal of R. The definition of (σ,τ)-Jordan ideal of R is introduced in [7] as follows: (i) U is called a right (σ,τ)-Jordan ideal of R if $(U,R)_{\sigma,\tau}\subset U$, (ii) U is called a left (σ,τ)-Jordan ideal if $(R,U)_{\sigma,\tau}\subset U$. (iii) U is called a (σ,τ)-Jordan ideal if U is both right and left (σ,τ)-Jordan ideal of R. Every Jordan ideal of R is a (1,1)-Jordan ideal of R, where 1:R→R is the identity map. The following example is given in [7]. Let Z be the set of integers. If R=$\left\{\begin{pmatrix} x & y \\ 0 & 0 \end{pmatrix} \mid x,y\in Z\right\}$, U=$\left\{\begin{pmatrix} x & 0 \\ 0 & 0 \end{pmatrix} \mid x\in Z\right\}$, $\sigma\left(\begin{pmatrix} x & y \\ 0 & 0 \end{pmatrix}\right)$ = $\left(\begin{pmatrix} x & 0 \\ 0 & 0 \end{pmatrix}\right)$ and $\tau\left(\begin{pmatrix} x & y \\ 0 & 0 \end{pmatrix}\right)$ = $\left(\begin{pmatrix} x & -y \\ 0 & 0 \end{pmatrix}\right)$, then U is (σ,τ)-right Jordan ideal but not a Jordan ideal of R.

A derivation d is an additive mapping on R which satisfies d(rs)=d(r)s+rd(s), ∀r, s∈R. The notion of generalized derivation was introduced by Brešar [2] as follows. An additive mapping h: R→R will be called a generalized derivation if there exists a derivation d of R such that h(xy)=h(x)y+xd(y), for all x, y∈R.

An additive mapping d:R→R is said to be a (σ,τ)-derivation if d(rs)=d(r)σ(s)+τ(r)d(s) for all r, s∈R. Every derivation d:R→R is a (1,1)-derivation. Chang [3] gave the following definition. Let R be a ring, σ and τ automorphisms of R and d:R→R a (σ,τ)-derivation. An additive mapping h:R→R is said to be a right generalized (σ,τ)-derivation of R associated with d if h(xy)=h(x)σ(y)+τ(x)d(y), for all x,y∈R and h is said to be a left generalized (σ,τ)-derivation of R associated with d if h(xy)=d(x)σ(y)+τ(x)h(y), for all x, y∈R. h is said to be a generalized (σ,τ)-derivation of R associated with d if it is both a left and right generalized (σ,τ)-derivation of R associated with d.

According to Chang's definition, every (σ,τ)-derivation $d:R\to R$ is a generalized (σ,τ)-derivation associated with d and every derivation $d:R\to R$ is a generalized (1,1)-derivation associated with d. A generalized (1,1)-derivation is simply called a generalized derivation. Every right generalized (1,1)-derivation is a right generalized derivation and every left generalized (1,1)-derivation is a left generalized derivation.

The definition of generalized derivation which is given in [2] is a right generalized derivation associated with derivation d according to Chang's definition.

The mapping $h(r)=(a,r)_{\sigma,\tau}$ for all $r\in R$ is a left-generalized (σ,τ)-derivation associated with (σ,τ)-derivation $d_1(r)=[a,r]_{\sigma,\tau}$ for all $r\in R$ and right-generalized (σ,τ)-derivation associated with (σ,τ)-derivation $d(r)=-[a,r]_{\sigma,\tau}$ for all $r\in R$.

In this paper we generalized some results which are given in [6, 8, 9, 10].

Throughout the paper, R will be a prime ring with center Z, characteristic not 2 and σ, τ, α, β, λ, μ, γ automorphisms of R. We set $C_{\sigma,\tau}(R)=\{c\in R \mid c\sigma(r)=\tau(r)c,\ \forall r\in R\}$ and shall use the following relations frequently.

$$[rs,t]_{\sigma,\tau}=r[s,t]_{\sigma,\tau}+[r,\tau(t)]s=r[s,\sigma(t)]+[r,t]_{\sigma,\tau}s$$

$$[r,st]_{\sigma,\tau}=\tau(s)[r,t]_{\sigma,\tau}+[r,s]_{\sigma,\tau}\sigma(t)$$

$$(rs,t)_{\sigma,\tau}=r(s,t)_{\sigma,\tau}-[r,\tau(t)]s=r[s,\sigma(t)]+(r,t)_{\sigma,\tau}s.$$

$$(r,st)_{\sigma,\tau}=\tau(s)(r,t)_{\sigma,\tau}+[r,s]_{\sigma,\tau}\sigma(t)=-\tau(s)[r,t]_{\sigma,\tau}+(r,s)_{\sigma,\tau}\sigma(t)$$

2. Results

We begin with the following known results, which will be used to prove our theorems.

Lemma 1 [5, Lemma 7] Let I be a nonzero ideal of R and a, $b\in R$. If $h:R\to R$ is a nonzero left-generalized (σ,τ)-derivation associated with (σ,τ)-derivation $d:R\to R$ such that $[h(I)a,b]_{\lambda,\mu}=0$, then $a[a,\lambda(b)]=0$ or $d(\tau^{-1}(\mu(b)))=0$.

Lemma 2 [4, Lemma 2.6] Let $h:R\to R$ be a nonzero right-generalized (σ,τ)-derivation associated with a nonzero (σ,τ)-derivation d and I be a nonzero ideal of R. If $a, b\in R$ such that $[ah(I),b]_{\lambda,\mu}=0$, then $[a,\mu(b)]a=0$ or $d(\sigma^{-1}(\lambda(b)))=0$.

Lemma 3 [7, Lemma 4] Let U be a nonzero (σ,τ)-right Jordan ideal of R and $a\in R$. (i) If $U\subset C_{\sigma,\tau}(R)$ then R is commutative. (ii) If $U\subset Z$ then R is commutative. (iii) If $aU=0$ or $Ua=0$, then $a=0$.

Lemma 4 [7, Lemma 5] Let U be a nonzero (σ,τ)-right Jordan ideal of R and $a, b\in R$. If $aUb=0$ then $a=0$ or $b=0$.

Lemma 5 [7, Lemma 2] If R is a ring and U a nonzero (σ,τ)-right Jordan ideal of R then $2\tau([R,R])U\subset U$ and $2U\sigma([R,R])\subset U$.

Lemma 6 [1, Lemma 1] Let R be a prime ring and $d:R\to R$ be a (σ,τ)-derivation. If U is a nonzero right ideal of R and $d(U)=0$ then $d=0$.

Lemma 7 Let $d:R\to R$ be a nonzero (α,β)-derivation. If $d(\gamma([R,R]))=0$ then R is commutative.

Proof. If $d(\gamma([R,R]))=0$ then we have, for all $r,s\in R$

$$0=d(\gamma([r,rs]))=d(\gamma(r)\gamma([r,s]))=d(\gamma(r))\alpha(\gamma([r,s]))+\beta(\gamma(r))d(\gamma([r,s]))$$

$$=d(\gamma(r))\alpha(\gamma([r,s]))$$

and so for all $r,s\in R$

$$d(\gamma(r))\alpha(\gamma([r,s]))=0. \qquad (2.1)$$

Replacing s by st, $t\in R$ in (2.1) for any $r\in R$, we get $d(\gamma(r))=0$ or $r\in Z$. Let $K=\{r\in R|d(\gamma(r))=0\}$ and $L=\{r\in R|r\in Z\}$. Then K and L are subgroups of R and $R=K\cup L$. Given the fact that a group can not be the union of two proper subgroups, Brauer's Trick, then we have $R=K$ or $R=L$. That is, $d(\gamma(R))=0$ or $R\subset Z$. Since $d\neq 0$ then $d(\gamma(R))\neq 0$ by Lemma 6. On the other hand, $R\subset Z$ means that R is commutative.

Remark 1 Let U be a nonzero right (σ,τ)-Jordan ideal of R. Lemma 5 gives that $2\tau([R,R])U\subset U$ and $2U\sigma([R,R])\subset U$. Since σ and τ are automorphisms of R then we will use the relations $2[R,R]U\subset U$ and $2U[R,R]\subset U$.

Theorem 1 Let U be a nonzero right (σ,τ)-Jordan ideal of R and $b\in R$, let $h:R\to R$ be a nonzero left-generalized (α,β)-derivation associated with a nonzero (α,β)-derivation $d:R\to R$.

(i) If $h(\gamma(U))=0$ then R is commutative.

(ii) If $h(\gamma(U))b=0$ then $b=0$ or R is commutative.

Proof. We can use that $2[r,s]v\in U$ for all $r, s\in R$, $v\in U$ by Remark 1.

(i) If $h(\gamma(U))=0$ then we have, for all $r, s\in R$, $v\in U$

$$0=h(\gamma(2[r,s]v))=h(2\gamma([r,s])\gamma(v))=2d(\gamma([r,s]))\alpha(\gamma(v))+2\beta(\gamma([r,s]))h(\gamma(v))$$

$$=2d(\gamma([r,s]))\alpha(\gamma(v)).$$

That is $\gamma^{-1}(\alpha^{-1}(d(\gamma([r,s]))))U=0$, for all $r, s\in R$. This means that $d(\gamma([R,R]))=0$ by Lemma 3 (iii). Using Lemma 7, we obtain R is commutative.

(ii) If $h\gamma(U)b=0$, then we get, for all $r, s\in R$, $v\in U$

$$0=h(\gamma(2[r,s]v))b=2d(\gamma([r,s]))\alpha(\gamma(v))b+2\beta(\gamma([r,s]))h(\gamma(v))b=2d(\gamma([r,s]))\alpha(\gamma(v))b$$

so $\gamma^{-1}(\alpha^{-1}(d(\gamma([R,R]))))U\gamma^{-1}(\alpha^{-1}(b))=0$. This means that $b=0$ or $d(\gamma([R,R]))=0$ by Lemma 4. If $d(\gamma([R,R]))=0$ then R is commutative by Lemma 7.

Theorem 2 Let U be a nonzero right (σ,τ)-Jordan ideal of R, $b\in R$ and let $h:R\to R$ be a nonzero right-generalized (α,β)-derivation associated with a nonzero (α,β)-derivation d.

(i) If $h(\gamma(U))=0$, then R is commutative.

(ii) If $bh(\gamma(U))=0$, then $b=0$ or R is commutative.

Proof. Remark1 gives that $2v[r,s]\in U$, for all $r, s\in R$, $v\in U$.

(i) If $h(\gamma(U))=0$ then we have, for all $r, s\in R$, $v\in U$

$$0=h(\gamma(2v[r,s]))=h(2\gamma(v)\gamma([r,s]))=2h(\gamma(v))\alpha(\gamma([r,s]))+2\beta(\gamma(v))d(\gamma([r,s]))$$

$$=2\beta(\gamma(v))d(\gamma([r,s])).$$

That is $U\gamma^{-1}(\beta^{-1}(d(\gamma([r,s]))))=0$, for all $r,s\in R$. This means that $d(\gamma([R,R]))=0$ by Lemma 3 (iii). Applying Lemma 7 to the last relation, we obtain that R is commutative.

(ii) If $bh(\gamma(U))=0$, then we get, for all $r, s\in R$, $v\in U$

$$0=bh(2\gamma(v)\gamma([r,s]))=2bh(\gamma(v))\alpha(\gamma([r,s]))+2b\beta(\gamma(v))d(\gamma([r,s]))=2b\beta(\gamma(v))d(\gamma([r,s])).$$

That is $\gamma^{-1}(\beta^{-1}(b))U\gamma^{-1}(\beta^{-1}(d(\gamma([R,R]))))=0$ so $b=0$ or $d(\gamma([R,R]))=0$ by Lemma 4. If $d(\gamma([R,R]))=0$ then we obtain R is commutative by Lemma 7.

Corollary 1 [6, Lemma 5] Let $d:R\to R$ be a nonzero derivation and $a\in R$. If $d(U)a=0$ or $ad(U)=0$ then $a=0$ or R is commutative**.**

Proof. Since d is a derivation and so left (and right)-generalized derivation associated with d then using Theorem 1 (ii) and Theorem 2 (ii) we get the result.

Theorem 3 Let U be a nonzero right (σ,τ)-Jordan ideal of R and $a\in R$.

(i) If $[a,U]_{\lambda,\mu}=0$ then $a\in Z$ or $a\in C_{\lambda,\mu}(R)$.

(ii) If $[U,a]_{\lambda,\mu}=0$ then $a\in Z$.

(iii) If $b[a,U]_{\lambda,\mu}=0$ or $[a,U]_{\lambda,\mu}b=0$ then $b=0$ or $a\in Z$ or $a\in C_{\lambda,\mu}(R)$.

(iv) If $b[U,a]_{\lambda,\mu}=0$ or $[U,a]_{\lambda,\mu}b=0$ then $b=0$ or $a\in Z$.

Proof. Let us consider the mappings defined by $d(r)=[a,r]$, for all $r\in R$ and $g(r)=[r,a]_{\lambda,\mu}$ for all $r\in R$. Then d is a (λ,μ)-derivation and so left (and right)-generalized (λ,μ)-derivation associated with d. If $d=0$ then $a\in C_{\lambda,\mu}(R)$. On the other hand, g is a left-generalized derivation associated with derivation $d_1(r)=[r,\mu(a)]$, for all $r\in R$. If $g=0$ then we obtain $d_1=0$ and so $a\in Z$. Let $g\neq 0$.

(i) If $[a,U]_{\lambda,\mu}=0$ then we have $d(U)=0$. This means that R is commutative by Theorem 1 (i). That is $a\in Z$. Consequently, we obtain $a\in Z$ or $a\in C_{\lambda,\mu}(R)$ for any case.

(ii) If $[U,a]_{\lambda,\mu}=0$ then $g(U)=0$. Since $g\neq 0$ then we have R is commutative by Theorem 1 (i) and so $a\in Z$.

(iii) If $b[a,U]_{\lambda,\mu}=0$ then we have $bd(U)=0$. This means that $b=0$ or R is commutative by Theorem 2 (ii). That is $b=0$ or $a\in Z$. Finally, we obtain $b=0$ or $a\in Z$ or $a\in C_{\lambda,\mu}(R)$. If $[a,U]_{\lambda,\mu}b=0$ then $d(U)b=0$ and so $b=0$ or R is commutative is obtained by Theorem 1 (ii). Again we obtain that $b=0$ or $a\in Z$ or $a\in C_{\lambda,\mu}(R)$ for any case.

(iv) If $b[U,a]_{\lambda,\mu}=0$ then $bg(U)=0$. Using Theorem 2 (ii) we obtain $b=0$ or R is commutative and so $b=0$ or $a\in Z$. Similarly if $[U,a]_{\lambda,\mu}b=0$ then $g(U)b=0$. Hence, $b=0$ or R is commutative by Theorem 1 (ii). Considering as above, we have $b=0$ or $a\in Z$ for any case.

Corollary 2 [10, Lemma 2.7] Let R be a 2-torsion free prime ring and U be a nonzero Jordan ideal of R. If U is a commutative then $U\subseteq Z$.

Proof. Every Jordan ideal is a right (1,1)-Jordan ideal of R, where $1:R\rightarrow R$ is an identity map. If U is commutative then we have $[U,U]_{1,1}=0$. Using Theorem 3 (ii), we obtain $U\subseteq Z$.

Corollary 3 Let U, V be nonzero right (σ,τ)-Jordan ideals of R. If $U\subset C_{\lambda,\mu}(V)$ then R is commutative.

Proof. If $U\subset C_{\lambda,\mu}(V)$ then $[U,V]_{\lambda,\mu}=0$. Using Theorem 3 (ii), we obtain $V\subset Z$. Hence, R is commutative by Lemma 3 (ii).

Theorem 4 Let U be a nonzero right (σ,τ)-Jordan ideal of R and $a, b\in R$.

(i) If $(a,U)_{\lambda,\mu}=0$ then $a\in Z$ or $a\in C_{\lambda,\mu}$.

(ii) If $(U,a)_{\lambda,\mu}=0$ then $a\in Z$.

(iii) If $b(a,U)_{\lambda,\mu}=0$ or $(a,U)_{\lambda,\mu}b=0$ then $b=0$ or $a\in Z$ or $a\in C_{\lambda,\mu}$.

(iv) If $b(U,a)_{\lambda,\mu}=0$ or $(U,a)_{\lambda,\mu}b=0$ then $b=0$ or $a\in Z$.

Proof. Let us consider the mappings defined by $h(r)=(a,r)_{\lambda,\mu}$ for all $r\in R$ and $g(r)=(r,a)_{\lambda,\mu}$ for all $r\in R$. Then h is a left-generalized (λ,μ)-derivation associated with (λ,μ)-derivation $d_1(r)=[a,r]_{\lambda,\mu}$, for all $r\in R$ and right-generalized (λ,μ)-derivation associated with (λ,μ)-derivation $d(r)=-[a,r]_{\lambda,\mu}$, for all $r\in R$. If $h=0$ then $d=0=d_1$ and so $a\in C_{\lambda,\mu}$ is obtained. Let $h\neq 0$. On the other hand g is a left-generalized derivation associated with derivation $d_2(r)=-[r,\mu(a)]$, for all $r\in R$ and right-generalized derivation associated with derivation $d_3(r)=[r,\lambda(a)]$, for all $r\in R$. If $g=0$, then $d_2=0=d_3$ and we obtain $a\in Z$.

(i) If $(a,U)_{\lambda,\mu}=0$ then we have $h(U)=0$. Using Theorem 1 (i) we get $a\in Z$. Finally, we obtain that $a\in Z$ or $a\in C_{\lambda,\mu}$.

(ii) If $(U,a)_{\lambda,\mu}=0$ then $g(U)=0$. Similarly Theorem 1 (i) gives that $a\in Z$.

(iii) If $b(a,U)_{\lambda,\mu}=0$ then we have $bh(U)=0$. Hence, $b=0$ or R is commutative by Theorem 2 (ii). That is $b=0$ or $a\in Z$. Finally, we obtain that $b=0$ or $a\in Z$ or $a\in C_{\lambda,\mu}$. If $(a,U)_{\lambda,\mu}b=0$ then we have $h(U)b=0$. Using Theorem 1 (ii) we get $b=0$ or R is commutative. Consequently, we have $b=0$ or $a\in Z$ or $a\in C_{\lambda,\mu}$ for any case.

(iv) If $b(U,a)_{\lambda,\mu}=0$ then $bg(U)=0$. Considering as in the proof of (iii) and using Theorem 2 (ii) we arrive $b=0$ or $a\in Z$. If $(U,a)_{\lambda,\mu}b=0$ then $g(U)b=0$. Using Theorem 1 (ii), we get the same result.

Theorem 5 Let U be a nonzero right (σ,τ)-Jordan ideal of R, $b\in R$ and let $h:R\rightarrow R$ be a nonzero right-generalized (α,β)-derivation associated with a nonzero (α,β)-derivation d and I nonzero ideal of R. If $bh(I)\subset C_{\lambda,\mu}(U)$ then $b\in Z$.

Proof. Let $bh(I)\subset C_{\lambda,\mu}(U)$. This means that $[bh(I),v]_{\lambda,\mu}=0$, for all $v\in U$. Using Lemma 2 we obtain that, for any $v\in U$,

$$[b,\mu(v)]b=0 \text{ or } d\alpha^{-1}\lambda(v)=0.$$

Let $K=\{v\in U \mid [b,\mu(v)]b=0\}$ and $L=\{v\in U \mid d(\alpha^{-1}(\lambda(v)))=0\}$. Using Brauer's Trick, we get $[b, \mu(U)]b=0$ or $d(\alpha^{-1}(\lambda(U)))=0$.The mapping $d_1(r)=[b,r]$, for all $r\in R$ is a derivation and so left (and right)-generalized derivation associated with derivation d_1. If $d_1=0$ then $b\in Z$ is obtained. Let $d_1\neq 0$. If $[b, \mu(U)]b=0$ then we can write $d_1(\mu(U))b=0$. Since d_1 is a left-generalized derivation, then we have $b=0$ or R is commutative by Theorem 1 (ii). Finally, we obtain $b\in Z$ for any case. If $d(\alpha^{-1}(\lambda(U)))=0$ then we have R is commutative by Theorem 1 (i) and so $b\in Z$.

Theorem 6 Let U be a nonzero right (σ,τ)-Jordan ideal of R, $h:R\rightarrow R$ be a nonzero left-generalized (α,β)-derivation associated with a nonzero (α,β)-derivation $d:R\rightarrow R$ and I be a nonzero ideal of R. If $b\in R$ such that $h(I)b\subset C_{\lambda,\mu}(U)$ then $b\in Z$.

Proof. If $h(I)b\subset C_{\lambda,\mu}(U)$ then we have $[h(I)b,v]_{\lambda,\mu}=0$, for all $v\in U$. This means that for any $v\in U$ $d(\beta^{-1}(\mu(v))=0$ or $b[b,\lambda(v)]=0$ by Lemma 1. Let $K=\{v\in U|d(\beta^{-1}(\mu(v)))=0\}$ and $L=\{v\in U|b[b,\lambda(v)]=0\}$. According to Brauer's Trick, we get $d(\beta^{-1}(\mu(U)))=0$ or $b[b,\lambda(U)]=0$. Since d is an (α,β)-derivation then d is a right (and left)-generalized (α,β)-derivation associated with d. If $d(\beta^{-1}(\mu(U)))=0$ then we have R is commutative by Theorem 1 (i). That is $b\in Z$. On the other hand, the mapping defined by $d_1(r)=[b,r]$, for all $r\in R$ is a derivation and so right (and left)-generalized derivation associated with derivation d_1. If $d_1=0$ then $b\in Z$ is obtained. If $d_1\neq 0$ then $b[b,\lambda(U)]=0$ gives that $b=0$ or R is commutative by Theorem 2 (ii). Finally, we obtain that $b\in Z$ for any case.

Corollary 4 Let U be nonzero right (σ,τ)-Jordan ideal of R and I be a nonzero ideal of R. If $b(a,I)_{\alpha,\beta}\subset C_{\lambda,\mu}(U)$ or $(a,I)_{\alpha,\beta}b\subset C_{\lambda,\mu}(U)$ then $b\in Z$ or $a\in C_{\alpha,\beta}(R)$ for all $a,b\in R$.

Proof. The mapping defined by $h(r)=(a,r)_{\alpha,\beta}$, for all $r\in R$ is a left-generalized (α,β)-derivation associated with (α,β)-derivation $d_1(r)=[a,r]_{\alpha,\beta}$ for all $r\in R$ and right-generalized (α,β)-derivation associated with (α,β)-derivation $d(r)=-[a,r]_{\alpha,\beta}$, for all $r\in R$. If $h=0$ then $d=0=d_1$ and so $a\in C_{\alpha,\beta}(R)$ is obtained. If $b(a,I)_{\alpha,\beta}\subset C_{\lambda,\mu}(U)$ then we have

$bh(I)\subset C_{\lambda,\mu}(U)$. Since h is a right-generalized (α,β)-derivation, then we obtain $b\in Z$ by Theorem 5.

Similarly, if $(a,I)_{\alpha,\beta}b\subset C_{\lambda,\mu}(U)$ then $h(I)b\subset C_{\lambda,\mu}(U)$. Since h is a left-generalized (α,β)-derivation, then we have $b\in Z$ by Theorem 6. Finally, we obtain that $b\in Z$ or $a\in C_{\alpha,\beta}(R)$ for any case.

Corollary 5 Let U, V be nonzero right (σ,τ)-Jordan ideals of R and $b\in R$. If $bV\subset C_{\lambda,\mu}(U)$ or $Vb\subset C_{\lambda,\mu}(U)$ then $b\in Z$.

Proof. If $bV\subset C_{\lambda,\mu}(U)$ then we have $b(V,R)_{\sigma,\tau}\subset C_{\lambda,\mu}(U)$. Hence

$$b\in Z \text{ or } V\subset C_{\lambda,\mu}(R) \qquad (2.2)$$

by Corollary 4. If $V\subset C_{\lambda,\mu}(R)$ in (2.2) then we can write $[V,R]_{\lambda,\mu}=0$. Using Theorem 3 (ii) we get $R\subset Z$, and so we obtain $b\in Z$. If $Vb\subset C_{\lambda,\mu}(U)$ then we have $(V,R)_{\sigma,\tau}b\subset C_{\lambda,\mu}(U)$. Using Corollary 4 and considering as above we obtain that $b\in Z$. This completes the proof.

The following Lemma is a generalization of [8] and [9].

Lemma 8 Let U be nonzero right (σ,τ)-Jordan ideal of R and a, $b\in R$. If b, $ba\in C_{\lambda,\mu}(U)$ or b, $ab\in C_{\lambda,\mu}(U)$ then $b=0$ or $a\in Z$.

Proof. If b, $ba\in C_{\lambda,\mu}(U)$ then, for all $v\in U$ we get

$$0=[ba,v]_{\lambda,\mu}=b[a,\lambda(v)]+[b,v]_{\lambda,\mu}a=b[a,\lambda(v)]$$

so $\lambda^{-1}(b)[\lambda^{-1}(a),U]=0$. This means that $b=0$ or $a\in Z$ or $a\in C_{1,1}(R)$ by Theorem 3 (iii). That is $b=0$ or $a\in Z$. If b, $ab\in C_{\lambda,\mu}(U)$, then for all $v\in U$, the relation $0=[ab,v]_{\lambda,\mu}=a[b,v]_{\lambda,\mu}+[a,\mu(v)]b=[a,\mu(v)]b$ gives that $[\mu^{-1}(a),U]\mu^{-1}(b)=0$. Smilary using Theorem 3 (iii), we get $b=0$ or $a\in Z$.

Theorem 7 Let U be nonzero right (σ,τ)-Jordan ideal of R, let I be ideal of R and a, $b\in R$. If $b\gamma([I,a]_{\alpha,\beta})\subset C_{\lambda,\mu}(U)$ or $\gamma([I,a]_{\alpha,\beta})b\subset C_{\lambda,\mu}(U)$ then $b=0$ or $a\in Z$.

Proof. If $b\gamma([I,a]_{\alpha,\beta})\subset C_{\lambda,\mu}(U)$ then we get, for all $x\in I$

$$b\gamma([x\alpha(a),a]_{\alpha,\beta})=b\gamma(x)\gamma([\alpha(a),\alpha(a)])+b\gamma([x,a]_{\alpha,\beta})\gamma(\alpha(a))=b\gamma([x,a]_{\alpha,\beta})\gamma(\alpha(a))\in C_{\lambda,\mu}(U)$$

and so

$$b\gamma([I,a]_{\alpha,\beta})\gamma(\alpha(a))\subset C_{\lambda,\mu}(U). \tag{2.3}$$

If we use hypothesis and Lemma 8 in (2.3), then we get $\gamma^{-1}(b)[I,a]_{\alpha,\beta}=0$ or $a\in Z$. If $\gamma^{-1}(b)[I,a]_{\alpha,\beta}=0$ then we obtain that $b=0$ or $a\in Z$ by Theorem 3 (iv). If $\gamma([I,a]_{\alpha,\beta})b\subset C_{\lambda,\mu}(U)$, then we have for all $x\in I$

$$\gamma([\beta(a)x,a]_{\alpha,\beta})b=\gamma(\beta(a))\gamma([x,a]_{\alpha,\beta})b+\gamma([\beta(a),\beta(a)])\gamma(x)b=\gamma(\beta(a))\gamma([x,a]_{\alpha,\beta})b\in C_{\lambda,\mu}(U).$$

That is

$$\gamma(\beta(a))\gamma[(I,a]_{\alpha,\beta})b\subset C_{\lambda,\mu}(U). \tag{2.4}$$

If we use Lemma 8 and hypothesis then (2.4) gives that $[I,a]_{\alpha,\beta}\gamma^{-1}(b)=0$ or $a\in Z$. If $[I,a]_{\alpha,\beta}\gamma^{-1}(b)=0$ then we obtain that $b=0$ or $a\in Z$ by Theorem 3 (iv). This completes the proof.

Theorem 8 Let U be nonzero right (σ,τ)-Jordan ideal of R, I be an ideal of R and $a,b\in R$. If $b\gamma(I,a)_{\alpha,\beta}\subset C_{\lambda,\mu}(U)$ or $\gamma(I,a)_{\alpha,\beta}b\subset C_{\lambda,\mu}(U)$ then $b=0$ or $a\in Z$.

Proof. If $b\gamma(I,a)_{\alpha,\beta}\subset C_{\lambda,\mu}(U)$ then we get, for all $x\in I$

$$b\gamma((x\alpha(a),a)_{\alpha,\beta})=b\gamma(x)\gamma([\alpha(a),\alpha(a)])+b\gamma((x,a)_{\alpha,\beta})\gamma(\alpha(a))=b\gamma((x,a)_{\alpha,\beta})\gamma(\alpha(a))\in C_{\lambda,\mu}(U)$$

and so

$$b\gamma((I,a)_{\alpha,\beta})\gamma(\alpha(a))\subset C_{\lambda,\mu}(U). \tag{2.5}$$

If we use hypothesis and Lemma 8 in above relation,then we get $\gamma^{-1}(b)((I,a)_{\alpha,\beta})=0$ or $a\in Z$. If $\gamma^{-1}(b)(I,a)_{\alpha,\beta}=0$ then we obtain that $b=0$ or $a\in Z$ by Theorem 4 (iv). If $\gamma((I,a)_{\alpha,\beta})b\subset C_{\lambda,\mu}(U)$ then we have, for all $x\in I$

$$\gamma((\beta(a)x,a)_{\alpha,\beta})b=\gamma(\beta(a))\gamma((x,a)_{\alpha,\beta})b-\gamma([\beta(a),\beta(a)])\gamma(x)b=\gamma(\beta(a))\gamma((x,a)_{\alpha,\beta})b\in C_{\lambda,\mu}(U).$$

That is

$$\gamma(\beta(a))\gamma((I,a)_{\alpha,\beta})b\subset C_{\lambda,\mu}(U). \tag{2.6}$$

If we use Lemma 8 and hypothesis, then (2.6) gives that $(I,a)_{\alpha,\beta}\gamma^{-1}(b)=0$ or $a\in Z$. If $(I,a)_{\alpha,\beta}\gamma^{-1}(b)=0$ then we obtain that b=0 or $a\in Z$ by Theorem 4 (iv).

References

[1] Aydın, N. and Kaya, K., *Some Generalizations in Prime Rings with (σ,τ)-Derivation*, Tr. J. Math., 16(3), 169-176, 1992.

[2] Bresar, M., *On the Distance of the Composition of Two Derivation to Generalized Derivations*, Glasgow Math. J., 33, 89-93, 1991.

[3] Chang, J. C., *On the Identity h(x)=af(x)+g(x)b*, Taiwanese J. Math., 7(1), 103-113, 2003.

[4] Güven, E., *One Sided (σ,τ)-Lie Ideals and Generalized Derivations in Prime Rings*, Palestine Journal of Mathematics, 7(2), 479-486, 2018.

[5] Güven, E., *One Sided Generalized (σ,τ)-Derivations in Rings*, Boletim Sociedade Paranaense de Matemática, 38(2), 41-50, 2020 (to appear).

[6] Kandamar, H. and Kaya, K., *(σ,τ)-Right Jordan Ideals*, Cumhuriyet Üniversitesi Fen Bilimleri Dergisi, 1993.

[7] Kaya, K., Kandamar, H. and Aydın, N., *Generalized Jordan Structure of Prime Rings*, Tr. J. Math., 251-258, 1993.

[8] Kaya, K., *On a (σ,τ)-Derivations of Prime Rings*, Doğa TU Mat. D. C., 12(2), 42-45, 1988.

[9] Mayne, J. H., *Centralizing Mappings of Prime Rings*, Canad. Math. Bull., 27, 122-126, 1984.

[10] Zaidi, S. M. A., Ashraf, M. and Ali, S., *On Jordan Ideals and Left (θ,θ)-Derivations, in Prime Rings*, Int. J. Math. Math. Sci., 37, 1957-1964, 2004.