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Research Article

Approximate Solutions of the Time-Fractional Kadomtsev-Petviashvili Equation with Conformable Derivative

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Abstract

In this study, residual power series method, namely RPSM, is applied to solve time-fractional Kadomtsev-Petviashvili (K-P) differential equation. In the solution procedure, the fractional derivatives are explained in the conformable sense. The model is solved approximately and the obtained results are compared with exact solutions obtained by the sub-equation method. The results reveal that the present method is accurate, dependable, simple to apply and a good alternative for seeking solutions of nonlinear fractional partial differential equations.
Keywords: Fractional partial differential equations, Fractional Kadomtsev-Petviashvili equation, conformable fractional derivative, residual power series method

Zaman-Kesirli Kadomtsev- Petviashvili Denkleminin Conformable Türev ile Yaklaşık Çözümleri

Öz

Bu çalışmada, zaman-kesirli Kadomtsev-Petviashvili (K-P) diferansiyel denklemini çözmek için Rezidual Kuvvet Serisi Metodu (RPSM) kullanılmıştır. Çözüm prosedüründe, kesirli türevler, conformable kesirli türev tanımına göre hesaplanmıştır. Bu model yaklaşık olarak çözülmüş ve elde edilen sonuçlar, sub-equation metodu ile elde edilen tam çözümlerle karşılaştırılmıştır. Sonuçlar, mevcut yöntemin doğru, güvenilir, uygulanmasının basit olduğunu ve doğrusal olmayan kısmi diferansiyel denklemlerin çözümü için iyi bir alternatif olduğunu ortaya koymaktadır.
Anahtar Kelimeler: Kesirli kısmi diferansiyel denklemler, Kesirli Kadomtsev-Petviashvili denklemi, conformable kesirli türev, residual kuvvet serisi metodu

1. Introduction

The history of the studies of fractional order calculus is nearly old as classical integer order analysis. However, it was not used in physical sciences for many years. But, at the last decades, applications of the fractional calculus in applied mathematics, viscoelasticity (Zhaosheng and Jianzhong 1998), control (Yeroglu and Senol 2013), electrochemistry (Oldham 2010), electromagnetic (Heaviside 2008) have

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become more and more evident. The development of the symbolic calculation programs also helped this improvement. Various interdisciplinary applications could be expressed by the help of fractional derivatives and integrals. Some fundamental descriptions and applications of fractional calculus are given in (Carpinteri and Mainardi 2014) and (Podlubny 1997). The existence of the fractional differential equations is also examined in (Yakar and Köksal 2012).

In parallel to these studies, fractional order partial differential equations (FPDEs) also gave scientists the chance of describing and modeling many important and useful physical problems.

Hereby, a considerable effort has been expended to construct numerical and analytical methods for solving FPDEs, in recent years. Some of them are, homotopy analysis method (Ghazanfari and Veisi 2011; Song and Zhang 2007), fractional variational iteration method [Guo and Mei 2011; Wu and Lee 2010], Adomian decomposition method (Jafari and Daftardar 2006; Momani and Shawagfeh; Song and Wang 2013), fractional differential transform method (Arikoglu and Ozkol 2009; El-Sayed et al. 2014; Momani et al. 2007) and perturbation-iteration algorithm (Şenol and Dolapci 2016; Şenol et al. 2018).

In this study, an earlier proposed method, RPSM is studied. This method is established by a Jordanian mathematician Omar Abu Arqub (Arqub 2013a and b). By choosing proper initial conditions, it can be applied through to problem without discretization, linearization, or any special transformation.

The primary aim of this study is to achieve approximate solutions of time-fractional K-P equation of the form

$$\frac{\partial}{\partial x}\frac{\partial^{\alpha}}{\partial t^{\alpha}}u+\frac{1}{4}\frac{\partial^{4}u}{\partial x^{4}}+\frac{3}{2}\frac{\partial u}{\partial x}\frac{\partial^{2}u}{\partial x^{2}}+\frac{3}{4}\frac{\partial^{2}u}{\partial y^{2}}=0, \tag{1}$$

where $u=u(x,y,t), t\geq 0$ and $0<\alpha\leq 1$.

This equation is proposed by Soviet physicists, Boris Borisovich Kadomtsev and Vladimir Iosifovich Petviashvili (Kadomtsev and Petviashvili 1970). It is actually a generalized form of the KdV equation. However, while the waves are strictly one dimensional in KdV equation, in K-P equation this limitation is relaxed and it allowed scientists to study with higher dimensions.

The K-P equation is a convenient tool to model water waves with frequency dispersion and weakly nonlinear restoring forces that travel in the positive x-direction with long wavelength. It is also used to model interaction of shallow or long water waves with two and three-dimensional cases. Moreover, it has numerous applications arise in ion-acoustic waves in dusty plasmas, ferromagnetics and dynamical systems of water waves.

2. Preliminaries

Several fractional or arbitrary order derivative definitions are exist in the literature. Riemann-Liouville and Caputo fractional derivatives are the most common used ones. In addition to these well-known definitions, we will also present the conformable fractional derivative that is used to achieve approximate solutions in this study.

2.1. Definition

The Riemann-Liouville fractional derivative operator $D^{\alpha}f(x)$ for $\alpha > 0$ and $q-1<\alpha<q$ defined as (Ahmad 2015; Das 2011; Diethelm 2010):

$$D^{\alpha}f(x) = \frac{d^q}{dx^q}\left[\frac{1}{\Gamma(q-\alpha)}\int_{a}^{x}\frac{f(t)}{(x-t)^{\alpha+1-q}}dt\right]. \tag{2}$$

2.2. Definition

The Caputo fractional derivative of order $\alpha > 0$ that is D_{*}^{α} for $\in \mathbb{N}, n-1<\alpha<n$, is defined as [Caputo 1967]:

$$D_{*}^{\alpha}\mathrm{f}(x) = J^{n-\alpha}D^{n}f(x) = \frac{1}{\Gamma(n-\alpha)}\int_{a}^{x}(x-t)^{n-\alpha-1}\left(\frac{d}{dt}\right)^{n}f(t)dt. \tag{3}$$

2.3. Definition

For all $t > 0$ and $\alpha \in (0,1)$ an α-th order "conformable fractional derivative" of a function is defined by (Khalil et al. 2014) as

$$T_{\boldsymbol{\alpha}}(f)(t) = \lim_{\varepsilon\to 0}\frac{f(t+\varepsilon t^{1-\alpha})-f(t)}{\varepsilon}, \tag{4}$$

for $f: [0,\infty) \to \mathbb{R}$.

The following theorem gives the properties of the definition (Khalil et al.2014).

2.4. Theorem

If f, g are α-differentiable functions at point $t > 0$ for $\alpha \in (0,1]$, then

i. $T_{\boldsymbol{\alpha}}(mf+ng) = mT_{\boldsymbol{\alpha}}(f)+nT_{\boldsymbol{\alpha}}(g)$ for $m, n \in \mathbb{R}$.

ii. $T_{\boldsymbol{\alpha}}(t^{p}) = pt^{t-\alpha}$ for all $p \in \mathbb{R}$.

iii. $T_{\boldsymbol{\alpha}}(f.g) = fT_{\boldsymbol{\alpha}}(g)+gT_{\boldsymbol{\alpha}}(f)$.

iv. $T_{\boldsymbol{\alpha}}(f/g) = \frac{T_{\alpha}(f)-fT_{\alpha}(g)}{g^{2}}$.

v. $T_{\boldsymbol{\alpha}}(c) = 0$ when c is a constant.

vi. Also, f is differentiable, then $T_{\boldsymbol{\alpha}}(f)(t) = t^{1-\boldsymbol{\alpha}}\frac{df}{dt}$.

2.5. Definition

Let f be a function with $x_1, x_2, \ldots, x_n$ variables. The conformable partial derivative of f order $\alpha \in (0,1]$ in x_i is defined as (Atangana et al. 2015).

$$\frac{d^{\alpha}}{dx_i^{\alpha}}f(x_1,x_2,\ldots,x_n) = \lim_{\varepsilon\to 0}\frac{f(x_1,x_2,\ldots,x_{i-1},x_i+\varepsilon x_i^{1-\alpha},\ldots,x_n)-f(x_1,x_2,\ldots,x_n)}{\varepsilon}. \tag{5}$$

2.6. Definition

Starting from $a \geq 0$, the conformable integral of an f function is defined as (Taşbozan et al. 2016)

$$I_{\alpha}^{a}(f)(s) = \int_{a}^{s}\frac{f(t)}{t^{1-\alpha}}dt. \tag{6}$$

Now we present some definitions and theorems that are crucial for residual power series.

2.7. Theorem

Let f be an infinitely α-differentiable function at a neighborhood of a t_0 point for some $0 < \alpha \leq 1$, then f has the fractional power series expansion of the form:

$$f(t) = \sum_{k=0}^{\infty} \frac{\left(T_{\alpha}^{t_0} f\right)^{(k)}(t_0)(t-t_0)^{k\alpha}}{\alpha^k k!}, \qquad t_0 < t < t_0 + R^{\frac{1}{\alpha}}, \qquad R > 0. \tag{7}$$

Here $\left(T_{\alpha}^{t_0} f\right)^{(k)}(t_0)$ expresses the implementation of the conformable derivative k-times (Abdeljawad 2015).

2.8. Definition

$\sum_{n=0}^{\infty} f_n(x) t^{n\alpha}$ for $0 \leq m-1 < \alpha < m$, is called a multiple fractional power series (FPS) about $t_0 = 0$, where $f_n(x)$ are the coefficients of the series and t is a variable (Alabsi 2017; El-Ajou et al. 2013).

2.9. Theorem

Suppose that

$$u(x,t) = \sum_{n=0}^{\infty} f_n(x) t^{n\alpha}, \qquad 0 < m-1 < \alpha \leq m, \qquad x \epsilon I, \qquad 0 < t < R^{\frac{1}{\alpha}}. \tag{8}$$

is the multiple FPS representation of $u(x,t)$ at $t_0 = 0$ If $u_t{}^{(n\alpha)}(x,t), n = 0,1,2,\ldots$ are continuous on $I \times (0, R^{\frac{1}{\alpha}})$, then $f_n(x) = \frac{u_t{}^{(n\alpha)}(x,0)}{\alpha^n n!}$ (Alabsi 2017).

3. Basic idea of the residual power series method

To illustrate the basic idea of RPSM (Alquran 2015a and b; Arqub 2013a and b), consider the nonlinear fractional differential equation below (Kumar et al. 2016):

$$\begin{aligned} &T_\alpha u(x,y,t) + N[x,y]u(x,y,t) + L[x,y]u(x,y,t) = g(x,y,t), \qquad t > 0, x \in \mathbb{R}, \\ &\qquad n-1 < n\alpha \leq n, \end{aligned} \tag{9}$$

expressed by the initial condition

$$f_0(x,y) = u(x,y,0) = f(x,y), \tag{10}$$

where $N[x,y]$ and $L[x,y]$ are nonlinear and linear operators respectively and $g(x,y,t)$ are continuous functions.

In RPS method, the solution of the equation (9) subject to (10), is constituted of stating it as a FPS expansion around $t = 0$.

$$f_{n-1}(x,y) = T_t^{(n-1)\alpha} u(x,y,0) = h(x,y). \tag{11}$$

The FPS expansion of the solution is given by

$$u(x,y,t) = f(x,y) + \sum_{n=0}^{\infty} f_n(x,y) \frac{t^{n\alpha}}{\alpha^n n!}. \tag{12}$$

Next, the k-th truncated series of $u(x,t)$, that is $u_k(x,t)$ can be written as:

$$u_k(x,y,t) = f(x,y) + \sum_{n=0}^{k} f_n(x,y) \frac{t^{n\alpha}}{\alpha^n n!}. \tag{13}$$

If the first RPSM approximate solution $u_1(x,y,t)$ is written as $u_1(x,y,t) = f(x,y) + f_1(x,y)\frac{t^\alpha}{\alpha^n}$ then we can write

$$u_k(x,y,t) = f(x,y) + f_1(x,y)\frac{t^\alpha}{\alpha^n} + \sum_{n=2}^{k} f_n(x,y) \frac{t^{n\alpha}}{\alpha^n n!}, k = 2,3,4,\ldots \tag{14}$$

for $0 < \alpha \le 1,\ x \in I,\ 0 \le t < R.$ Initially we express the residual function

$$Resu(x,y,t) = T_\alpha u(x,y,t) + N[x,y]u(x,y,t) + L[x,y]u(x,y,t) - c(x,y,t), \tag{15}$$

and the k-th residual function

$$Resu_k(x,y,t) = T_\alpha u_k(x,y,t) + N[x,y]u_k(x,y,t) + L[x,y]u_k(x,y,t) - c(x,y,t),\quad k = 1,2,3,\ldots \tag{16}$$

respectively. Obviously, $Res(x,y,0) = 0$ and $\lim_{k\to\infty} Resu_k(x,y,t) = Resu(x,y,t)$for each $x \in I$ and $t \ge 0$. As long as the fractional derivative of a constant is zero (Arqub 2013a and b; Jaradat et al. 2016) in conformable sense,

$\frac{\partial^{(n-1)\alpha}}{\partial t^{(n-1)\alpha}} Resu_k(x,y,t) = 0$ for

$n = 1,2,3,\ldots,k.$ Solving this equation produces the required $f_n(x,y)$ coefficients. Actually in our case we will have differential equations. Thus, the $u_n(x,y,t)$ RPSM approximate solutions can be calculated in this manner respectively.

4. Approximate Solutions for Fractional Kadomtsev-Petviashvili Equation

Now we present nonlinear time-fractional K-P equation (Kaya 2003) as

$$\frac{\partial}{\partial x}\frac{\partial^\alpha}{\partial t^\alpha} u + \frac{1}{4}\frac{\partial^4 u}{\partial x^4} + \frac{3}{2}\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2} + \frac{3}{4}\frac{\partial^2 u}{\partial y^2} = 0, \tag{17}$$

where $u = u(x,y,t),\ t \ge 0$ and $0 < \alpha \le 1$. The initial condition obtained from the exact solution that is obtained by the sub-equation method (Durur 2019) is

$$u(x,y,0) = f(x,y) = 1 + \frac{2}{3+x+y}. \tag{18}$$

For residual power series

$u(x,y,t) = f(x,y) + \sum_{n=1}^{\infty} f_n(x,y)\frac{t^{n\alpha}}{\alpha^n n!}$,(19)

the k −th truncated series of it is defined as

$$u_k(x,y,t) = f(x,y) + \sum_{n=1}^{k} f_n(x,y)\frac{t^{n\alpha}}{\alpha^n n!}, \qquad k = 1,2,3,\dots \tag{20}$$

Therefore, the k −th residual function of time-fractional K-P equation can be written as

$$Resu_k(x,y,t) = (\partial_t^\alpha u_k)_x + \frac{1}{4}(u_k)_{xxxx} + \frac{3}{2}(u_k)_x(u_k)_{xx} + \frac{3}{4}(u_k)_{yy}, \tag{21}$$

To determine the $f_1(x,y)$ coefficient, in $u_1(x,y,t)$, we should replace the first truncated series $u_1(x,y,t) = f(x,y) + f_1(x,y)\frac{t^\alpha}{\alpha}$ into the first truncated residual function as

$$\begin{aligned} Resu_1(x,y,t) = {} & \big(f_1(x,y)\big)_x + \frac{3}{4}\left(\frac{4}{(3+x+y)^3} + \frac{t^\alpha\big(f_1(x,y)\big)_{yy}}{\alpha}\right) \\ & + \frac{3}{2}\left(-\frac{2}{(3+x+y)^2} + \frac{t^\alpha\big(f_1(x,y)\big)_x}{\alpha}\right)\left(\frac{4}{(3+x+y)^3} + \frac{t^\alpha\big(f_1(x,y)\big)_{xx}}{\alpha}\right) \\ & + \frac{1}{4}\left(\frac{48}{(3+x+y)^5} + \frac{t^\alpha\big(f_1(x,y)\big)_{xxxx}}{\alpha}\right). \end{aligned} \tag{22}$$

Substitution of $t = 0$ into the equation $Resu_1(x,y,t)$ gives

$$\big(f_1(x,y)\big)_x = \frac{1}{4}\big(-3f(x,y)\big)_{yy} - 6\big(f(x,y)\big)_x\big(f(x,y)\big)_{xx} - \big(f(x,y)\big)_{xxxx}. \tag{23}$$

Solving this differential equation gives the first unknown parameter as

$$f_1(x,y) = \frac{3}{2(3+x+y)^2}. \tag{24}$$

Thus, the first RPSM approximate solutions of time-fractional K-P equation is calculated as

$$u_1(x,y,t) = 1 + \frac{2}{3+x+y} + \frac{3t^\alpha}{2\alpha(3+x+y)^2}. \tag{25}$$

Similarly, to obtain$f_2(x,y)$ coefficient, we replace $u_2(x,y,t) = f(x,y) + f_1(x,y)\frac{t^\alpha}{\alpha} + f_2(x,y)\frac{t^{2\alpha}}{2\alpha^2}$ into the 2nd residual function and get

$$Resu_2(x,y,t) = \left(f_1\,(x,y)\right)_x + \frac{t^\alpha\left(f_2(x,y)\right)_x}{\alpha} + \frac{3}{4}\left(\frac{4}{(3+x+y)^3} + \frac{t^\alpha\left(f_1(x,y)\right)_{yy}}{\alpha} + \frac{t^{2\alpha}\left(f_2(x,y)\right)_{yy}}{2\alpha^2}\right) + \frac{3}{2}\left(-\frac{2}{(3+x+y)^2} + \frac{t^\alpha\left(f_1(x,y)\right)_x}{\alpha} + \frac{t^{2\alpha}\left(f_2(x,y)\right)_x}{2\alpha^2}\right) \times \left(\frac{4}{(3+x+y)^3} + \frac{t^\alpha\left(f_1(x,y)\right)_{xx}}{\alpha} + \frac{t^{2\alpha}\left(f_2(x,y)\right)_{xx}}{2\alpha^2}\right) + \frac{1}{4}\left(\frac{48}{(3+x+y)^5} + \frac{t^\alpha\left(f_1(x,y)\right)_{xxxx}}{\alpha} + \frac{t^{2\alpha}\left(f_2(x,y)\right)_{xxxx}}{2\alpha^2}\right). \tag{26}$$

Taking T_α conformable derivative of both sides of $Resu_2(x,y,t)$ and evaluating it for $t = 0$ gives

$$\left(f_2(x,y)\right)_x = \frac{1}{4}\left(-3f_1(x,y)\right)_{yy} - 6\left(f_1(x,y)\right)_x\left(f(x,y)\right)_{xx} - 6\left(f(x,y)\right)_x\left(f_1(x,y)\right)_{xx} - \left(f_1(x,y)\right)_{xxxx}. \tag{27}$$

Solving this differential equation gives

$$f_2(x,y) = \frac{9}{4(3+x+y)^3}. \tag{28}$$

So the 2nd RPSM approximate solution of time-fractional K-P equation is:

$$u_2(x,y,t) = 1 + \frac{2}{3+x+y} + \frac{3t^\alpha}{2\alpha(3+x+y)^2} + \frac{9t^{2\alpha}}{8\alpha^2(3+x+y)^3}. \tag{29}$$

Similarly, applying the same scheme for $n = 3$, the following results are obtained.

$$f_3(x,y) = \frac{81}{16(3+x+y)^4}, \tag{30}$$

$$u_3(x,y,t) = 1 + \frac{2}{3+x+y} + \frac{3t^\alpha}{2\alpha(3+x+y)^2} + \frac{9t^{2\alpha}}{8\alpha^2(3+x+y)^3} + \frac{27t^{3\alpha}}{32\alpha^3(3+x+y)^4}. \tag{31}$$

Table 1. Numerical results of the third approximate solutions for $y = 1$ and $t = 0.1$.

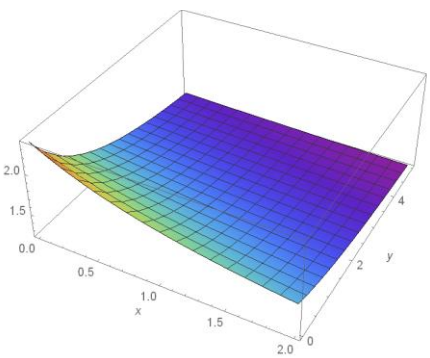
	$\alpha = 0.25$			$\alpha = 0.50$			$\alpha = 0.75$		
x	$RPSM$	$Exact$	$Abs.Error$	$RPSM$	$Exact$	$Abs.Error$	$RPSM$	$Exact$	$Abs.Error$
$\mathbf{0.0}$	1.83733	1.86469	2.7359E-2	1.56716	1.56727	1.1218E-4	1.52326	1.52326	2.0439E-6
$\mathbf{0.1}$	1.59586	1.60369	7.8237E-3	1.44189	1.44192	3.5795E-5	1.41475	1.41475	6.6360E-7
$\mathbf{0.2}$	1.46082	1.46372	2.8982E-3	1.36193	1.36195	1.4138E-5	1.34351	1.34351	2.6505E-7
$\mathbf{0.3}$	1.37517	1.37644	1.2699E-3	1.30648	1.30648	6.4621E-6	1.29316	1.29316	1.2210E-7
$\mathbf{0.4}$	1.31618	1.31681	6.2650E-4	1.26575	1.26576	3.2846E-6	1.25568	1.25568	6.2422E-8
$\mathbf{0.5}$	1.27315	1.27349	3.3763E-4	1.23458	1.23459	1.8100E-6	1.22670	1.22670	3.4552E-8
$\mathbf{0.6}$	1.24039	1.24059	1.9487E-4	1.20996	1.20996	1.0629E-6	1.20362	1.20362	2.0362E-8
$\mathbf{0.7}$	1.21464	1.21475	1.1881E-4	1.19001	1.19001	6.5701E-7	1.18481	1.18481	1.2622E-8
$\mathbf{0.8}$	1.19385	1.19393	7.5754E-5	1.17353	1.17353	4.2364E-7	1.16917	1.16917	8.1584E-9
$\mathbf{0.9}$	1.17674	1.17679	5.0137E-5	1.15967	1.15967	2.8302E-7	1.15598	1.15598	5.4612E-9
$\mathbf{1.0}$	1.16240	1.16243	3.4248E-5	1.14787	1.14787	1.9486E-7	1.14470	1.14470	3.7665E-9

Table 1 represents the approximate RPSM solutions of time-fractional K-P equation of third-order that are compared with the exact solution

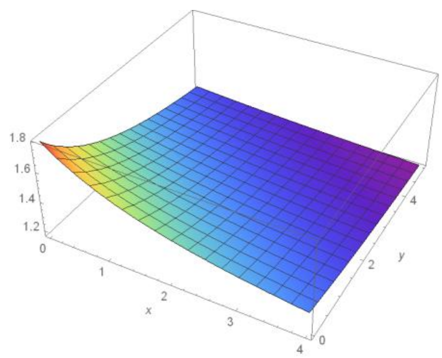
$$u(x, y, t) = 1 + \frac{2}{3 + x + y - \frac{3}{4}\frac{t^{\alpha}}{\alpha}}. \quad (32)$$

For $\alpha = 0.25$, $\alpha = 0.50$ and $\alpha = 0.75$ values, the absolute errors are demonstrated. As seen, the absolute errors decrease while the x values increase. Likewise, the absolute errors decrease while the α values increase. Besides, Table 1 indicates RPSM solutions are in great agreement with the exact solutions. Also, in Figure 1, the 3-dimensional illustrations of the RPSM solutions are presented for $\alpha = 0.25$, $\alpha = 0.50$, $\alpha = 0.75$ and $\alpha = 0.95$.

a.)



b.)



c.) d.)

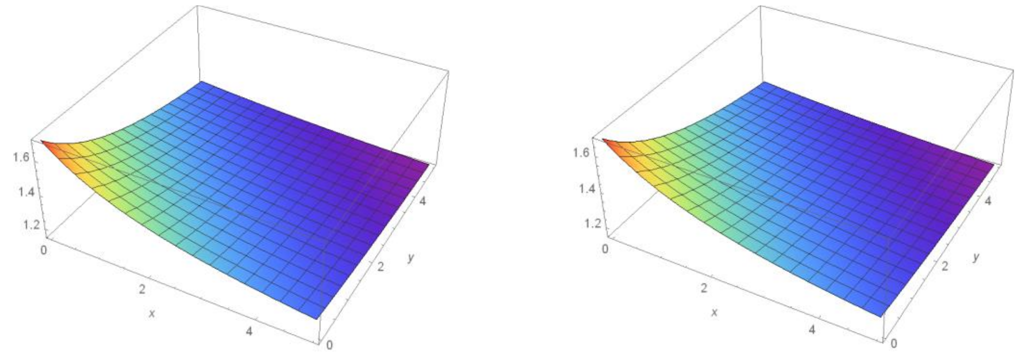


Figure 1. Surface plots of the third approximate solutions for $y = 1$ and $t = 0.1$ and a.) $\alpha = 0.25$, b.) $\alpha = 0.50$, c.) $\alpha = 0.75$ and d.) $\alpha = 0.95$.

5. Results

In this study, the residual power series method (RPSM) has been applied to time-fractional Kadomtsev-Petviashvili equation with conformable derivative. The main advantage of the present method is that the necessity of special assumptions or transformations is eliminated.

In application part, K-P equation is solved by RPSM approximately and some solutions are obtained. In Table 1, the RPSM results are shown with the exact solutions for the values of $\alpha = 0.25$, $\alpha = 0.50$ and $\alpha = 0.75$. It is clearly seen that RPSM gives very near results. Also in Figure 1, the obtained results are illustrated graphically. All these results indicate that RPSM is a very simple, reliable and convenient method.

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