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The Portfolio Optimization Based on Sharp Performance Ratio

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A B S T R A C T

In recent years, investors evaluate their portfolio using modern portfolio theory developed by Markowitz while in the past they evaluated portfolio types according to the traditional portfolio theory based on simple diversification. In modern portfolio theory, it has been defended that the relationships among financial assets included in the portfolio should be taken into account. In addition, the return and risk of the portfolio can be calculated by the mean-variance model. Investors always expect the maximum return and the minimum risk. Therefore, they want to choose the optimum one. In Economics literature there are some measurements to evaluate the performances of the different portfolios. In this study, it is aimed at the portfolio analysis to do for the data of the BIST 30 index. For portfolio optimization, some Artificial Intelligence techniques such as the Genetic Algorithm and Particle Swarm Optimization were used for the data belonging to the year 2018. In these algorithms, different values for the parameters were tried and Sharp Performance Ratio (SPR) was used as a performance criterion. The portfolio found with the maximum SPR has been determined as the optimum portfolio. Finally, the risk and the expected return of the portfolio, the included financial assets and their weights have been obtained. The values of the parameters of the final result are considered as the best.

1. Introduction

The portfolio is the entire assets which are owned by an investor who thinks to be returned back to him/her as profit. In other words, the portfolio is a set of assets consisting of at least two assets and created to obtain the highest return based on the investor's risk. The portfolio becomes a problem to be solved because when you construct your portfolio there are some questions, such as which assets are included in the portfolio, what their weights will be, what is the expected return and the risk of it. It means that obtaining an optimum portfolio can be considered as the portfolio optimization problem. For this reason, there have been so many researches on portfolio optimization so far and some techniques have been revealed. There are two theories about it; one is Conventional Portfolio Theory and the second is Modern Portfolio Theory. Conventional portfolio theory has been acknowledged up to the Second World War by economics and financial circles [1]. The conventional theory based on the diversity of the financial assets actually ignores the relationships among the assets and defends that the risk can be reduced by increasing the number of assets in the portfolio and the financial assets with high return should be added to the portfolio. Of course, every investor wants to have the portfolio which gives him/her profit but every investment for the future has the risk because of

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uncertainty. Since it is very hard to measure this risk, an investor cannot be get rid of taking the risk of the portfolio [2].

On the other side, modern portfolio theory is actually Markowitz' theory, which defines the risk and returns by statistically a model. This theory tells that the risk cannot be reduced only by the diversity of the portfolio but also the relationships among the assets must be taken consideration [3]. Markowitz's mean-variance model has been raised for this reason. With this model, the risk of the portfolio might be smaller than the risk of each financial asset. As the correlation between the assets included in the portfolio is decreasing the main risk of the portfolio can be decreasing. Furthermore, the investor will prefer to choose the portfolio with more return among the portfolios with the same risk. In modern portfolio theory, the aim of reducing the risk cannot be reached only by the diversity of the portfolio but also to take into account the return from the assets is playing an important role [4]. Taking consideration, the correlations and including the assets with the exact negative correlation in the same portfolio can be reduced the risk of the portfolio without giving up the targeted expectation return.

In conventional portfolio theory while it is defended that the more the assets are included the less the risk is to have but the risk cannot be explained numerically. In 1952, the risk and the expected return of the portfolio were firstly calculated with a model proposed by Harry Markowitz. After a certain amount of the assets added to the portfolio, adding more and more assets does not reduce the risk of the portfolio [5]. In the mean-variance model of Markowitz, the mean of the portfolio is the sum of the multiplication of the weight with the return of each asset. The risk of the portfolio is the standard deviation of the portfolio. The aim of the model is making the risk minimum and the return maximum. Then the risk and the expected return are given by the following formulas, respectively.

$$\sigma_p^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \sigma_{ij} \tag{1}$$

σ_p^2: The risk of the portfolio

σ_{ij}: The covariance between the i^{th} and j^{th} financial asset

x_i: The weight of the i^{th} financial asset in the portfolio

x_j: The weight of the j^{th} financial asset in the portfolio

N: the number of the assets included in the portfolio

$$E(R) = \mu = \sum_{i=1}^{N} x_i \mu_i \tag{2}$$

μ: The expected return of the portfolio invested by the investor

μ_i: the expected return of the i^{th} financial asset

The objective function of Markowitz's Mean and Variance Model is defined as;

$$Min \sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \sigma_{ij} \tag{3}$$

Subject with

$$\mu = \sum_{i=1}^{N} x_i \mu_i$$

$$\sum_{i=1}^{N} x_i = 1$$

$$0 \le x_i \le 1 \quad i = 1,2,\dots,N, j=1,2,\dots,N$$

The performance of the invested portfolio has to be continuously evaluated since the investor wants to know the performances of his/her portfolio compared to the other portfolio [6]. There are some performance criteria in the literature. In this study, Sharp performance ratio (SPR) has been used as a criterion in order to evaluate how the portfolio performs. This criterion has been developed to measure the relationship between the risk and the return of the portfolio [7]. It is defined as given below.

$$S_p = \frac{E(R_p) - R_f}{\sigma_p} \tag{4}$$

Here, S_p is SPR, $E(R_p)$ is the expected return of the portfolio, R_f is the risk-free interest rate and σ_p is the risk of the portfolio. Generally, if the portfolio has with high SPR its probability of being preferable is high.

Recently in many research, artificial intelligence procedures have been used for solving the optimization problems. Since then, this idea of using artificial intelligence techniques is attracted by the researchers who are seeking methods for finding the optimum portfolio. [8] provided some choices for portfolio variety by developing decision support systems and combining it with a genetic algorithm. [9] observed that the genetic algorithm has been successful to find the optimum portfolio for the data of the Korean Stock Exchange. In the article done by [10], the portfolio optimization problem has been distinguished into two stages. In the first stage, the qualified financial assets are determined and in the second stage the weights are defined by Markowitz's mean-variance model; that is, the qualified assets were conveniently allocated within the portfolio for reaching the optimum portfolio. [11] proposed a method in which the weights of the assets to be included are being calculated by using the ratio of the expected return to the risk. In this paper the problem is actually the multi-dimensional portfolio optimization problem and genetic algorithm has been used.

In the paper of [12], a genetic algorithm model was designed with the different constraints used for some certain periods and the results were found satisfactory in the sense of achieving the optimum portfolio. [13] introduced three possible models for portfolio selection problems with minimum transaction lots using with genetic algorithm.

[14] introduces a heuristic approach for the portfolio optimization problem in which the Genetic algorithm is used with different risk measures. In the paper by [15], a portfolio selection model which is based on Markowitz's portfolio selection problem including three of the most important limitations is considered. The problem considered as a mixed-integer nonlinear programming (NP-Hard) and is solved by a corresponding genetic algorithm. [16] proposed a method mainly based on the index funds constructing methods of [9] and [10] adjusted-GA model. Then, a new model is constructed based on the Taiwan market environment and its characteristics.

[17] pointed out that the non-linear constrained portfolio optimization problem with multi-objective functions cannot be efficiently solved using traditional approaches and therefore presents a meta-heuristic approach to portfolio optimization problem using particle swarm optimization (PSO) technique. The model was tested on various restricted and unrestricted risky investment portfolios and compared to Genetic Algorithms. The PSO model demonstrated high computational efficiency in constructing optimal risky portfolios.

[18] proposed a novel two-level particle swarm optimization (TLPSO) to solve the credit portfolio management problem. The objective of the manager is to minimize the maximum expected loss of the portfolio subject to a given consulting budget constraint. The captured problem is very challenging due to its hierarchical structure and its time complexity, so the TLPSO is designed for the credit portfolio management model. The TLPSO has two searching processes, namely, "internal-search", the searching process of the maximization problem and "external-search", the searching process of the minimization problem. The performance of TLPSO is then compared with both the GA and the PSO, in terms of efficient frontiers, fitness values, convergence rates, computational consumption and reliability. The experiment results show that TLPSO is more efficient and reliable for the credit portfolio management problem than the other tested methods.

[19] present a novel heuristic method for solving an extended Markowitz mean-variance portfolio selection model. The extended model includes four sets of constraints: bounds on holdings, cardinality, minimum transaction lots and sector (or market/class) capitalization constraints. The extended model is classified as a quadratic mixed-integer programming model necessitating the use of efficient heuristics to find the solution. Then they propose a heuristic based on the PSO method. The proposed approach is compared with GA. The computational results show that the proposed PSO effectively outperforms GA, especially in large-scale problems.

[20] pointed out Solving the multi-stage portfolio optimization (MSPO) problem is very challenging due to nonlinearity of the problem and its high consumption of computational time therefore many heuristic methods have been employed to tackle the problem. In this paper, they propose a novel variant of PSO called drift particle swarm optimization (DPSO), and apply it to the MSPO problem-solving. The experiment results show that DPSO is more efficient and effective in MSPO problem solving than other tested optimization tools.

[21] proposed a new admissible efficient portfolio selection model and design an improved PSO algorithm because traditional optimization algorithms fail to work efficiently for their proposed problem. In this paper the results of a numerical example illustrated the proposed approached is effective.

After reviewing the past researches in this part of the study we can talk about what we did in this article. The main difference is to use Sharp performance ratio as an objective function. With this objective, the algorithms based on GA and PSO were proposed. From the application of these proposed approaches, we have obtained a number of portfolios. These portfolios were compared in terms of SPR and the portfolio with the maximum SPR have been chosen as optimum.

2. Genetic Algorithm

A genetic algorithm introduced by Holland in 1960 is a heuristic algorithm. Genetic algorithms are generally used to generate solutions for an optimization problem. This algorithm usually starts to get a population of randomly generated individuals [22]. The smallest unit of a GA is called a "gene". Genes coming together constitute a chromosome. Each chromosome actually represents an alternative solution. The number of genes and chromosomes in a GA are defined by the researcher studying a certain problem. In evolution theory, if an individual has got good genes, he/she will survive, otherwise vanishes. In a GA if the chromosome has got good genes it can be an optimal solution otherwise it should be discarded from the memory. The value of the objective function of an algorithm is actually the fitness value of each chromosome. In order to have a qualified chromosome, the elitism method can be used. Then for increasing the variety of solutions the operators of mutation and cross over are used [23]. *Crossover*, which is actually re-combinations of genes from different chromosomes, is a genetic operator used to combine the genetic information of two parents to generate new offspring. *The mutation* is also a genetic operator used to maintain genetic diversity from one generation of a population of genetic algorithm chromosomes to the next. The mutation alters one or more gene values in a chromosome from its initial state. In mutation, the solution may change entirely from the previous solution. Hence GA can come to a better solution by using mutation.

The formal steps of a GA are given as follows;

Step 1: The initial population is generated.

Step 2: The fitness value is calculated for each chromosome of the initial population.

Step 3: The matching points are defined for genetic operators.

Step 4: Cross-over and mutation are applied with respect to the probabilities previously defined and a new population is generated.

Step 5: The fitness value is calculated for the new population.

Step 6: The steps from 1 to 5 are repeated as much as the number previously defined as iteration otherwise the algorithm is stopped.

3. Particle Swarm Optimization

PSO is a numerical technique which can be used for solving optimization problems in computational science. It has been developed by Dr. Eberhart and Dr. Kennedy in 1995, inspired by the social behaviour of bird flocking or fish schooling [24]. In PSO, each single solution in search space is called as a particle, which represents a "bird" in the flocking. The general purpose of PSO is to achieve the optimum result by developing social information sharing between birds or solutions in the flocking. The particles are moving around in the search space according to the particle's position and velocity. Each particle's movement is influenced by its local best-known position, but is also guided toward the best-known positions in the search-space, which are updated as better positions are found by other particles.

Step 1: Generate the initial population with the initials for the position and velocity of each particle generated randomly from the pre-specified interval.

Step 2: Calculate the fitness value of each particle in the initial population.

Step 3: Find the best local (*pbest*) and the best global (*gbest*) fitness value.

Step 4: Update the positions and velocities

Step 5: The steps from 2 to 4 are repeated until the stopping criteria is satisfied.

4. Application

In this study, the portfolio optimization problem has been solved by using algorithms based on both genetic and PSO which we proposed. Our data set is the BIST 30 index of 2018. That means the data are the closing prices of 30 financial assets at the end of the day the dates between 02/01/2018 and 31/12/2018. The analysis has been done by using the MATLAB code for both proposed approaches. In both approaches, the Sharp performance ratio is considered as the objective function and the constraints are defined as in mean-variance model constraints.

$$Max\ S_p = \frac{E(R_p)-r_f}{\sigma_p} \frac{\sum_{i=1}^{n} x_i\mu_i - r_f}{\sum_{i=1}^{n}\sum_{j=}^{n} x_i x_j \sigma_{ij}} \tag{5}$$

With subject to

$\sum_{i=1}^{n} x_i\mu_i \geq R$ for $\sum_{i=1}^{n} x_i = 1$ and $0 \leq x_i \leq 1$

Here S_p is the value of Sharp Performance Ratio of the portfolio, $E(R_p)$ is the expected return, R_f is the interest ratio without risk, σ_p is the risk of the portfolio, x_i is the weight of the ith asset in the portfolio μ_i is the mean return of the ith asset and $\sigma_{ij}$is the covariance between the ith and jth asset. In the originality of this study is that SPR has been used to evaluate the risk and the return of the portfolio. The values recommended in the literature have been chosen as values for the parameters of algorithms based on PSO. For the approach based on GA, the number of chromosomes represents the number of portfolios and the number of gene in each chromosome represents the weights of the assets included in the portfolio. In addition, the constraint that the sum of weights is 1 has been added to the algorithm. The initial values of the genes are defined randomly. There are some recommended values for the parameters of the GA by [25], [26] and [27] according to their experimental studies. These values are presented in Table 1.

Table 1. The most commonly used parameter values.

The parameter	**De Jong**	**Schaffer**	**Grefenstette**
The # of chromosomes	50-100	20-30	30
The crossover ratio	0,60	0,75-0,95	0,95
The mutation ratio	0,001	0,005-0,01	0,01
Elitism ratio	%30	%30	%30

Firstly, we applied the proposed algorithm based on GA. In that application, 5 different iterations such as 1000, 2500, 5000 and 10000 have been done for each parameter and the obtained portfolios were saved at each iteration. Then these portfolios were evaluated according to their SPR. The portfolio with the greatest SPR was defined as the resulting optimum portfolio obtained by GA. This portfolio obtained from this application is represented in Table 2. As you can see this table summarizes which assets are included and what their weights will be, in the final portfolio. The parameter values where the optimum portfolio found by GA are given in Table 3.

Table 2. The optimum portfolio found by GA

Code of the asset	**Weight**	**Code of the asset**	**Weight**
AKBNK	0	OTKAR	0
ARCLK	0	PETKM	0
ASLSN	0,029412	PGSUS	0
BIMAS	0,102941	SAHOL	0,014706
DOHOL	0,102941	SISE	0,132353
ECLC	0	SKBNK	0
EKGYO	0	TAVHL	0,102941
EREGL	0	TCELL	0,073529
GARAN	0	THYAO	0,073529
HALKB	0	TKFEN	0,132353
ISCTR	0,029412	TOASO	0
KCHOL	0,029412	TTKOM	0
KOZAA	0,073529	TUPRS	0,073529
KRDMR	0,029412	VAKBN	0
MAVI	0	YKBNK	0
SPR	**0,034717**		
The expected return	**0,045667**		
The risk	**1,315428**		
The # of assets	**14**		

Table 3. The parameter values for the optimum portfolio obtained from GA.

The # of Chromosomes	**Crossover Ratio**	**Mutation Ratio**	**The # of iterations**
50	0,60	0,001	10000

Secondly, the same data were analyzed by the proposed approach based on PSO. In the problem of portfolio optimization, the unknown parameters of the proposed approach based on PSO are the number of a particle, inertia weight and cognitive and social coefficients. In the algorithm, each particle represents a portfolio. It is generally recommended to use 20-60 particles in the algorithm [28]. It is believed that using (0.2, 0.5) for the inertia weight increases the speed of locational seeking [23]. Cognitive and social coefficients were taken as 2 in our approach. In our application, the groups were constructed corresponding to different parameters. For each group, 1000 iterations have been done in order to get a number of varieties of solutions. More than 1000 iterations gave the same solution. In Table 4 SPR values at different weights and the different number of particles are presented. As can be seen from Table 4, many options have achieved almost the same SPR as we consider the best. That is, we have concluded that the optimum portfolio has been the portfolios with the highest SPR. Table 5 presents the optimum portfolio found by the proposed approach based on PSO.

Table 4. SPR at different weights and the number of particles.

	Weights			
The # of Particle	0,2	0,3	0,4	0,5
20	0,0677	0,0683	0,0684	0,0684
30	0,0684	0,0659	0,0684	0,0684
40	0,0659	0,0684	0,0684	0,0684
50	0,0684	0,0684	0,0684	0,0684
60	0,0684	0,0684	0,0684	0,0684

Table 5. The optimum portfolio found by PSO

Code of the asset	**Weight**	**Code of the asset**	**Weight**
AKBNK	0	OTKAR	0
ARCLK	0	PETKM	0
ASLSN	0	PGSUS	0
BIMAS	0,2238	SAHOL	0
DOHOL	0,1548	SISE	0,2389
ECLC	0	SKBNK	0
EKGYO	0	TAVHL	0
EREGL	0	TCELL	0
GARAN	0	THYAO	0
HALKB	0	TKFEN	0,2630
ISCTR	0	TOASO	0
KCHOL	0	TTKOM	0
KOZAA	0,0830	TUPRS	0,0366
KRDMR	0	VAKBN	0
MAVI	0	YKBNK	0
SPR	**0,0684**		
The expected return	**0,0937**		
The risk	**1,3702**		
The # of assets	**6**		

In order to see the two resulting portfolios comparatively, Table 6 was prepared. Which assets have been included and what their weights have been can be seen in Table 6, therefore we can easily compare them.

5. Conclusion

An investor always wishes to have the financial assets which have the minimum risk and bring the highest return. Therefore, the portfolio optimization problem is an important issue as a financial phenomenon. Recently many artificial intelligence techniques have been using for solving optimization problems. In this study, we propose two new algorithms for solving this problem. Sharp performance ratio, which evaluates portfolio performance, is used to reach the optimum portfolio. SPR must be maximum to reach the optimum portfolio. therefore, the optimal portfolio in both algorithms is maximum SPR. The new algorithms are based on GA and PSO and for the SPR has been used for the objective function of these algorithms. Of course, we wonder how they perform for solving portfolio optimization. Finally, according to the application results, we concluded that PSO provided a better result when we evaluate according to SPR. According to the GA method of the PSO algorithm has reached a solution in a short time. In addition, the number of parameters in the PSO algorithm is less than GA. In this study, it is proved that the PSO algorithm is better for portfolio optimization. In addition to that, it has been observed that PSO also provided better result in terms of the expected return. It has been observed that the risk of PSO is very close to the risk from GA.

Moreover, the portfolio provided by PSO includes much fewer financial assets than the ones of the optimum portfolio from GA. This can be considered as advantages for the investors since the tracking of the portfolio with fewer assets is much easier.

We can finally say that artificial intelligence techniques can be very useful for solving the optimization problems because they work with a principle of seeking the optimum in a big solution set. Of course, for the different data sets, these techniques should be tried separately to see which one performs well for the data set.

Table 6. The comparison the results from both approaches

	GA	PSO
AKBNK	0	0
ARCLK	0	0
ASLSN	0,029412	0
BIMAS	0,102941	0,2235
DOHOL	0,102941	0,1548
ECLC	0	0
EKGYO	0	0
EREGL	0	0
GARAN	0	0
HALKB	0	0
ISCTR	0,029412	0
KCHOL	0,029412	0
KOZAA	0,073529	0,0831
KRDMR	0,029412	0
MAVI	0	0
OTKAR	0	0
PETKM	0	0
PGSUS	0	0
SAHOL	0,014706	0
SISE	0,132353	0,239
SKBNK	0	0
TAVHL	0,102941	0
TCELL	0,073529	0
THYAO	0,073529	0
TKFEN	0,132353	0,2631
TOASO	0	0
TTKOM	0	0
TUPRS	0,073529	0,0365
VAKBN	0	0
YKBNK	0	0
SPR	**0,034717**	**0,0684**
The expected return	**0,045667**	**0,0937**
The risk	**1,315428**	**1,3702**
The # of assets	**14**	**6**

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