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Permutation Test Approach for Ordered Alternatives in Randomized Complete Block Design: A Comparative Study

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ABSTRACT

Randomized complete block design is one of the most used experimental designs in statistical analysis. For testing ordered alternatives in randomized complete block design, parametric tests are used if random sample are drawn from Normal distribution. If normality assumption is not provide, nonparametric methods are used. In this study, we are interested nonparametric tests and we introduce briefly the nonparametric tests, such as Page, Modified Page and Hollander tests. We also give Permutation Version of Page test. We compare the performance of these tests in terms of the empirical type I error rates and powers of tests.

Key words: Permutation test, Page test, Hollander test, Modified Page test, Simulation study

1. INTRODUCTION

Randomized complete block design (RCBD) is one of the most widely used experimental designs in statistical analysis. Also it has a wide range of application in agriculture, veterinary medicine, medical research and etc. This design increases accuracy of between treatments by reducing variation between blocks. Each treatment is randomly assigned to the experimental units within each block. If the experimental units within blocks are similar, RCBD tests the differences between the means of treatments better than other designs.

RCBD is used as similar to one-way ANOVA when the number of factors is one. Furthermore, unlike one-way ANOVA, there are some systematic differences between the experimental units. The effect of these systematic differences can be solved by using blocks that are homogeneous in themselves and heterogeneous among themselves. Blocking enables to increase sensitivity of experiment by reducing the experimental error. Mathematical model for the completed randomized block design is;

$$X_{ij} = \mu + \tau_i + \beta_j + \varepsilon_{ij} \quad , \quad i = 1,...,t \quad , \quad j = 1,...,b$$

where X_{ij} 's are observation value of the *t*th treatment on block j, μ is an overall effect, τ_i are treatment effects, β_j are block effects and ε_{ij} are independent and identically distributed random variables with mean zero.

To test the differences between the effects of the treatments, null and alternative hypothesis are as seen below:

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$$H_0 : \tau_1 = \tau_2 = \ldots = \tau_t = 0$$

$$H_1 : \tau_i \neq 0, \quad \text{for at least one } i.$$

In this alternative hypothesis, it is assumed that treatments are not equal to each other or at least one of the other in alternative hypothesis. However, sometimes we have prior knowledge of this treatments that increase or diminish. For example, dose-response studies are frequently used in animal experiments or clinical trials. Animals are assigned to k groups corresponding to k dosages of an experimental drug. So the drug effect on animals is likely to increase (or decrease) with increasing doses. In this case, an ordered alternative is considered.

In this study we examine ordered alternatives that more specific alternative. Therefore null and alternative hypothesis of ordered alternatives are

$$H_0 : \tau_1 = \tau_2 = \ldots = \tau_t = 0$$

$$H_1 : \tau_1 \leq \tau_2 \leq \ldots \leq \tau_t \quad \text{or}$$

$$H_1 : \tau_1 \geq \tau_2 \geq \ldots \geq \tau_t \qquad (1)$$

where at least one of the inequalities is strict.

In order to use parametric tests, random sample should be drawn from Normal distribution or large enough sample size. Non-parametric methods are used when normality assumptions are not provided. Testing the null hypothesis against ordered alternatives was made firstly by Jonckheere and Terpstra [1,2]. Their test statistic based on the Mann-Whitney test was used in many studies. One of the most important test for ordered alternatives in RCBD was developed by Page [3]. Hollander [4] introduced a test statistic based on Wilcoxon signed-rank test for ordered alternatives. Skillings and Wolfe [5] suggested a test statistic by generalizing the Jonckheere statistic for ordered alternatives. Alvo and Cabilio [6] proposed a test statistic for ordered alternatives in incomplete block design. Best and Rayner [7] developed a test statistic by using Lancaster partition for ordered alternatives in incomplete block design. Also Best and Rayner [8] introduced a test statistic modifying Page test by the help of orthogonal contrasts in incomplete block design. In recent years, computer intensive methods including Permutation, Bootstrap and Monte Carlo are used in many nonparametric testing procedure [9-11]. In this study, we develop test based on Permutation Version of Page test statistic.

This article is organized as follows. The Page, Modified Page, Hollander and Permutation Version of Page test statistics are introduced briefly. After this section, we compare the performance of these tests with simulation study in terms of the type I error rates and powers of tests. Eventually, we give summary our founding.

2. MATERIAL AND METHODS

In this section, we examine Page, Modified Page, Hollander and Permutation version of Page test statistics given as follows.

2.1. Page Test

Page [3] developed a statistic for ordered alternative for complete block design. Test statistic is given by

$$L = \sum_{i=1}^{t} iR_i \qquad (2)$$

Page test statistic can be applied using the following steps;

1. $R_i = \sum_{j=1}^{b} r_{ij}$ is calculated. Let r_{ij} denotes the rank of X_{ij} (For example R_2 is sum of rank second treatment).
2. Compute $L = \sum_{i=1}^{t} iR_i = 1R_1 + 2R_2 + \ldots + tR_t$.

Here, t is a number of treatments, R_i is the sum of rank values that assigned treatments.

3. At the α level of significance, H_0 is rejected if $L \geq l(\alpha, t, b)$.

The value of $l(\alpha, t, b)$ can be found in the Hollander and Wolfe [12]'s paper, where b is the number of blocks. The Page statistic approximates the standardized normal distribution as sample size increases Thus we need to obtain the expected value and variance of the Page statistic. Expected value and variance of the Page statistic are as follows:

$E(L) = bt(t+1)^2 / 4$ and $V(L) = bt^2(t+1)(t^2-1)/144$.

The standardized Page test is as shown below:

$$Z_L = \frac{L - E(L)}{\sqrt{V(L)}}.$$

If $Z_L > z_\alpha$ or P<α, is rejected.

2.2. Hollander Test

The Hollander test based on Wilcoxon signed-rank test statistic [4] is used for testing null hypothesis against ordered alternatives in RCBD. For each pair of (u, v) treatment and each of ($1 < u < v < t$), it is defined that T_{uv} signed-rank statistic is given as follows:

$$T_{uv} = \sum_{i=1}^{b} R_{uv}^{i} \psi_{uv}^{i} \ ,$$

where

$$\psi_{uv}^{i} = \begin{Bmatrix} 1 & X_{iu} < X_{iv} \\ 1/2 & X_{iu} = X_{iv} \\ 0 & X_{iu} > X_{iv} \end{Bmatrix}$$

H statistic depended on the statistic of T_{uv} is as shown below:

$$H = \sum_{u=1}^{t-1} \sum_{v=u+1}^{t} T_{uv} \ . \quad (3)$$

The expected value and variance of the statistic H are given as follows:

$$E(H) = \frac{bt(t-1)(b+1)}{8} \quad (4)$$

$$V(H) = \frac{bt(b+1)(2b+1)(t-1)\left\{3 + 2(t-2)p_{U}^{b}\right\}}{144} \quad (5)$$

where P_{u}^{b}, depend on b, value of the correlation between Wilcoxon signed-ranked test statistic. Using Eq. (4) and Eq.(5), Z_H statistic is calculated as follows:

$$Z_H = \frac{H - E(H)}{\sqrt{V(H)}} .$$

If $Z_H > Z_\alpha$ or P<α, H_0 is rejected.

2.3. Modified Page Statistic

Best and Rayner [8] defined the Modified Page (MP) test for ordered alternatives. Test statistic is given by

$$MP = \frac{CL^2}{\sum_{i=1}^{t} \lambda_j^2} \ . \quad (6)$$

This test statistic is obtained by using orthogonal trend contrasts. Here $L = \sum_{j=1}^{t} \lambda_j \overline{R}$ is orthogonal trend contrast, λ_j is linear coefficient, $\overline{R}_j = \frac{R_j}{b}$, b is number of block and R_j is the sum of rank values that assigned treatments. The C and V terms are as follows:

$$C = \begin{cases} \dfrac{b(t-1)}{tV} & \text{, ties} \\ \dfrac{12b}{t(t+1)} & \text{,no ties} \end{cases}$$

$$V = \begin{cases} \left\{ \sum_{s=1}^{q} r_s^2 c_s / (bt) \right\} - (t+1)^2/4 \ , & \text{ties} \\ \dfrac{(t^2-1)}{12} \ , & \text{no ties} \end{cases}$$

where r_s and c_s denote the sth ranking and its associated count, respectively. If $MP > \chi_1^2$ or P<α, H_0 is rejected.

2.4. Permutation Version of Page Test

Permutation test has some advantages according to parametric and nonparametric tests. One of the most important advantages of Permutation test does not require any assumption about sample drawn from population distribution such parametric tests. Another important advantage of Permutation test is that real values are used instead of rank numbers of data [9]. Normality approximation which is used for Page test is not adequate for small sample sizes. For this reason, we investigate the permutation approach for Page test.

The algorithm for calculating p-value using the Permutation approach could be given as shown below:

1. Compute Page test statistic in Eq. (2) for the original data.

2. Choose permutation resample from the data without replacement in a way that is consistent with the null hypothesis of the test and with the study design. By the same way, generate artificial sample a large number of times (say N times).

3. For each of these replicated samples, recalculated Page test statistic in Eq. (2).

4. Let these recalculated Page test statistic values be $L_1^*, \ldots, L_N^*$. So the permutation distribution of the test statistic is obtained.

5. Calculate the *p-value* as:

$P - value = \dfrac{\#(L_i^* > L)}{N}$, *i=1,…,N*. Reject the null hypothesis of no treatment effects if P<α and accept null hypothesis otherwise.

3. SIMULATION

The Page test (PT), Hollander test (HT), Modified Page test (MPT) and Permutation Version of Page test (PPT) are compared in terms of the empirical type I error and power values.

In simulation study, block numbers are taken 5, 10, 15 and treatment numbers are taken 3, 4 and 5. Random numbers are generated from binomial distribution. Test statistics given in Section 2 are computed for these generated random numbers. This procedure is repeated 5000 times.

To obtain type I error rates of tests, we generated the random numbers from binomial distribution with (n, p)=(5, 0.5) parameters. Also, to obtain power values of tests, we generated the random numbers from binomial distribution with different parameters given in Table 2-4. In addition, we used N=5000 for calculation of the p-value given in the Permutation Version of Page test. MATLAB R2009A program is used for simulation study. The empirical type I error rates and powers of tests are given Table 1 and Table 2-4, respectively.

Table 1. The Empirical type I errors rates of tests for $\alpha = 0.05$.

		Test Statistics			
t	***b***	**P**	**MP**	**H**	**PP**
	5	0.0308	0.0458	0.0400	0.0650
3	10	0.0362	0.0560	0.0454	0.0624
	15	0.0274	0.0494	0.0446	0.0556
	5	0.0322	0.0450	0.0428	0.0520
4	10	0.0308	0.0496	0.0460	0.0548
	15	0.0394	0.0538	0.0498	0.0546
	5	0.0370	0.0418	0.0414	0.0514
5	10	0.0320	0.0468	0.0438	0.0498
	15	0.0388	0.0480	0.0464	0.0550

Table 2. The power values of tests for t=3.

		p		Test Statistics P	MP	H	PP
b=5	0.1	0.15	0.2	0.1184	0.1218	0.1682	0.2636
	0.1	0.2	0.3	0.3532	0.3280	0.4374	0.5308
	0.45	0.5	0.55	0.1052	0.0846	0.1294	0.1764
	0.4	0.5	0.6	0.2744	0.2262	0.3262	0.3914
	0.7	0.75	0.8	0.1212	0.1106	0.1568	0.2142
	0.7	0.8	0.9	0.3622	0.3346	0.4420	0.5376
b=10	0.1	0.15	0.2	0.2550	0.2526	0.3454	0.4008
	0.1	0.2	0.3	0.6632	0.6292	0.7576	0.7748
	0.45	0.5	0.55	0.1904	0.1530	0.2328	0.2582
	0.4	0.5	0.6	0.4000	0.4500	0.5000	0.5838
	0.7	0.75	0.8	0.2068	0.1760	0.2624	0.3000
	0.7	0.8	0.9	0.6674	0.6344	0.7568	0.7746
b=15	0.1	0.15	0.2	0.3164	0.3470	0.4590	0.4998
	0.1	0.2	0.3	0.8104	0.8048	0.8958	0.8980
	0.45	0.5	0.55	0.2152	0.1894	0.3066	0.3168
	0.4	0.5	0.6	0.6570	0.6236	0.7728	0.7614
	0.7	0.75	0.8	0.2558	0.2480	0.3586	0.3818
	0.7	0.8	0.9	0.8080	0.8084	0.9000	0.8972

Table 3. The power values of tests for t=4.

			p		Test Statistics P	MP	H	PP
b=5	0.1	0.15	0.2	0.25	0.2936	0.2718	0.3752	0.4204
	0.1	0.2	0.3	0.4	0.7094	0.6532	0.7774	0.8026
	0.4	0.45	0.5	0.55	0.2040	0.1614	0.2526	0.2772
	0.4	0.5	0.6	0.7	0.5792	0.5126	0.6502	0.6688
	0.6	0.7	0.8	0.9	0.7272	0.6694	0.7890	0.8140
	0.75	0.8	0.85	0.9	0.2786	0.2564	0.3538	0.4078
b=10	0.1	0.15	0.2	0.25	0.5034	0.4922	0.6158	0.6422
	0.1	0.2	0.3	0.4	0.9494	0.9352	0.9748	0.9748
	0.4	0.45	0.5	0.55	0.3504	0.3010	0.4286	0.4422
	0.4	0.5	0.6	0.7	0.8642	0.8298	0.9170	0.9120
	0.6	0.7	0.8	0.9	0.9484	0.9352	0.9730	0.9706
	0.75	0.8	0.85	0.9	0.5218	0.5028	0.6344	0.6606
b=15	0.1	0.15	0.2	0.25	0.7062	0.6758	0.7844	0.7938
	0.1	0.2	0.3	0.4	0.9936	0.9904	0.9970	0.9966
	0.4	0.45	0.5	0.55	0.5056	0.4296	0.5788	0.5730
	0.4	0.5	0.6	0.7	0.9744	0.9614	0.9880	0.9816
	0.6	0.7	0.8	0.9	0.9948	0.9914	0.9982	0.9970
	0.75	0.8	0.85	0.9	0.6960	0.6586	0.7870	0.7936

Table 4. The power values of tests for *t*=5.

	p					Test Statistics			
						P	MP	H	PP
b=5	0.1	0.15	0.2	0.25	0.3	0.5066	0.4496	0.5512	0.5982
	0.1	0.2	0.3	0.4	0.5	0.9348	0.8980	0.9496	0.9536
	0.4	0.45	0.5	0.55	0.6	0.3802	0.3014	0.4140	0.4444
	0.3	0.4	0.5	0.6	0.7	0.8962	0.8394	0.9160	0.9210
	0.7	0.75	0.8	0.85	0.9	0.5026	0.4392	0.5540	0.5922
	0.5	0.6	0.7	0.80	0.9	0.9414	0.9100	0.9560	0.9610
b=10	0.1	0.15	0.2	0.25	0.3	0.7936	0.7642	0.8494	0.8580
	0.1	0.2	0.3	0.4	0.5	0.9994	0.9988	0.9996	0.9996
	0.4	0.45	0.5	0.55	0.6	0.6204	0.5486	0.6810	0.6824
	0.3	0.4	0.5	0.6	0.7	0.9930	0.9906	0.9964	0.9956
	0.7	0.75	0.8	0.85	0.9	0.7920	0.7650	0.8500	0.8586
	0.5	0.6	0.7	0.80	0.9	0.9990	0.9980	0.9996	0.9996
b=15	0.1	0.15	0.2	0.25	0.3	0.9252	0.9046	0.9582	0.9536
	0.1	0.2	0.3	0.4	0.5	1.0000	1.0000	1.0000	1.0000
	0.4	0.45	0.5	0.55	0.6	0.7948	0.7332	0.8426	0.8342
	0.3	0.4	0.5	0.6	0.7	0.9994	0.9994	0.9998	1.0000
	0.7	0.75	0.8	0.85	0.9	0.9222	0.9056	0.9534	0.9502
	0.5	0.6	0.7	0.80	0.9	1.0000	1.0000	1.0000	1.0000

In Table 1, one can see that the Page test seems to have a type I error rate lower than the nominal level. The empirical type I error rate of Modified Page, Hollander and Permutation Version of Page tests are close to the nominal level.

We observe the following from the numerical results in Table 2-4. The powers of Page, Hollander and Modified Page tests are not high enough, especially in small block and treatment numbers. The Permutation Version of Page test is superior to other tests, especially when numbers of block and treatments are small. Also, the Hollander test appears to be more powerful than the Page and Modified Page tests in this situation. When the block number increases, it is seen that the power of all tests are getting higher. In large block and treatment numbers, the Permutation Version of Page and Hollander tests have close power values and appear to be more powerful than the other tests.

4. CONCLUSION

We studied some nonparametric tests for ordered alternatives under randomized complete block design. Also, Permutation version for Page test was given. We compared it with the Page, Modified Page and Hollander tests in terms of the type I error rate and power of the tests.

Consequently, it can be said that the Permutation version of Page test appears to be more powerful than the other tests, especially small treatment and block numbers. When numbers of treatment and block increases, the Permutation Version of Page and Hollander tests have almost same values of power. So, one of these two tests is preferable in large treatment and block numbers.

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