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Complex hyperbolic traveling wave solutions of Kuramoto-Sivashinsky equation using $(1/G')-$ expansion method for nonlinear dynamic theory

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Abstract

In this paper, it is $(1/G')-$expansion method which are used to obtain new complex hyperbolic traveling wave solutions of the non-linear Kuramoto-Sivashinsky equation. Special values are given to the parameters in the solutions obtained and graphs are drawn. These graphs are presented using special package program. This method is employed to achieve the goals set for this study.

Keywords: *Kuramoto-Sivashinsky equation, $(1/G')$-expansion method, complex hyperbolic traveling wave solutions.*

Lineer olmayan dinamik teorisi için $(1/G')$-açılım metodunu kullanarak Kuramoto-Sivashinsky denkleminin karmaşık hiperbolik yürüyen dalga çözümleri

Özet

Bu makalede lineer olmayan Kuramoto–Sivashinsky denkleminin yeni karmaşık hiperbolik yürüyen dalga çözümlerini elde etmek için $(1/G')$metodunu kullanılmıştır. Elde edilen çözümlerdeki parametrelere özel değerler verilmiş ve grafikler çizilmiştir. Bu grafikler özel paket programı kullanılarak sunulmuştur. Bu yöntem, bu çalışma için belirlenen hedeflere ulaşmak için kullanılmıştır.

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Anahtar Kelimeler: *Kuramoto-Sivashinsky denklemi, $(1/G')$-açılım metodu, karmaşık hiperbolik yürüyen dalga çözümleri.*

1. Introduction

Nonlinear partial differential equations have been studied in many areas in recent years, such as in physics, engineering, fluid dynamics, plasma physics and chemistry, the search of exact solutions of PDEs which appeared. The exact solution have been proposed for solving PDEs. For this aim many variety analytical methods are used such as (G'/G)-expansion method, jacobi elliptic function expansion method, homotopy analysis method, extended trial equation method, auxiliary equation method, the functional variable method, generalized auxiliary equation method, the first integral method [1-8], $(1/G')$-expansion method [9-11], inverse Laplace homotopy technique [12], solutions of partial differential equations [13-16], the homotopy perturbation method (HPM) [17], multistage Adomian decomposition method (MADM)[18] and so on.

The Kuramoto-Sivashinsky (K-S) equation appears in many work such as Weiss-Tabor-Carnevale method [19], extensive numerical simulation [20], several alternative methods for the approximation of inertial manifolds [21], “viscous shocks” and periodic solutions [22], the Painleve test [23], a new topological method [24], a generalized tanh function method [25], the normal form analysis of nonlinear dynamic theory [26], detailed crossover analysis [27], the homotopy analysis method (HAM) [28], galerkin method [29] and so on.

In this study, our aim is to obtain the traveling wave solution of the Kuramoto-Sivashinsky equation by using $(1/G')$-expansion method. The K-S equation can be shown in the form of

$$u_t - \alpha u^2 u_x - \gamma u_{xx} - \beta u^3 u_x + u_{xxxx} = 0, \tag{1.1}$$

where α, γ, β arbitrary constants.

The KS equation is derived from many physical events. These physical phenomena are mainly: plasma instabilities in the diffusion system in the reaction, heat dissipation, phase turbulence, etc. [30].

2. $(1/G')$-expansion method

Firstly, in order to explain this method, think two-variable general form of NPDEs

$$F\left(u, \frac{\partial^\alpha u}{\partial t^\alpha}, \frac{\partial u}{\partial x}, \frac{\partial^2 u}{\partial x^2}, \ldots\right) = 0, \tag{2.1}$$

in the general form. Here, let $u=u(x,t)=u(\xi)$, $\xi=x+vt$, $v\neq 0$, and where v is a constant. Then, we can be converted into following nonlinear ODE for $u(\xi)$:

$$\Omega(u',u'',u''',...)=0. \tag{2.2}$$

The solution of (2. 2) Eq. is assumed to have the form

$$u(\xi)=a_0+\sum_{i=1}^{m}a_i\left(\frac{1}{G'}\right)^i, \tag{2.3}$$

where a_i are constants, m is a pozitif integer which is balancing term. This term is found in the Eq. (2.2) by equalizing the linear terms with the largest order and the non-linear terms having the largest order. $G=G(\xi)$ is also considered as the solution of the following differential equation

$$G''+\lambda G'+\mu=0, \tag{2.4}$$

where λ and μ represent the fixed number. If the desired derivatives of the Eq. (2.3) are calculated and replaced in the equation (2.2), a polynomial with the argument $(1/G')$ is obtained. An algebraic equation system is created by equalizing the coefficients of this polynomial to zero. This equation system is solved with the help of the computer package program and put into place in the default (2.2) solution function. As a result, the solution of the Eq. (1.1) is found.

3. Solution of Kuramoto-Sivashinsky equation

We consider Kuramoto-Sivashinsky Eq. (1.1). Therefore, using transformation $u=u(x,t)=u(\xi)$, $\xi=x+vt, v\neq 0$, of Eq. (1.1) we obtain

$$vu'-\alpha u^2u'-\gamma u''-\beta u^3u'+u^{(4)}=0, \tag{3.1}$$

where v represents the speed of the wave. Considering the Eq. (3.1), balancing $u^{(4)}$ with u^3u' gives m = 1, and we obtain the following form of the solution

$$u=u(\xi)=a_0+a_1\left(\frac{1}{G'}\right). \tag{3.2}$$

If we replace the Eq. (3.2) in the Eq. (3.1) and the coefficients of the algebraic equation are equal to zero, we can construct the following algebraic equation system.

$$\frac{1}{G'[\xi]}:\quad v\lambda a_1-\gamma\lambda^2a_1+\lambda^4a_1-\alpha\lambda a_0^2a_1-\beta\lambda a_0^3a_1=0,$$

$$\frac{1}{G'[\xi]^2}:\quad v\mu a_1-3\gamma\lambda\mu a_1+15\lambda^3\mu a_1-\alpha\mu a_0^2a_1-\beta\mu a_0^3a_1-2\alpha\lambda a_0a_1^2-3\beta\lambda a_0^2a_1^2=0,$$

$$\frac{1}{G'[\xi]^3}: \quad -2\gamma\mu^2 a_1 + 50\lambda^2\mu^2 a_1 - 2\alpha\mu a_0 a_1^2 - 3\beta\mu a_0^2 a_1^2 - \alpha\lambda a_1^3 - 3\beta\lambda a_0 a_1^3 = 0, \tag{3.3}$$

$$\frac{1}{G'[\xi]^4}: \quad 60\lambda\mu^3 a_1 - \alpha\mu a_1^3 - 3\beta\mu a_0 a_1^3 - \beta\lambda a_1^4 = 0,$$

$$\frac{1}{G'[\xi]^5}: \quad 24\mu^4 a_1 - \beta\mu a_1^4 = 0,$$

where a_0, a_1, α, γ, β and v are constants, aim with computer package program, finding the solutions of system (3.3) and we obtained the following stations.

Case 1. If

$$v = \frac{2\left(\gamma\sqrt{\left(\gamma+2\lambda^2\right)\mu^4} + 2\lambda^2\sqrt{\left(\gamma+2\lambda^2\right)\mu^4}\right)}{9\mu^2},$$

$$a_0 = \frac{-\frac{3\sqrt{3}\lambda\mu^2\left(\left(\gamma+2\lambda^2\right)\mu^4\right)^{1/4}}{\sqrt{\alpha}} + \frac{\sqrt{3}\left(\left(\gamma+2\lambda^2\right)\mu^4\right)^{3/4}}{\sqrt{\alpha}}}{3\mu^3}, \tag{3.4}$$

$$a_1 = -\frac{2\sqrt{3}\left(\gamma\mu^4 + 2\lambda^2\mu^4\right)^{1/4}}{\sqrt{\alpha}}, \quad \beta = \frac{\alpha^{3/2}\left(\left(\gamma+2\lambda^2\right)\mu^4\right)^{1/4}}{\sqrt{3}\left(-\gamma\mu - 2\lambda^2\mu\right)},$$

substituting values (3.4) into (3.2) and we have the following wave solutions for Eq. (1.1):

$$\xi = x + \left(\frac{2\left(\gamma\sqrt{\left(\gamma+2\lambda^2\right)\mu^4} + 2\lambda^2\sqrt{\left(\gamma+2\lambda^2\right)\mu^4}\right)}{9\mu^2}\right)t,$$

$$u_1(x,t) = \frac{-\frac{3\sqrt{3}\lambda\mu^2\left(\left(\gamma+2\lambda^2\right)\mu^4\right)^{1/4}}{\sqrt{\alpha}} + \frac{\sqrt{3}\left(\left(\gamma+2\lambda^2\right)\mu^4\right)^{3/4}}{\sqrt{\alpha}}}{3\mu^3} - \frac{2\sqrt{3}\left(\gamma\mu^4 + 2\lambda^2\mu^4\right)^{1/4}}{\sqrt{\alpha}\left(-\frac{\mu}{\lambda} + A\cosh[\lambda\xi] - A\sinh[\lambda\xi]\right)}, \tag{3.5}$$

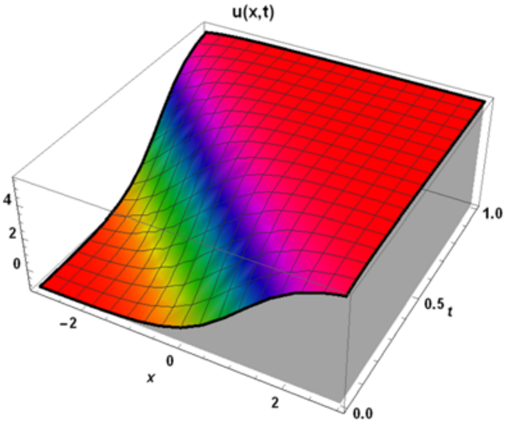

Figure 1. The solution representing the stationary *hyperbolic traveling wave solution* in the $u_1(x,t)$ obtained of the Eq. (1.1) for $\lambda=1.6,\ \mu=1,\ \gamma=4,\ \alpha=2,\ A=-4.$

Case 2. If

$$v=\frac{2\left(-\gamma\sqrt{\left(\gamma+2\lambda^2\right)\mu^4}-2\lambda^2\sqrt{\left(\gamma+2\lambda^2\right)\mu^4}\right)}{9\mu^2},$$

$$a_0=-\frac{\mathrm{i}\left(\dfrac{3\sqrt{3}\lambda\mu^2\left(\left(\gamma+2\lambda^2\right)\mu^4\right)^{1/4}}{\sqrt{\alpha}}+\dfrac{\sqrt{3}\left(\left(\gamma+2\lambda^2\right)\mu^4\right)^{3/4}}{\sqrt{\alpha}}\right)}{3\mu^3}, \tag{3.6}$$

$$a_1=-\frac{2\mathrm{i}\sqrt{3}\left(\gamma\mu^4+2\lambda^2\mu^4\right)^{1/4}}{\sqrt{\alpha}},\quad \beta=-\frac{\mathrm{i}\backslash\alpha^{3/2}\backslash\left(\left(\gamma+2\backslash\lambda^2\right)\backslash\mu^4\right)^{1/4}}{\sqrt{3}\backslash\left(\gamma\backslash\mu+2\backslash\lambda^2\backslash\mu\right)},$$

substitute (3.6) into (3.2), obtain two types of traveling wave solutions of Eq. (1.1):

$$\xi=x+\left(\frac{2\left(-\gamma\sqrt{\left(\gamma+2\lambda^2\right)\mu^4}-2\lambda^2\sqrt{\left(\gamma+2\lambda^2\right)\mu^4}\right)}{9\mu^2}\right)t,$$

$$u_2(x,t)=-\frac{\mathrm{i}\left(\dfrac{3\sqrt{3}\lambda\mu^2\left(\left(\gamma+2\lambda^2\right)\mu^4\right)^{1/4}}{\sqrt{\alpha}}+\dfrac{\sqrt{3}\left(\left(\gamma+2\lambda^2\right)\mu^4\right)^{3/4}}{\sqrt{\alpha}}\right)}{3\mu^3} -\frac{2\mathrm{i}\sqrt{3}\left(\gamma\mu^4+2\lambda^2\mu^4\right)^{1/4}}{\sqrt{\alpha}\left(-\dfrac{\mu}{\lambda}+A\cosh\left[\lambda\xi\right]-A\sinh\left[\lambda\xi\right]\right)}, \tag{3.7}$$

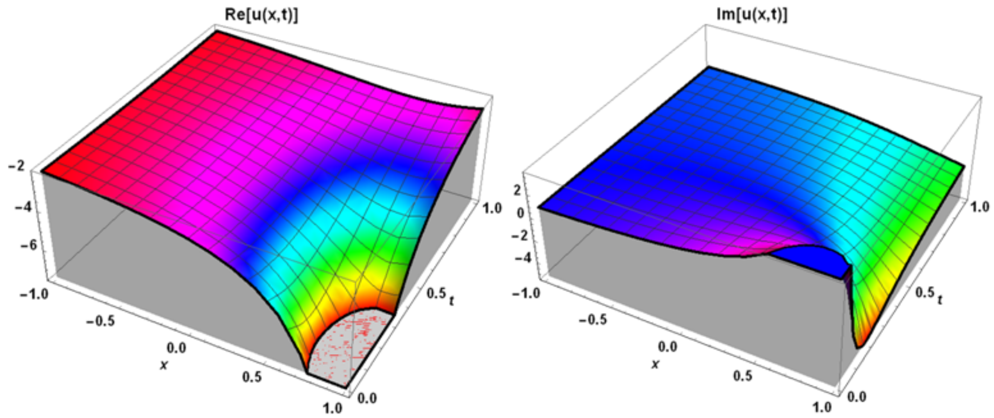

Figure 2. The solution representing the stationary *complex hyperbolic traveling wave solution* in the $u_2(x,t)$ obtained of the Eq. (1.1) for $\lambda=1, \mu=-1, \gamma=-5, \alpha=2, A=-3.$

Case 3. If

$$v=\frac{2\left(-\gamma\sqrt{\left(\gamma+2\lambda^2\right)\mu^4}-2\lambda^2\sqrt{\left(\gamma+2\lambda^2\right)\mu^4}\right)}{9\mu^2},$$

$$a_0=\frac{\mathrm{i}\left(\frac{3\sqrt{3}\lambda\mu^2\left(\left(\gamma+2\lambda^2\right)\mu^4\right)^{1/4}}{\sqrt{\alpha}}+\frac{\sqrt{3}\left(\left(\gamma+2\lambda^2\right)\mu^4\right)^{3/4}}{\sqrt{\alpha}}\right)}{3\mu^3}, \tag{3.8}$$

$$a_1=\frac{2\mathrm{i}\sqrt{3}\left(\gamma\mu^4+2\lambda^2\mu^4\right)^{1/4}}{\sqrt{\alpha}},\quad \beta=\frac{\mathrm{i}\alpha^{3/2}\left(\left(\gamma+2\lambda^2\right)\mu^4\right)^{1/4}}{\sqrt{3}\left(\gamma\mu+2\lambda^2\mu\right)}$$

substituting values found (3.8) into (3.2), obtain two types of wave solutions for Eq. (1.1):

$$\xi=x+\left(\frac{2\left(-\gamma\sqrt{\left(\gamma+2\lambda^2\right)\mu^4}-2\lambda^2\sqrt{\left(\gamma+2\lambda^2\right)\mu^4}\right)}{9\mu^2}\right)t,$$

(3.9)

$$u_3(x,t)=\frac{\mathrm{i}\left(\frac{3\sqrt{3}\lambda\mu^2\left(\left(\gamma+2\lambda^2\right)\mu^4\right)^{1/4}}{\sqrt{\alpha}}+\frac{\sqrt{3}\left(\left(\gamma+2\lambda^2\right)\mu^4\right)^{3/4}}{\sqrt{\alpha}}\right)}{3\mu^3}+\frac{2\mathrm{i}\sqrt{3}\left(\gamma\mu^4+2\lambda^2\mu^4\right)^{1/4}}{\sqrt{\alpha}\left(-\frac{\mu}{\lambda}+A\cosh\left[\lambda\xi\right]-A\sinh\left[\lambda\xi\right]\right)},$$

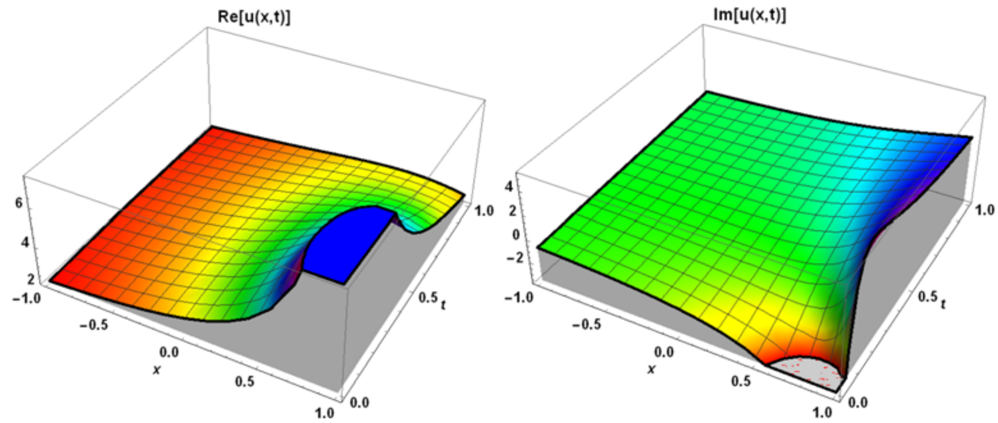

Figure 3. The solution representing the stationary *complex hyperbolic traveling wave solution* in the $u_3(x,t)$ obtained of the Eq. (1.1) for $\lambda=1,\ \mu=-1,\ \gamma=-5,\ \alpha=2,\ A=-3.$

Case 4. If

$$v=\frac{2\left(\gamma\sqrt{\left(\gamma+2\lambda^2\right)\mu^4}+2\lambda^2\sqrt{\left(\gamma+2\lambda^2\right)\mu^4}\right)}{9\mu^2},$$

$$a_0=\frac{\dfrac{3\sqrt{3}\lambda\mu^2\left(\left(\gamma+2\lambda^2\right)\mu^4\right)^{1/4}}{\sqrt{\alpha}}-\dfrac{\sqrt{3}\left(\left(\gamma+2\lambda^2\right)\mu^4\right)^{3/4}}{\sqrt{\alpha}}}{3\mu^3}, \tag{3.10}$$

$$a_1=\frac{2\sqrt{3}\left(\gamma\mu^4+2\lambda^2\mu^4\right)^{1/4}}{\sqrt{\alpha}},\quad \beta=\frac{\alpha^{3/2}\left(\left(\gamma+2\lambda^2\right)\mu^4\right)^{1/4}}{\sqrt{3}\left(\gamma\mu+2\lambda^2\mu\right)},$$

substituting values found (3.10) into (3.2), obtain two types of wave solutions for Eq. (1.1):

$$\xi=x+\left(\frac{2\left(\gamma\sqrt{\left(\gamma+2\lambda^2\right)\mu^4}+2\lambda^2\sqrt{\left(\gamma+2\lambda^2\right)\mu^4}\right)}{9\mu^2}\right)t,$$

$$u_4(x,t)=\frac{\dfrac{3\sqrt{3}\lambda\mu^2\left(\left(\gamma+2\lambda^2\right)\mu^4\right)^{1/4}}{\sqrt{\alpha}}-\dfrac{\sqrt{3}\left(\left(\gamma+2\lambda^2\right)\mu^4\right)^{3/4}}{\sqrt{\alpha}}}{3\mu^3}+\frac{2\sqrt{3}\left(\gamma\mu^4+2\lambda^2\mu^4\right)^{1/4}}{\sqrt{\alpha}\left(-\dfrac{\mu}{\lambda}+A\cosh[\lambda\xi]-A\sinh[\lambda\xi]\right)}, \tag{3.11}$$

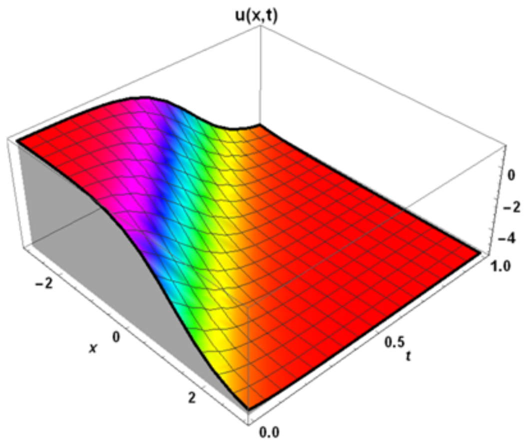

Figure 4. The solution representing the stationary *hyperbolic traveling wave solution* in the $u_4(x,t)$ obtained of the Eq. (1.1) for $\lambda = 1.6, \mu = 1, \gamma = 4, \alpha = 2, A = -4.$

4. Conclusion

In this study, $(1/G')$-expansion method is successfully employed to establish a series of new exact travelling wave solutions for K-S equation. The shock wave structure is obtained by the (1 /G')- expansion method, in particular the wave solutions exhibiting asymptotic behavior are obtained. The process is less complicated than other methods. As a result, new series of exact traveling wave solutions are obtained. These solutions include $(1/G')$-expansion method, which may be useful to further understand the mechanisms of PDEs. Also, Mathematica has been used for computations and programming in this paper. The results obtained here show that the method is effective and reliable, can be further used applied to other PDE's.

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