Investigation of Empirical Abstraction Processes for Slope Concept According to Students' Attitude, Anxiety, Motivation and Self Efficacy Perceptions towards Mathematics[1]

Aslıhan Özmen, Ordu University, ORCID ID:0000-0002-3314-4647
Hayal Yavuz Mumcu, Ordu University, ORCID ID:0000-0002-6720-509X

Abstract

The aim of this study is to investigate the empirical abstraction processes of 7th grade students in relation to the variables of attitude, anxiety, self-efficacy and motivation towards mathematics. This is a mixed method correlational study using a sequential explanatory design with two distinct stages. The study group consist of 91 students attending public schools in a city center who were selected by using criterion sampling technique. The data collection tools used in the study are Abstraction Test (AT) which was developed by the researchers and consisting of 8 open ended, gap filling and multiple-choice questions, Clinical Interview Processes conducted with the students, Mathematics Attitude Scale, Math Anxiety Scale, Mathematical Self-Efficacy Scale and Mathematics Motivation Scale. In the analysis of the qualitative data obtained from the study, Action Sequence formed by the researchers and the Clinical Interviews conducted with students were used while Pearson-Correlation analysis techniques were used for the analysis of the quantitative data of the study. As a result of the study, it was seen that students generally do not perform well in the process of knowledge construction and they have misconceptions about the concept of slope. In addition, a significant positive relationship was found between students' knowledge construction processes and attitude-self-efficacy scores towards mathematics, while a significant negative relationship was found between knowledge construction processes and anxiety levels. In the light of the results obtained from the study, some suggestions were made for the literature and teaching environments.

Keywords: *empirical abstraction, the concept of slope, attitude, anxiety, self-efficacy*

İnönü University
Journal of the Faculty of Education
Vol 21, No 2, 2020
pp. 785-800
DOI: 10.17679/inuefd.657175

Article type:
Research article

Received : 09.12.2019
Accepted : 24.06.2020

Suggested Citation

Özmen, A. & Mumcu, H, Y. (2020). Investigation of Empirical Abstraction Processes for Slope Concept According to Students' Attitude, Anxiety, Motivation and Self Efficacy Perceptions towards Mathematics, *Inonu University Journal of the Faculty of Education*, *21*(2), 785-800. DOI: 10.17679/inuefd.657175

[1] This study was presented as an oral presentation at the 4th International Symposium of Turkish Computer and Mathematics Education (TURCOMAT-4), 26-28 September 2019 in Çeşme/İzmir.

INTRODUCTION

The fact that mathematics is considered as the science of abstraction and mathematical concepts are obtained as a result of abstraction (Altun, Arslan & Yazgan, 2008) reveal the importance of abstraction in the formation of mathematical concepts. Piaget (1974) states that every new knowledge in mathematics occurs at the end of the abstraction process. The concept of abstraction, which has a very important place in mathematics education, has been defined by different researchers with different perspectives. Yılmaz (2011) defines mathematical abstraction as the process of extracting the essence of the features that form the basis of a mathematical concept, removing its devotion to real world objects and generalizing to wider applications.

Today, the idea of abstraction has been interpreted in two different meanings from a cognitive and sociocultural perspective. Researchers, such as Piaget, Dienes and Skemp, who discuss abstraction from a cognitive point of view, claim that learning will be based on similarities in the presented examples. Researchers with a view of abstraction from a sociocultural perspective such as Noss and Hoyles, Van Oers, Ohlsson and Lehtinen, think that learning cannot take place separately from the environment, from using tools, from social interaction and from the conditions surrounding the environment. Both perspectives, however, agree that abstraction is a process occurs in the mind of the individual. In this section, cognitive abstraction theory will be elaborated in accordance with the scope of the study.

Cognitive researchers generally think that abstraction process consists of a series of mathematical objects and that these objects are classified under similar characteristics as a result of the learner's mental activities, thus achieving more advanced mathematical objects (Herskowitz, Scwarz, & Dreyfus, 2001). Adopting the idea of cognitive abstraction, Dienes (1961) considers abstraction as a process, not as a finished product, and defines it as the process of extracting common traits from a group of different situations. Skemp (1986) looks abstraction as a product of the abstraction process and states that abstraction allows the individual to become aware of new experiences of similarities. Researchers who consider abstraction with a cognitive approach, emphasize three important common expressions. These; i) Generalization achieved by recognizing the common points of a number of specific examples, ii) Climbing from low concrete levels to abstract levels of thought, iii) A process that takes place independently from the conditions surrounding the environment (Özmantar, 2005).

According to the cognitive abstraction approach, Piaget discusses and explains mathematical abstraction in two different types. The first is empirical abstraction, which is related to the apparent properties of objects, and the other is deep-reflective abstraction, which is related to the actions of individuals and the results of these actions. Of these two abstraction processes, empirical abstraction is defined as the process of creating information by using the properties of objects (Beth & Piaget, 1966). In this process, the individual chooses the common properties of a series of visible objects, analyzes them with using his/her mental activities, and isolate (abstracts) these characteristics from the objects. In the process of empirical abstraction, the individual experiences observable features, but the knowledge he constructs with using these features is internal (Dubinsky, 1991). According to Piaget, this type of abstraction allows to reveal the common properties of objects and make dimensional generalizations (Piaget & Garcia, 1983). Reflective abstraction represents a more complex process. While the individual receives only information with empirical abstraction, he begins to apply actions on objects with reflective abstraction. Reflective abstraction is referred to by Piaget (1985) as the overall coordination of actions, so its origin is subjective and entirely internal. In this process, the individual tries to reach the information by observing the results of his actions and relating his actions with each other accordingly. The process of reflective abstraction is purely constructivist since it is based on the internal construction of knowledge by the individual (Beth & Piaget, 1966). This process leads to very different type of generalization that is constructive and results in new syntheses in which certain rules turn into new concepts (Sopamena, Nusantara, Irawan, & Sisworo, 2016).

Mitchelmore and White (2004) discussed empirical abstraction in terms of learning processes and mentioned the importance of empirical abstraction especially for learning basic mathematical concepts. Accordingly, students learn basic and abstract mathematical concepts in schools firstly relating them with real life. The main feature of these processes is recognizing similarity. The concept of similarity mentioned here is not the apparent similarity, but rather the structures created by the student in mental processes. As students experience objects and situations they do not already recognize, they can make internal connections and create meaningful relationships between structures which seem meaningless to them before. An example of this is the learning of numbers, spaces and relationships that form the basis of mathematics. As students experience situations previously conceived as disconnected, they can discover new relationships and they become able to do things they were not able to do before. The process of recognizing similarity, which forms the basis for the establishment of these relations, is called the empirical abstraction process by Mitchelmore and White (2004). Therefore, it can be said that empirical abstraction leads to knowledge construction processes in most cases. Similarly, Piaget states that the actions that serve reflective abstraction processes are performed only on objects that individuals discover only through empirical abstraction (Piaget, 1985). In this context, it can be said that empirical abstraction processes have a very important role especially for mathematical knowledge construction processes carried out at early ages. Mitchelmore and White (2004) stated that students learn abstract concepts in general without going through the abstraction process, therefore the concepts are not fully understood, learned weakly and quickly forgotten, so empirical abstraction processes should be shaped to provide full learning. So, this study can also be evaluated in relevant context.

Although there are different factors affecting learning of mathematics, the effect of affective factors on students is of indisputable importance today. According to Bloom (1995), approximately 25% of the difference between individuals' learning is due to affective characteristics (cited in Baykul, 2009). Among the students whose cognitive input behavior levels are equal, the learning speeds and levels of those who are interested in learning, are higher than those who are reluctant, uninterested and excited towards learning (Kesici & Aşılıoğlu, 2017). Therefore, in order to realize a high level of learning, affective characteristics such as interest, attitude and academic self should be taken into consideration as well as cognitive input characteristics such as knowledge, skills and competence (Demir, 2011). So, the aim of this study is to examine the empirical abstraction processes which can be expressed as the first step of knowledge construction processes in relation to students' attitude, self-efficacy, anxiety and motivation levels towards mathematics. The concept of slope was used for the examination of these processes. Some of the reasons for this situation are the abstract structure of the slope concept and students have some learning difficulties related to this concept (Knuth, 2000; Moschkovich, Schoenfeld, & Arcavi, 1993; Newburgh, 2001; Postelnicu, 2011). Also, this concept constitutes the basis of many different concepts in mathematics (Deniz & Kabael, 2017). The sub-problems of the study are as follows.

• What are the performance of 7th grade students in the process of empirical abstraction for the concept of slope?
• Are there any significant relationships between the empirical abstraction performances of 7th grade students and their anxiety, attitude, motivation and self-efficacy levels towards mathematics?

METHOD

Research Model

This is a mixed method correlational study using a sequential exploratory design with two distinct stages. Scientific studies are defined as descriptive, relational and interventional studies according to their level (Fraenkel & Wallen, 2006). In this study, since it is aimed to examine the abstraction performances of students in relation to some affective factors, the study can be considered in the context of correlational study. When the study process is taken into consideration, it can be said that mixed method is used because both quantitative and qualitative methods are used in the study (Creswell, 2003; Johnson & Onwuegbuzie, 2004; Tashakkori, Teddlie & Teddlie, 1998). Sequential exploratory design was used for the mixed method in the study because before the quantitative stage, the researcher begins to explore the subject qualitatively in this design. The explorer pattern is a two-stage sequential pattern and a tool is generally developed and used to collect quantitative data in these studies (Creswell & Plano-Clark, 2007). The data obtained from the first stage of the study (the qualitative stage), is used to develop and provide the second stage (the quantitative stage) (Greene, Caracelli, & Graham, 1989).

In the first stage of this study, *Abstraction Test* (AT) was developed and qualitative analysis was performed in order to determine the students' abstraction scores. The data obtained from this stage were converted into quantitative data depending on the theoretical structure formed within the scope of the study and abstraction scores were determined for each student in the study. Afterwards, statistical analyzes were conducted for the sub-problems of the study and abstraction performances of the students were examined in relation to the variables of attitude, self-efficacy, motivation and anxiety towards mathematics.

Study Group

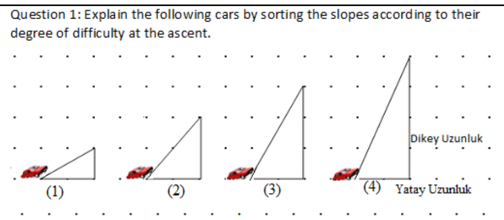
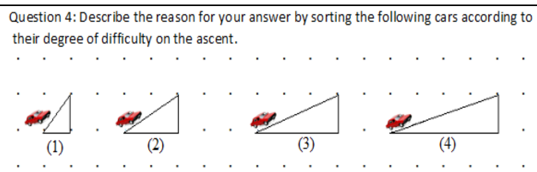
The study group of this study consisted of 91 students attending 7th grade in secondary schools in Ordu city center. In the selection of students, two public schools with an academic achievement level above the provincial average were determined in line with the opinions of school principals and mathematics teachers in the city center, and then the students were selected by using criterion sampling technique. The criterion was determined as being above the school average of academic achievement level and being willing to participate in the study. Because abstraction is a high-level mathematical thinking skill (Russell, 1926) it was paid attention to the mathematics achievements of the students to be included in the study are above the provincial average. Otherwise, it would be difficult to access valid and reliable data on students' abstraction performances.

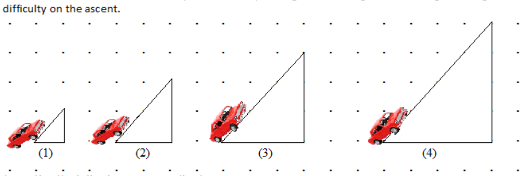
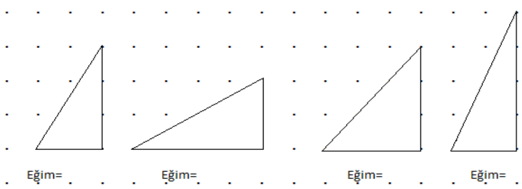
Data Collection Tools

Abstraction Test (AT) developed by the researchers was used in the analysis of the students' knowledge construction processes. In this test, the concept of slope was used specially. The students are expected to construct the knowledge that “the slope of a line is the ratio of the vertical length of a right triangle which accepts subject line as hypotenuse, to the horizontal length”. In this process, it is aimed that the students realize the meaning of slope as “height belongs to the unit length” for a right triangle. So, first of all three special cases were presented to the students about the mathematical knowledge aimed to be constructed in AT. The students were expected to make mathematical inferences depending on these special situations and to achieve the general knowledge step by step. In the last case coming after these special cases, students were expected to construct mathematical knowledge about the meaning of the slope concept. In this step, it was predicted that the students will form mathematical generalizations based on the inferences obtained from the special cases and will express their knowledge mathematically.

Depending on the theoretical structure and purpose of the study, the empirical abstraction behaviors expected from the students in the knowledge construction processes are given in Table 1 in relation to the questions in AT.

Table 1
Contents of the Questions in AT

	Aim/content of the question		Activity of empirical abstraction
	Comparing the slopes of right triangles with the same horizontal length (EB)		
Case 1	Noticing that the slope in right triangles with the same horizontal length depends on the vertical length (EB)	Question 2: Explain the reason for your answer by filling the space below with the appropriate option. “The difficulty experienced by the cars in question 1 depends on...............” A)Horizontal length B) Vertical length C) Both horizontal and vertical length	Making mathematical connections and cognitive inferences based on physical properties of the objects.
	Noticing that in right triangles with the same horizontal length, the slope changes in direct proportion to the vertical length (TB)	Question 3: As the .... (vertical / horizontal) length increases, the slope .... (increases / decreases).	
Case 2	Comparing the slopes of right triangles of the same vertical length (EB)		

	Noticing that the slope depends on the horizontal length in right triangles with the same vertical length (EB)	Question 5: Explain the reason for your answer by filling the space below with the appropriate option. "The difficulty experienced by the cars in question 4 depends on..............." A) Horizontal length B) Vertical length C) Both horizontal and vertical length	Making mathematical connections and cognitive inferences based on physical properties of the objects.
	Notice that in right triangles with the same vertical length, the slope changes inversely proportional to the horizontal length (TB)	Question 6: As the .... (vertical / horizontal) length increases, the slope .... (increases / decreases).	
Case 3	Noticing that the slopes of the right triangles with the same horizontal and vertical lengths are equal (unchanged) (TB)	Question 7: Describe the reason for your answer by sorting the following cars according their degree of difficulty on the ascent.  	Making mathematical connections and cognitive inferences based on physical properties of the objects.
KC	Noticing that the slope in a right triangle is calculated with the formula of: vertical length / horizontal length (TB)	Question 8: Calculate the slope of the triangles given below. Explain your answer.  	Reaching mathematical generalizations based on the inferences obtained, expressing the obtained knowledge mathematically.

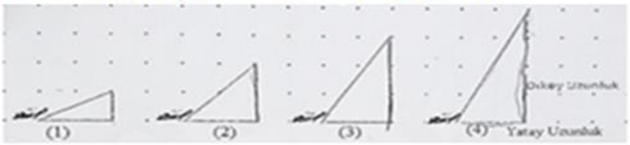
EB: Early behaviour, TB: Targeted behavior, KC: Knowledge Construction.

In this study, Math Attitude Scale (MAS) developed by Duatepe and Çilesiz (1999) to determine students' attitudes towards mathematics, Math Anxiety Scale (MAS*) developed by Erol (1989) to determine math anxiety levels for mathematics, Mathematics Self-efficacy Scale (MSS) developed by Umay (2001) to determine self-efficacy beliefs towards mathematics and Mathematical Motivation Scale (MMS), which was developed by Githua and Mwangi (2003) and adapted to Turkish by Ayan (2014), was used to determine motivation levels of students for mathematics were used as data collection tools in addition to AT additionally.

Another data collection tool of the research is the *Interview Processes* conducted with the students in the study group. These processes were used for the transformation of qualitative data to the quantitative, which were obtained from AT. The interviews conducted on the situations determined for the student responses that were unclear on paper and lasted for 20-30 minutes for each student. These processes were also made recordable and stored. In addition, interviews were conducted with each student who successfully completed all steps of the abstraction process within the scope of the study and thus the validity of the study was tried to be increased. During these interviews, it was determined that 3 students knew the concept of slope before and therefore gave correct answers to all questions in AT, so the data of these students was excluded from the study.

One of the mentioned interview processes is given below. The following example shows the interview process with the $Ö_3$ code student. The reason for the interview with this student is the inconsistency of the student's answers. The student gave contradictory answers to the related questions in special cases of AT. For this reason, this student was interviewed and the reason of inconsistent answers was found to be inattention of the student, and the abstraction score of the student was determined accordingly. In the following interview process, it is seen that the answers given by the student to the 1st and 3rd questions are contradictory, and by the intervention of the researcher, he accepted and changed his answer for the third question consistent with his other response.

R: Please explain your answer for the first question.



Ö3: The longer the vertical length, the harder it is for the vehicle to go up. That is, if the horizontal length is short and the vertical length is long, it becomes more difficult for the vehicle to go up. But the horizontal length is long and the vertical length is short, the easier it is for the vehicle to go up.
R: So in question 3, you said, "As the vertical length increases, the slope decreases." Why did you answer this?
Ö3: ....(the student thinks for a while). I actually gave the wrong answer here. The slope increases as the vertical length increases.
R: Ok, is there anything else you want to add?
Ö3: No.
R: Ok, lets see question 4.

Ö3: Here, the longer the horizontal length, the simpler it goes up. The slope decreases as the horizontal length increases.....according to this figure...
R: Let's go to question 7.

Ö3: Here both the vertical and horizontal lengths have increased, so it will be difficult. Number 4 is the hardest.
R: So why is it forced, what is the reason for this difficulty?
Ö3: Vertical and horizontal length is long and slope is high.
R: Why is the slope here too high?
Ö3: Due to the vertical and horizontal length. That is because both are long.

The validity and reliability of the *Abstraction Test* (AT) was ensured by expert opinions and pilot studies. The test was finalized by referring to the ideas of two faculty members who are experts in the field of mathematics education for this purpose. During the pilot study, 30 different students were selected from the schools in the sample and it was tried to determine whether the questions served their purpose. During the interview process after the pilot study, it was seen that some of the questions in the AT evoked the answers of the other questions and therefore the structure of these questions was changed. In addition to this, additional questions were added to the test as some of the expected answers could not be given by the students. In the light of the regulations made after the pilot study, the main study was started. In addition, Cronbach-Alpha reliability coefficients were calculated for the reliability of the data collection instruments in the study. The calculated reliability values are .971, .960, .853, and .945 for MAS, MAS*, MSS and MMS, respectively.

Data Analysis

The structure created by the researchers was used to convert the qualitative data obtained from the Abstraction Test (AT) into quantitative data. Accordingly, there are basically four behaviors expected from students. Three of these behaviors are related to special situations and one is related to the knowledge construction. These behaviors are;

• *Noticing that in vertical triangles of the same horizontal length, the slope changes proportionally to the vertical length (Action 1)*
• *Noticing that in vertical triangles of the same vertical length, the slope changes inversely proportional to the horizontal length (Action 2)*
• *Realizing that the right triangles with equal horizontal and vertical lengths have equal slopes (Action 3)*
• *Noticing that the slope in a vertical triangle can be expressed as vertical length / horizontal length (Action 4).*

The above mentioned behaviors were named as *Action Sequence* (AS) for this research and was used in the evaluation of AT performances of the students. Each behaviour (targeted behaviours in Table 1) was evaluated as 1 point and the leading behaviours (early behaviours in Table 1) belong to the subject behaviour were not scored in the data analysis processes because of the aim of these leading behaviours are to direct the students to the action belongs to the abstraction process. Accordingly, each student in the study group obtained an abstraction score between 0 and 4 as his/her abstraction performance.

In this process, interviews were conducted on the situations determined in order to clarify the uncertain situations where the student responses did not include any explanation. Uncertain situations were tried to be overcome through these interviews and the scores of the students were tried to be determined.

Simple correlation analysis was applied to reveal the relationships between students' abstraction scores and mathematics attitude, self-efficacy, anxiety and motivation levels. Normality assumption of the correlation analysis were examined and the skewness coefficients for abstraction score, math self-efficacy, math anxiety, math motivation and math attitude variables were calculated as .052 -.129 -.362 -.156 -.479 respectively, while the kurtosis coefficients were as -.095 -.388 -.479, -.302, and -.277 respectively. Since the skewness and kurtosis coefficients for these variables were in the range of [-1, +1], it was observed that these data provided a normal distribution (Büyüköztürk, 2018). Accordingly, Pearson Correlation Coefficient was calculated between abstraction scores and mathematics anxiety, attitude, self-efficacy belief, motivation levels. Obtained data were analyzed with SPSS 21.0 program and the significance level was taken as .05 for the interpretation of the findings.

In the presentation of the *abstraction performances* of the students, "AP-n" categories were used according to the number of behaviours shown by the students in the *Action Sequence*. Where SP-n refers to the frequency of the student showing n behaviors which takes place in Action Sequence. As an example, Table 2 shows that 48 students are in the AP-2 category. This means that 48 students (52.74% of all students in the study group) in the study group can demonstrate 2 actions in the *Action Sequence* (AS). For this student, it can be said that he can show Action 1 and Action 2 but fail in Action 3 and Action 4. Besides in the quotations from the student responses, the students were coded with numbers, so *Sn* meant the *nth* student in study group. In addition, in some parts of the study, the expressions used by the students are exemplified and the frequency of the students who gave the same answer were given in brackets.

FINDINGS

Findings for the First Sub-Problem

The results obtained from Abstraction Test, which is prepared to measure the student abstraction performances on the concept of slope, are given in Table 2.

Table 2
Data Obtained from AT.

	Abstraction Performance (AP)					
	AP-O	AP-1	AP-2	AP-3	AP-4	Total
N	6	19	48	15	3	91
%	6.59	20.87	52.74	16.48	3.29	100

When Table 2 is examined, it is seen that more than half of the students could successfully complete only two steps of the Action Sequence. In addition, it can be said that one out of every five students can only perform one action successfully. The rates of students who received 3 and 4 points from AT were 16.48% and 3.29%, respectively. 6.59% of the students did not perform any action in the sequence. In this part of the study, findings from each category will be elaborated.

When the responses of the students in AP-0 category were examined, it was seen that these students generally made partial sense of special case 1, made incorrect comments in special case 2, and generally could not make any comment and gave empty answers in special case 3. Therefore, it can be said that these students generally fail in mathematical inferences about physical situations.

When the answers in AP-1 category were examined, it was seen that 14 (76.68%) of these students answered questions 1, 2 and 3 correctly in AT. Therefore, it can be said that these students do not have difficulty in making sense of the special case 1. While two of the other five students could not make sense of special case

1, gave correct answers to the questions in special case 2. Three of them answered the first question correctly and used contradictory expressions in questions 2 and 3. An example of this situation is given in Figure 1.

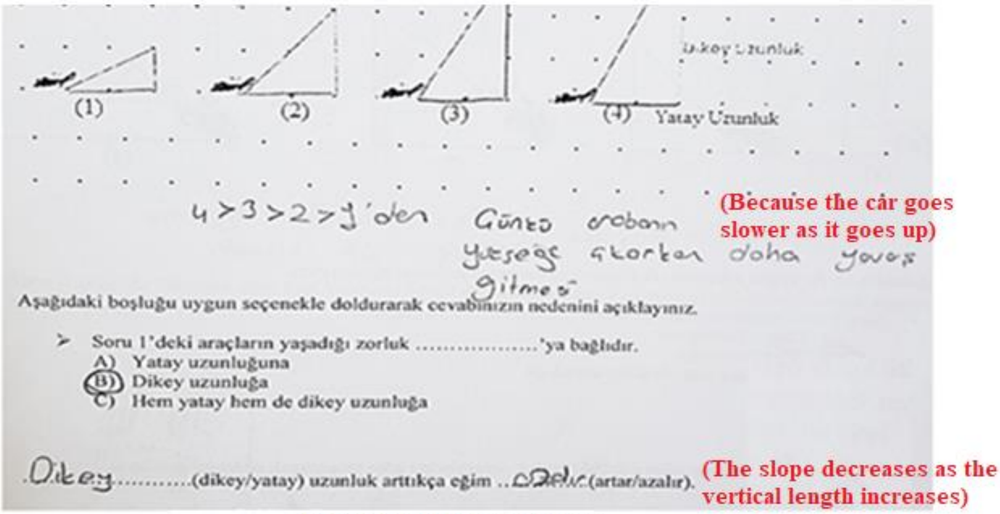


Figure 1: Response from S24

The contradictory expressions of the subject students can be interpreted as deficiencies in the knowledge of the meaning of the concept of slope. 12 (85.71%) of these 14 students had difficulty in answering the 4th question in AT (see figure 3). Therefore, it can be said that these students have different difficulties in making mathematical inferences about different visual shapes. This can be illustrated by the responses of the student coded as S_{10} (Figure 2 and Figure 3).

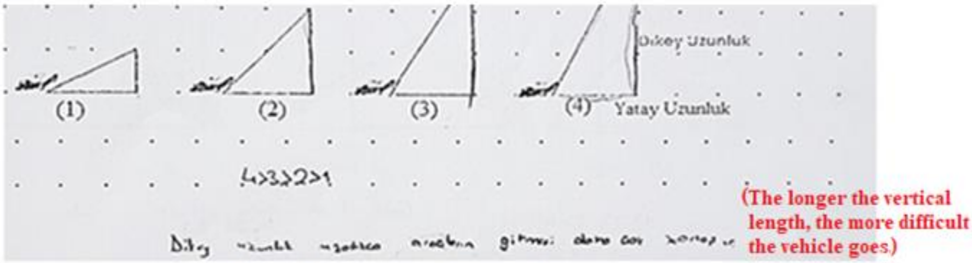


Figure 2: Response from S10 for the first question.

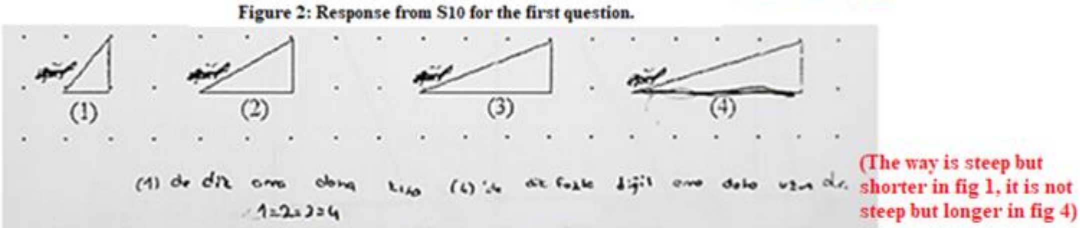


Figure 3: Response from S10 for the fourth question.

The other two students used incorrect and contradictory expressions in questions 5 or 6 even though they answered the 4th question correctly. Therefore, it can be said that these students could not make fully transition to special case 2.

When the answers in AP-2 category were examined, it was seen that 35 of these students (72.91%) gave the same answers. These students successfully completed special cases 1 and 2, but gave the wrong answers in special case 3 and made the same order for four different right triangles with equal slopes in question 7. Student expressions in this category were like as*, if vertical and horizontal lengths increase, the difficulty increases too* (9), *the more difficult the vertical length increases* (6), *the longer the length of the road, the more difficult the vehicle goes up* (11), *all have the same degree of steepness, but here the road is longer* (3). Some responses in this category are exemplified as follows (Figure 4 and Figure 5).

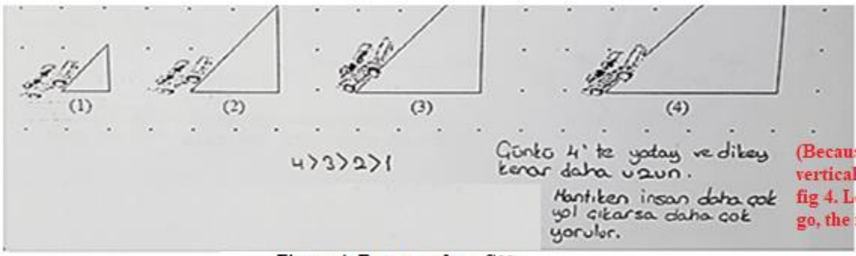


Figure 4: Response from S20.

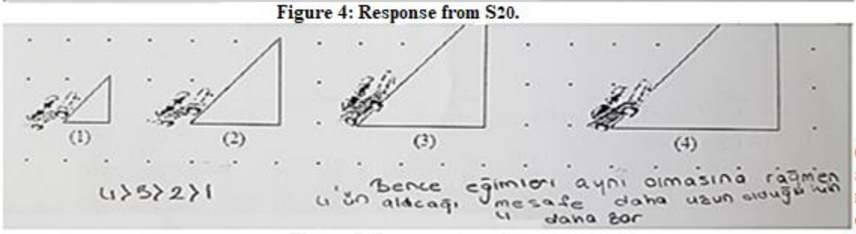


Figure 5: Response from S41.

6 students did not give any reason for their response. In this category, 5 students have made the opposite order (1> 2> 3> 4) for the question, and made the expressions like *the more the horizontal length decreases, the more difficult it is to come out* (S_8), *I think it comes out more easily* (S_{19}) *in long ways (meaning the length of the ramp), because the ramp in the first situation* (1) *is steeper* (S_{17}). In addition, three students stated that the slope was equal in all these situations (the students gave incorrect answers in either case 1 and case 2), while five students left this question unanswered. Therefore, it was seen that the majority of students in SP-2 category were successful in special cases 1 and 2 but could not make the correct inferences in special case 3 because they could not make the necessary mathematical connections belong to the situations. In particular, it can be said that most of the students in this category associate the difficulty of a car on a ramp with the distance that the car will take rather than the steepness of the road.

When AP-3 answers were examined, it was seen that eight students correctly interpreted the special cases 1, 2 and 3, and could make the right mathematical inferences, but they did not reach the required mathematical generalization in the last case. Although these students successfully completed all the exceptions and successfully completed the first three steps of the sequence of action, they could not complete the last step successfully and could not construct the targeted mathematical knowledge. It can be said that these students were able to interpret physical (apparently) information correctly in relation to real life, but had difficulty in expressing this information mathematically. Two of these students left the last question blank, five of them gave incorrect answers and one student could not calculate the slopes mathematically although he ordered them correctly. Incorrect responses can be illustrated as follows.

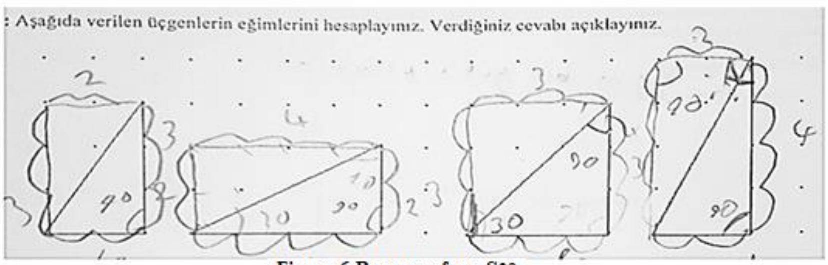


Figure 6:Response from S29.

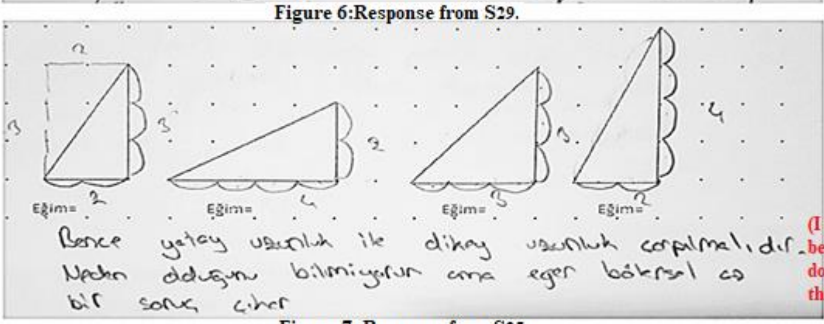


Figure 7: Response from S25.

In addition, seven students were able to accurately express the mathematical knowledge required in the last case, however, they could not make sense of the special case 3 and could not complete the action 3. The statements of some of these students as *I did but not sure* support this situation. Therefore, although it is possible to reach the mathematical knowledge targeted for these students, it can be said that their cognitive awareness is weak because their answers (for question 7 and question 8) are not consistent with each other. It was seen that three students in AP-4 category gave correct answers for all special situations and completed the knowledge construction process successfully. When the data obtained from abstraction performances is examined according to the action sequence, the following table is obtained.

Table 3

Data Belongs to The Action Sequence

Abstraction Level	Number of successful students	(%)
Action 1	70	76.92
Action 2	56	61.53
Action 3	14	15.38
Action 4	10	10.98

According to the data in Table 3, the actions in which the students were successful were action 1 (76.92%), action 2 (61.53%), action 3 (15.38%) and action 4 (3.29%), respectively. Therefore, it was seen that the majority of students were successful in special cases 1 and 2, while they had difficulty in special case 3 and very few could reach to special case 4.

Findings for the Second Sub-Problem

The table below presents the results of the correlation analysis results showing the relationship between the scores obtained from AT and the scores obtained from MAS, MAS*, MSS and MMS.

Table 4

Correlation Coefficients for the Relationships Between Abstraction Performances and Attitudes, Anxiety, Self-Efficacy and Motivation Variables Towards Mathematics.

	MAS*	MAS	MMS	MSS
AT	-.236*	.230*	.200	.244*

According to the data in Table 4, there is a positive and significant relationship between abstraction performances and attitudes and self-efficacy beliefs towards mathematics, and a negative and significant relationship between abstraction performances and anxiety levels of students. All of these relationships are low, but they are significant (Kalaycı, 2010).

RESULTS AND DISCUSSION

Empirical abstraction, which is a special type of mathematical abstraction, was investigated and related to some affective factors in this study and geometric form of slope concept is used in this process. Empirical abstraction is a process of knowledge construction which is very important especially for young children to learn basic mathematical concepts in abstract structure and these processes are mainly used in daily life scenarios (Mitchelmore & White, 2004). In this study, students' knowledge construction processes related to the concept of slope are examined through some scenarios established in relation to real life.

As a result of the study, it was seen that nearly half of the students successfully completed two steps of the action series. These students were able to predict that the slope of the vertical triangles with equal horizontal lengths was proportional to the vertical length and the slope of the vertical triangles with equal vertical lengths was inversely proportional to the horizontal length. However, in general, students (83.5%) could not give an idea about the slopes of the right triangles with equal horizontal and vertical lengths. The mentioned situation can be seen as the critical point of knowledge construction processes for the concept of slope (Aydın Çınar, 2019). Depending on this situation, while students can make accurate inferences from a single variable in interpreting real-life information and converting them into mathematical information, it can be said that they have difficulty in situations where multiple variables are simultaneously changed. The fact that the students cannot predict that the slope will not change for different triangles with equal slopes is accepted as an indicator of this situation. In different studies, while students can comment on a single variable in their mathematical thinking processes, they have difficulty in situations involving the simultaneous change of more than one variable (Kahraman, Kul, & Aydoğdu İskenderoğlu, 2019; Kertil, Erbaş, & Çetinkaya, 2017; Kocaman, 2017).

For the above-mentioned situation, it was also found that students were more successful in interpreting the effect of changes in vertical length on slope than in interpreting the effect of changes in horizontal length on slope. This situation can be interpreted as the students are more easily able to make mathematical inferences when there is a direct proportion between the variables, and they have difficulty in interpreting the situation with the variables in the presence of the inverse proportion. In different studies in the literature, it is stated that the students are more successful in using and interpreting the direct proportion compared to the situations involving the inverse proportion (Doğan & Çetin, 2009; Duatepe, Akkuş Çıkla, & Kayhan, 2005; Koçyiğit Gürbüz, 2018; Singh, 2000). In addition, it can be said that the students use proportional reasoning skills in the process of knowledge construction about the concept of slope. Similar studies (Cheng, 2010; Duncan & Chick, 2013) show that the ability to solve steepness problems for the concept of slope is related to proportional reasoning skills.

It was observed that the difficulties experienced by the students in the process which we call as the critical point of the knowledge construction process are cognitive difficulties related to the meaning of the concept of slope in general. In this process, it was noticed that student expressions were generally like as *slope increases as the length of the path increases* or *slope increases as the vertical length increases*. Similarly, Clement (1985) states that the students confuse the slope with the height, and that the slope of the line with a higher height would be greater. As a result of the study, only 16.5% of the students were able to demonstrate the critical behavior and predicted that the slope of the orthogonal triangles of different sizes with proportional lengths would be equal. Students who successfully completed all steps of the abstraction process constitute only 3.3% of all students in study group. Different studies also reveal that the basis of student difficulties for the concept of slope stems from cognitive situations (Chiu, Kessel, Moschkovich, & Munoz-Nunez, 2002; Stump, 2001; Zaslavsky, Sela, & Leron, 2002). Although students are introduced to the concept of slope from a very young age and encounter in many different areas throughout life, they have difficulty in understanding slope in both functional and physical situations (Bell & Janvier, 1981; Janvier, 1981; Orton, 1984; Simon & Blume, 1994; Stump, 2001). Studies show that students think slope as a ratio of two numbers (Barr, 1980,1981) and they have difficulty in mathematically structuring the concept of slope when they see it with different representations (Barr, 1981; Clement, 1985; Tabaghi, Mamolo, & Sinclair, 2009).

According to the results obtained from the second sub-problem of this study, it was found that there was a negative relationship between students' knowledge construction performance and mathematics anxiety, and there was a positive relationship between students' knowledge construction performance and mathematics attitudes and self-efficacy factors. Therefore, it can be said that students' perceptions of attitude, anxiety and self-efficacy towards mathematics are a significant predictor of their knowledge construction processes. Although there are no studies in the literature that examine knowledge construction processes with affective factors, there are many studies examining students' mathematical thinking processes and mathematics performances in relation to affective factors. Therefore, the discussion in this part of the study was conducted within the mentioned studies. Tekindal (2009) states that affective factors are an important component of the educational process, while Reynolds (2003) and Uysal (2007) demonstrate that affective factors affect cognitive performances such as problem solving, logical reasoning, and making connections. Kocaman (2017) found that there is a significant relationship between mathematical thinking skills and attitudes towards mathematics. Alkan and Bukova Güzel (2005) explained mathematical thinking as abstraction, estimation, generalization, hypothesis testing, reasoning, proving and describing with using mathematical concepts and knowledge that an individual has previously learned. Based on this definition, it can be said that the processes of knowledge construction require the use of mathematical thinking skills and the findings obtained from this study coincide with the results of Kocaman's (2017) study. As a different study conducted on the subject, Ateş (2016) revealed that eighth grade students' perceptions of self-efficacy, anxiety levels and attitudes towards mathematics caused differences on their math performance. In this study, as the attitude scores of the participants increased, their success scores increased too. Similarly, Tan (2015) found that there is a significant positive relationship between math achievement level and attitudes towards mathematics, a negative relationship between math achievement level and anxiety with learned helplessness. In addition, the factors affecting the mathematical performance of students in different studies were determined as self-efficacy (Akarsu, 2009; Alcı, 2007; Bandura, 1997; Kaya & Bozdağ, 2016; Pajares & Graham, 1999; Usher & Pajares, 2009; Yıldırım, 2011), reasoning skills (Brodie, 2010; Umay, 2003), attitude and interest (Doğan & Barış, 2010; Peker & Mirasyedioğlu, 2003; Yücel & Koç, 2011) and anxiety (İlhan & Öner Sünkür, 2013; Sad, Kıs, Demir, & Özer, 2016; Yıldırım, 2011). Therefore, it can be said that the results of this study are consistent with the literature and there are significant relationships between students' math performance and attitudes, anxiety and self-efficacy perceptions towards mathematics.

This study is valuable in that it relates to experimental abstraction processes that do not take much place in the literature and sheds light on these thinking processes of the students. However, the number of students in the study group can be shown as one of the limitations of the study. The current study can be carried out on larger samples and more comprehensive results can be achieved by the researchers in different studies.

SUGGESTIONS

The concept of slope is one of the concepts that students have difficulty in understanding and using in relation to real life. The conceptual knowledge of the slope includes understanding the relationships between the various slope representations (algebraic, geometric, trigonometric, and mathematical) seen at school and understanding that slope is a measure of the rate of change in the real world. In this context, when the study results are taken into consideration, it is thought that the courses planned to help the students to develop knowledge construction processes related to the concept of slope will support the learning processes positively and thus contribute to the permanent learning of the concepts. In addition, it is recommended that the students' affective as well as cognitive characteristics be taken into consideration in the mentioned processes and that the processes carried out are controlled together with these factors.

In addition, while there are many studies on deep (reflective) abstraction processes in the literature, it has been observed that the studies on empirical abstraction are very limited and most of the studies which could be reached by the researchers have theoretical structure generally. In this context, it is considered that there is a need for especially applied researches related to empirical abstraction processes in the literature and it is proposed to filling this gap in the literature with new and different studies.

Çıkar Çatışması Bildirimi
Yazar(lar), bu makalenin araştırılması, yazarlığı ve / veya yayınlanmasına ilişkin herhangi bir potansiyel çıkar çatışması beyan etmemiştir.

Destek/Finansman Bilgileri
Yazar(lar), bu makalenin araştırılması, yazarlığı ve / veya yayınlanması için herhangi bir finansal destek almamıştır.

REFERENCES

Akarsu, S. (2009). *A cross-national study on self-efficacy, motivation and Pısa 2003 mathematics literacy: Turkey and Finland*. (Unpublished master's thesis). Abant İzzet Baysal University, Bolu.

Alcı, B. (2007). *Yıldız Teknik Üniversitesi öğrencilerinin, matematik başarıları ile algıladıkları problem çözme becerileri, öz-yeterlik algıları, biliş üstü öz düzenleme stratejileri ve ÖSS sayısal puanları arasındaki açıklayıcı ve yordayıcı ilişkiler örüntüsü*. (Doctoral dissertation). Yıldız Technical University, İstanbul.

Alkan, H., & Bukova Güzel, E. (2005). Öğretmen adaylarında matematiksel düşünmenin gelişimi. *Gazi Üniversitesi Eğitim Fakültesi Dergisi, 25*(3), 221-236.

Altun, M., Arslan, Ç., & Yazgan, Y. (2004). Lise matematik ders kitaplarının kullanım şekli ve sıklığı üzerine bir çalışma. *Uludağ Üniversitesi Eğitim Fakültesi Dergisi, 17*(2), 131-147.

Ateş, F. (2016). *Assessment of the effects of math anxiety, attitude and self-efficacy beliefs on the 8th grade students' graph reading and interpretations* (Unpublished master's thesis). Kocatepe University, Afyon.

Ayan, A. (2014). *Ortaokul öğrencilerinin matematik öz yeterlik algıları, motivasyonları, kaygıları ve tutumları arasındaki ilişki* (Master's thesis). Balıkesir Üniversity, Balıkesir.

Aydın Çınar, B. (2019). *Examination of the abstraction process about slope knowledge of 8th grade students.* (Master's thesis). Balıkesir University, Balıkesir.

Bandura, A. (1997). *Self-Efficacy: The exercise of control.* New York, NY: W.H.

Barr, G. (1980). Graphs, gradients and intercepts. *Mathematics in School, 9*(1), 5-6.

Barr, G. (1981). Some student ideas on the concept of gradient. *Mathematics in School, 10*(1), 14–17.

Baykul, Y. (2009). *İlköğretimde matematik öğretimi 6-8. sınıflar* (1. Baskı). Ankara: Pegem Academy.

Bell, A., & Janvier, C. (1981). The interpretation of graphs representing situations. *For the Learning of Mathematics, 2*(1), 34-42.

Beth, E. W., & Piaget, J. (1966). *Mathematical epistemology and psychology.* Netherlands: D. Reidel Publishing Company.

Brodie, K. (2010). Pressing dilemmas: meaning-making and justification in mathematics teaching. *Journal of Curriculum Studies*, *42*(1), 27-50.

Büyüköztürk, Ş. (2018). *Sosyal bilimler için veri analizi el kitabı.* Ankara: Pegem.

Cheng, D. S. (2010). *Connecting proportionality and slope: Middle school students' reasoning about steepness.* (Unpublished Doctoral Dissertation). Boston University, Boston.

Chiu, M. M., Kessel, C., Moschkovich, J., & Muñoz-Nuñez, A. (2001). Learning to graph linear functions: A case study of conceptual change. *Cognition and Instruction, 19*(2), 215-252.

Clement, J. (1985). Misconceptions in graphing. In *Proceedings of the Ninth International Conference for The Psychology of Mathematics Education* (pp. 369-375). Utrecht, The Netherlands: Utrecht University.

Creswell, J. (2003). *Research design: Qualitative, quantitative and mixed methods approaches* (2nd ed.). Thousand Oaks, CA: SAGE Publications.

Creswell, J. W., & Plano Clark, V. L. (2007). *Designing and conducting mixed methods research.* Thousand Oaks, CA: Sage.

Demir, K. (2011). Tam öğrenme modeli. Demirel, Ö. (Ed.), *Eğitimde yeni yönelimler* (5. edt.) içinde (199-217). Ankara: Pegem Academy.

Deniz, Ö., & Kabael, T. (2017). 8th grade students' construction processes of the concept of slope. *Education and Science*, *42*(192), 139-172.

Dienes, Z. P. (1961). On abstraction and generalization. *Harvard Educational Review*, *31*(3), 281-301.

Doğan, A., & Çetin, İ. (2009). Doğru ve ters orantı konusundaki 7. ve 9. sınıf öğrencilerinin kavram yanılgıları. *Uşak Üniversitesi Sosyal Bilimler Dergisi*, *2*(2), 118-128.

Doğan, N., & Barış, F. (2010). Tutum, değer ve öz-yeterlik değişkenlerinin TIMSS-1999 ve TIMSS-2007 sınavlarında öğrencilerin matematik başarılarını yordama düzeyleri. *Eğitimde ve Psikolojide Ölçme ve Değerlendirme Dergisi*, *1*(1), 44-50.

Duatepe, A., & Çilesiz, Ş. (1999). Matematik tutum ölçeği geliştirilmesi. *Hacettepe University Journal of Education,* 16-17, 45-52.

Duatepe, A., Akkuş Çıkla, O., & Kayhan, M. (2005). Orantısal akıl yürütme gerektiren sorularda öğrencilerin kullandıkları çözüm stratejilerinin soru türlerine göre değişiminin incelenmesi. *Hacettepe University Journal of Education*, 28, 73-81.

Dubinsky, E. (1991). Constructive aspects of reflective abstraction in advanced mathematics. In: L.P. Steffe (Ed.), *Epistemological foundations of mathematical experience* (pp. 160–187). New York: Springer-Verlag.

Duncan, B., & Chick, H. L. (2013). How do adults perceive, analyse and measure slope? In *36th Annual Conference of the Mathematics Education Research Group of Australasia (MERGA 36)* (p. 258-265). Melbourne.

Erol, E. (1989). *Prevalence and correlates of math anxiety in Turkish high school students.* (Unpublished master's thesis). Bogazici University, İstanbul.

Fraenkel, J. R. & Wallen, N. E. (2006). *How to design and evaluate research in education*. New York: Mc Graw Hill.

Githua, B.N., & Mwangi, J.G. (2003). Students' mathematics self-concept and motivation to learn mathematics: relationship and gender differences among Kenya's secondary-school students in Nairobi and Rift Valley provinces. *International Journal of Educational Development*, *23*, 487-499.

Greene, J. C., Caracelli, V. J., & Graham, W. F. (1989). Toward a conceptual framework for mixed-method evaluation designs. *Educational Evaluation and Policy Analysis, 11*(3), 255-274.

Hershkowitz, R., Schwarz, B., & Dreyfus, T. (2001). Abstraction in contexts: epistemic actions. *Journal for Research in Mathematics Education*, *32*(2), 195-222.

İlhan, M., & Öner Sünkür, M. Ö. (2013). Investigation of predictive power of mathematics anxiety on mathematics achievement in terms of gender and class variables. *Gaziantep University Journal of Social Sciences, 12*(3), 427- 441.

Janvier, C. (1981). Use of situations in mathematics education. *Educational Studies in Mathematics*, *12*(1), 113-122.

Johnson, R. B., & Onwuegbuzie, A. J. (2004). Mixed methods research: A research paradigm whose time has come. *Educational Researcher*, *33*(7), 14-26.

Kahraman, H., Kul, E., & Aydoğdu-İskenderoğlu, T. (2019). Strategies Employed by 7th and 8th Graders for Quantitative Proportional Reasoning Problems. *Turkish Journal of Computer and Mathematics Education (TURCOMAT)*, *10*(1), 195-216.

Kalaycı, Ş. (Ed.) (2010). *SPSS uygulamalı çok değişkenli istatistik teknikleri*. Ankara: Asil Pub.

Kaya, D., & Bozdag, H. C. (2016). Resources of mathematics self-efficacy and perception of science self-efficacy as predictors of academic achievement. *European Journal of Contemporary Education*, *18*(4), 438-451.

Kertil, M., Erbaş, A. K., & Çetinkaya, B. (2017). Pre-service elementary mathematics teachers' ways of thinking about rate of change in the context of a modeling activity. *Turkish Journal of Computer and Mathematics Education (TURCOMAT)*, *8*(1), 188-217.

Kesici, A., & Aşılıoğlu, B. (2017). Ortaokul öğrencileri için stres ölçeğinin geliştirilmesi: Geçerlik ve güvenirlik çalışması. *Kastamonu Eğitim Dergisi*, *25*(6), 2413-2426.

Knuth, E. J. (2000). Student understanding of the Cartesian connection: An exploratory study. *Journal for Research in Mathematics Education, 31*(4), 500-508.

Kocaman, M. (2017). *Investigation of mathematical thinking and the reasoning skills of the 11th grade students.* (Master's thesis). Balıkesir University, Balıkesir.

Koçyiğit Gürbüz, M. (2018). *Investigation of the process of creating ratio-proportion concepts in the 7th grade students*' *activity based learning approach: APOS theory*. (Master's thesis). Eskişehir Osmangazi University, Eskişehir.

Mitchelmore, M., & White, P. (2004). Teaching mathematical concepts: Instruction for abstraction. Invited regular lecture presented at the *10th International Congress on Mathematical Education*. Copenhagen, Denmark.

Moschkovich, J., Schoenfeld, A. H., & Arcavi, A. (1993). Aspects of understanding: On multiple perspectives and representations of linear relations and connections among them. In T. A. Romberg, E. Fennema, & T. P. Carpenter (Eds.), *Integrating research on the graphical representation of functions* (pp. 69–100). Hillsdale, NJ: Lawrence Erlbaum.

Newburgh, R. (2001). An experiment to give physical meaning to the mathematical concept of slope. *Physics Education, 36* (4), 316-319.

Noss, R., & Hoyles, C. (1996). Windows on mathematical meanings: Learning cultures and computers. Dordrecht, The Netherlands: Kluwer Academic.

Ohlsson, S., & Lehtinen, E. (1997). Abstraction and the acquisition of complex ideas. *International Journal of Educational Research*, *27*(1), 37-48.

Orton, A. (1984). Understanding rate of change. *Mathematics in School*, *13*(5), 23-26.

Özmantar, M. F. (2005). *An investigation of the formation of mathematical abstractions through scaffolding.* (Doctoral dissertation). University of Leeds.

Pajares, F., & Graham, L. (1999). Self-efficacy, motivation constructs, and mathematics performance of entering middle school students. *Contemporary Educational Psychology*, *24*(2), 124-139.

Peker, M., & Mirasyedioğlu, Ş. (2003). Lise 2. sınıf öğrencilerinin matematik dersine yönelik tutumları ve başarıları arasındaki ilişki. *Pamukkale Üniversitesi Eğitim Fakültesi Dergisi,14*(14), 157-166.

Piaget, J. (1974). *Adaptation vitale et psychologie de l'intelligence [Vital adaptation and psychology of intelligence].* Paris: Hermann.

Piaget, J. (1985). *The equilibration of cognitive structures: The central problem of intellectual development.* Chicago: University of Chicago Press.

Piaget, J., & Garcia, R. (1983). *Psychogenese et histoire des sciences [Psychogenesis and the History of Science].* Paris: Flammarion.

Postelnicu, V. (2011). *Student difficulties with linearity and linear functions and teachers' understanding of student difficulties*. Arizona State University.

Reynolds, J. M. (2003). *The role of mathematics anxiety in mathematical motivation: A path analysis of the Cane model*. (Unpublished doctoral dissertation). University of Central Florida Orlando, Florida.

Russell, B. (1926). *Education and the good life*. New York: Liveright.

Sad, S. N., Kis, A., Demir, M., & Özer, N. (2016). Meta-analysis of the relationship between mathematics anxiety and mathematics achievement. *Pegem Eğitim ve Öğretim Dergisi, 6*(3), 371-392.

Simon, M. A., & Blume, G. W. (1994). Mathematical modeling as a component of understanding ratio-as-measure: A study of prospective elementary teachers. *Journal of Mathematical Behavior, 13*,183–197.

Singh, P. (2000). Understanding the Concepts of Proportion and Ratio among Grade Nine Students in Malaysia. *International Journal of Mathematical Education in Science and Technology*, 31, 579-599.

Skemp, R. (1986). *The psychology of learning mathematics*. (Second edition). Harmondsworth: Penguin.

Sopamena, P., Nusantara, T., Irawan, E. B., & Sisworo, S. (2016). Characteristics of thinking processes abstraction reflective of students in solving limits problems. *IOSR Journal of Research & Method in Education (IOSR-JRME)*, *6*(5), 27-35.

Stump, S. L. (2001). High school precalculus students' understanding of slope as measure. *School Science and Mathematics*, *101*(2), 81-89.

Tabaghi, S. G. Mamolo, A., & Sinclair, N. (2009). The effect of DGS on students' conception of slope. In proceedings of the *31st Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 5, pp. 226-234). Atlanta, GA: Georgia State University.

Tan, M. N. (2015). *The investigation of relationships among the levels of secondary school students' mathematics anxiety, learned helplessness and attitudes toward mathematics.* (Unpublished master's thesis). Necmettin Erbakan University, Konya.

Tashakkori, A., Teddlie, C., & Teddlie, C. B. (1998). Mixed methodology: Combining qualitative and quantitative approaches. In *Applied social research methods series* (Vol. 46). Thousand Oaks, CA: Sage.

Tekindal, S. (2009). *Duyuşsal özeliklerin ölçülmesi için araç oluşturma*. (Geliştirilmiş 2. Baskı). Ankara: Pegem Pub.

Umay, A. (2001). İlköğretim matematik öğretmenliği programının matematiğe karşı öz yeterlik algısına etkisi. *Journal of Qafqaz University*, *8*(1), 1-8.

Umay, A. (2003). Matematiksel muhakeme yeteneği. *Hacettepe University Journal of Education, 24*(24), 234-243.

Usher, E. L., & Pajares, F. (2009). Sources of self-efficacy in mathematics: A validation study. *Contemporary Educational Psychology*, *34(*1), 89-101.

Uysal, O. (2007). *Evaluation of relation between problem solving skills, anxiety and attitudes of secondary education grade ii students directed towards mathematics course.* (Master's thesis). Dokuz Eylül University, İzmir.

Van Oers, B. (2001). Contextualisation for abstraction. *Cognitive Science Quarterly*, *1*(3), 279-305.

Yıldırım, S. (2011). Self-efficacy, intrinsic motivation, anxiety and mathematics achievement: Findings from Turkey, Japan and Finland. *Necatibey Faculty of Education Journal of Electronic Science and Mathematics Education*, *5* (1), 277-291.

Yılmaz, R. (2011). *Visualization in mathematical abstraction and generalization processes and its role.* (Unpublished doctoral dissertation). Gazi University, Ankara.

Yücel, Z., & Koç, M. (2011). The relationship between the prediction level of elementary school students' math achievement by their math attitudes and gender. *Elementary Education Online*, *10*(1), 133-143.

Zaslavsky, O., Sela, H., & Leron, U. (2002). Being sloppy about slope: The effect of changing the scale. *Educational Studies in Mathematics*, *49*(1), 119-140.

İletişim/Correspondence
Dr. Lecturer Hayal YAVUZ MUMCU
hayalym52@gmail.com
Master student Aslıhan ÖZMEN
aslihanozmen1995@gmail.com