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# Stability of an Iterative Algorithm

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**Abstract:** We prove that iterative algorithm (1.7) of [7] is weak $w^2-$stable w.r.t. an operator $T$ in the class of weak contraction mappings.

**Keywords:** Fixed point, Iterative algorithm, Stability.

## 1 Introduction

Let $C$ be a nonempty closed convex subset of a Banach space $X$ and $T : C \to C$ a mapping. An element $x$ in $C$ is said to be a fixed point of $T$ if $Tx = x$.

**Definition 1.** *([1]) Let $(M, d)$ be a metric space. A mapping $T : M \to M$ is said to be weak-contraction if there exist $\delta \in [0, 1)$ and $L \geq 0$ such that*

$$d\left(Tx, Ty\right) \leq \delta d\left(x, y\right) + Ld\left(y, Tx\right), \textit{for all } x, y \in M.$$

**Theorem 1.** *([1]) Let $(M, d)$ be a complete metric space and $T : M \to M$ a weak-contraction for which there exist $\delta \in [0, 1)$ and $L_1 \geq 0$ such that*

$$d\left(Tx, Ty\right) \leq \delta d\left(x, y\right) + L_1 d\left(x, Tx\right), \textit{for all } x, y \in M. \tag{1}$$

*Then, $T$ has a unique fixed point.*

**Definition 2.** *([2]) Let $\{x_n\}_{n=1}^{\infty}$ and $\{y_n\}_{n=1}^{\infty}$ be sequences in $C$. We say that these sequences are equivalent if $\lim_{n\to\infty} \|x_n - y_n\| = 0$.*

**Definition 3.** *([3]) Let $\{x_n\}_{n=1}^{\infty}$ be an iterative sequence produced by operator $T$*

$$\begin{cases} x_1 \in C, \\ x_{n+1} = f\left(T, x_n\right), n \in \mathbb{N}, \end{cases}$$

*where $f$ is a function. Assume that $\{x_n\}_{n=1}^{\infty}$ converges to a $p^* = Tp^*$. If for any equivalent sequence $\{y_n\}_{n=1}^{\infty} \subseteq C$ of $\{x_n\}_{n=1}^{\infty}$,*

$$\lim_{n\to\infty} \|y_{n+1} - f\left(T, y_n\right)\| = 0 \Rightarrow \lim_{n\to\infty} y_n = p^*,$$

*then the iterative sequence $\{x_n\}_{n=1}^{\infty}$ is said to be weak $w^2-$stable w.r.t. $T$.*

Over the past few decades, many research papers are devoted to the study of stability of various well-known iterative algorithms for different classes of operators, see, e.g. [3–6] and references therein.

Recently, Karakaya et al. [7] introduced a three-step iterative algorithm as follows:

$$\begin{cases} x_1 \in C, \\ x_{n+1} = Ty_n, \\ y_n = \left(1 - \alpha_n\right) z_n + \alpha_n T z_n, \\ z_n = Tx_n, n \in \mathbb{N}, \end{cases} \tag{2}$$

where $\{\alpha_n\}_{n=1}^{\infty}$ is a real sequence in $[0, 1]$.

Karakaya et al. [7] showed that iterative algorithm (2) strongly converges to the fixed points of weak-contraction mappings. More precisely, they proved the following result.

**Theorem 2.** *([7]) Let $C$ be a nonempty closed convex subset of a Banach space $X$ and $T : C \to C$ a weak-contraction satisfying condition (1). Let $\{x_n\}_{n=1}^{\infty}$ be an iterative sequence generated by (2) with real sequence $\{\alpha_n\}_{n=1}^{\infty} \subseteq [0, 1]$ satisfying $\sum_{n=1}^{\infty} \alpha_n = \infty$. Then, $\{x_n\}_{n=1}^{\infty}$ converges to a unique fixed point $p^*$ of $T$.*

 

## 2 Main result

Here, we prove that iterative sequence generated by (2) is weak $w^2-$stable w.r.t. a weak-contraction mapping $T$ satisfying condition (1).

**Theorem 3.** *Let $C$ be a nonempty closed convex subset of a Banach space $X$ and $T : C \to C$ with $p^* = Tp^*$ a weak-contraction satisfying condition (1). Let $\{x_n\}_{n=1}^{\infty}$ be an iterative sequence generated by (2) with real sequence $\{\alpha_n\}_{n=1}^{\infty} \subseteq [0, 1]$ satisfying $\sum_{n=1}^{\infty} \alpha_n = \infty$. Let $\{r_n\}_{n=1}^{\infty}$ be an equivalent sequence of $\{x_n\}_{n=1}^{\infty}$ and $\epsilon_n = \|r_{n+1} - Ts_n\|$, $s_n = (1 - \alpha_n) p_n + \alpha_n Tp_n$, $p_n = Tr_n$ for all $n \in \mathbb{N}$. Suppose that $\lim_{n\to\infty} \epsilon_n = 0$. Then, the sequence $\{x_n\}_{n=1}^{\infty}$ is weak $w^2-$stable w.r.t. $T$.*

*Proof:* From (1) and (2), we have

$$\begin{aligned}
\|r_{n+1} - p^*\| &= \|r_{n+1} - x_{n+1}\| + \|x_{n+1} - p^*\| \\
&\leq \|r_{n+1} - Ts_n\| + \|Ts_n - x_{n+1}\| + \|x_{n+1} - p^*\| \\
&\leq \epsilon_n + \|Ts_n - Ty_n\| + \|x_{n+1} - p^*\| \\
&\leq \epsilon_n + \delta \|s_n - y_n\| + L \|y_n - Ty_n\| + \|x_{n+1} - p^*\| \\
&\leq \epsilon_n + \delta \|(1 - \alpha_n)(p_n - z_n) + \alpha_n (Tp_n - Tz_n)\| + L \|y_n - p^*\| \\
&\quad +L \|p^* - Ty_n\| + \|x_{n+1} - p^*\| \\
&\leq \epsilon_n + \delta \{(1 - \alpha_n) \|p_n - z_n\| + \alpha_n \|Tp_n - Tz_n\|\} \\
&\quad + (1 + \delta) L \|y_n - p^*\| + \|x_{n+1} - p^*\| \\
&\leq \epsilon_n + \delta \{(1 - \alpha_n) \|Tr_n - Tx_n\| + \alpha_n \|Tp_n - Tz_n\|\} \\
&\quad + (1 + \delta) L \|(1 - \alpha_n) z_n + \alpha_n Tz_n - p^*\| + \|x_{n+1} - p^*\| \\
&\leq \epsilon_n + \delta \{(1 - \alpha_n) \|Tr_n - Tx_n\| + \alpha_n \|Tp_n - Tz_n\|\} \\
&\quad + (1 + \delta) L [1 - \alpha_n (1 - \delta)] \|z_n - p^*\| + \|x_{n+1} - p^*\| \\
&\leq \epsilon_n + \delta^2 [1 - \alpha_n (1 - \delta)] \|r_n - x_n\| \\
&\quad +\delta L \left\{1 - \alpha_n + (1 - \alpha_n) \delta + \alpha_n \delta + \alpha_n \delta^2 + \alpha_n \delta (1 + \delta)\right\} \|x_n - p^*\| \\
&\quad (1 + \delta) L [1 - \alpha_n (1 - \delta)] \delta \|x_n - p^*\| + \|x_{n+1} - p^*\| \qquad (3)
\end{aligned}$$

Since sequences $\{x_n\}_{n=1}^{\infty}$ and $\{r_n\}_{n=1}^{\infty}$ are equivalent, therefore we have $\lim_{n\to\infty} \|r_n - x_n\| = 0$. By Theorem 2, we have $\lim_{n\to\infty} \|x_n - p^*\| = 0$. Now taking limit on both sides of (3) and then using the assumption $\lim_{n\to\infty} \epsilon_n = 0$ leads to $\lim_{n\to\infty} \|r_{n+1} - p^*\| = 0$. Thus $\{x_n\}_{n=1}^{\infty}$ is weak $w^2-$stable w.r.t. $T$. □

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