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Exponential Growth and Lower Bounds for the Blow-up Time of Solutions for a System of Kirchhoff-Type Equations

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ABSTRACT: This paper deals with the system of Kirchhoff-Type equations with a bounded domain $\Omega \subset R^n$. We prove exponential growth of solutions with negative initial energy. Later, we give some estimates for lower bounds of the blow up time.

Keywords — *Exponential growth, Lower bounds for the blow up time, Kirchhoff-Type equation*

1. Introduction

We consider the following the system of Kirchhoff-Type equations with weak and nonlinear damping terms

$$\begin{cases} u_{tt} - M(\|\nabla u\|^2)\Delta u + \gamma_2 u_t + |u_t|^p u_t = F_u(u,v), & (x,t) \in \Omega \times (0,T), \\ v_{tt} - M(\|\nabla v\|^2)\Delta v + \gamma_2 v_t + |v_t|^q v_t = F_v(u,v), & (x,t) \in \Omega \times (0,T), \\ u(x,t) = v(x,t) = 0, & (x,t) \in \partial\Omega \times (0,T), \\ u(x,0) = u_0(x),\ u_t(x,0) = u_1(x), & x \in \Omega, \\ v(x,0) = v_0(x),\ v_t(x,0) = v_1(x), & x \in \Omega, \end{cases} \tag{1.1}$$

where Ω is a bounded domain with smooth boundary $\partial\Omega$ in R^n $(n = 1,2,3)$; $p,q > 0$, $\gamma_2 > 0$. Let $\Delta = \sum\limits_{j=1}^{n} \frac{\partial^2}{\partial x_j^2}$ be the Laplace operator, and $M(s)$ be a nonnegative locally Lipschitz function, and $F : R^2 \to R$ is a C^1 function given by

$$F(u,v) = a|u+v|^{r+2} + 2b|uv|^{\frac{r+2}{2}}, \tag{1.2}$$

where $r \geq 2$, $a > 1$ and $b > 0$, which implies

$$\begin{aligned} F_u(u,v) &= (r+2)\left[a|u+v|^r(u+v) + b|u|^{\frac{r-2}{2}} u|v|^{\frac{r+2}{2}}\right], \\ F_v(u,v) &= (r+2)\left[a|u+v|^r(u+v) + b|v|^{\frac{r-2}{2}} v|u|^{\frac{r+2}{2}}\right]. \end{aligned}$$

Also, we have

$$uF_u(u,v) + vF_v(u,v) = (r+2)F(u,v) \quad \forall (u,v) \in R^2. \tag{1.3}$$

In the case of $M(s) = 1$, problem (1.1) becomes

$$\begin{cases} u_{tt} - \Delta u + \gamma_2 u_t + |u_t|^p u_t = F_u(u,v), \\ v_{tt} - \Delta v + \gamma_2 v_t + |v_t|^p v_t = F_v(u,v). \end{cases} \tag{1.4}$$

The system (1.4) was considered by some authors (see (Korpusov, 2012; Miranda and Medeiros, 1987; Pişkin, 2014a,1,1; Wu, 2012; Ye, 2013)). They studied the existence, blow up and decay of solutions. In addition to, in Agre and Rammaha (2006); Said-Houari (2010,1), authors studied the existence and the blow up of solutions (1.4) for $\gamma_2 = 0$.

In this study, we developed existing methods and applied them to system of Kirchhoff type equation with weak and nonlinear damping terms. Our results improved the results in the literature, see Peyravi (2017); Pişkin (2017). Our aim in this paper firstly is to find exponential growth and later lower bounds of the blow up time T^* for solutions of (1.1). The remaining of this paper is organized as follows: In the Section 2, we introduced some lemmas, notations and local existence theorem. In Section 3, we prove exponential growth of solutions. In the last section, we find lower bounds for the blow up time when the blow up occurs.

2. Preliminaries

In this section, we give some lemmas and assumptions which will be used along this work. $\|.\|$ and $\|.\|_p$ denotes the usual $L^2(\Omega)$ norm and $L^p(\Omega)$ norm, respectively.

We make the following assumptions:

(**A1**) $M(s)$ is a nonnegative C^1 function for $s \geq 0$ satisfying

$$M(s) = \alpha + \beta s^{\gamma},\ \gamma \geq 0,\ \alpha, \beta > 0.$$

(**A2**) Let

$$\begin{cases} p, q > 0,\ \ r > 0 \ \text{ if } n = 1, 2, \\ 0 < p, q \leq \frac{2}{n-2},\ \ r > 0 \ \text{ if } n = 3. \end{cases} \tag{2.1}$$

We define the energy functional as follows

$$\begin{aligned} E(t) &= \frac{1}{2}\left(\|u_t\|^2 + \|v_t\|^2\right) + \frac{\alpha}{2}(\|\nabla u\|^2 + \|\nabla v\|^2) \\ &\quad + \frac{\beta}{2(\gamma+1)}(\|\nabla u\|^{2(\gamma+1)} + \|\nabla v\|^{2(\gamma+1)}) - \int_{\Omega} F(u,v)\, dx. \end{aligned} \tag{2.2}$$

Lemma 2.1. *Messaoudi and Said-Houari (2010). There exist two positive constants c_0 and c_1 such that*

$$c_0\left(|u|^{r+2} + |v|^{r+2}\right) \leq F(u,v) \leq c_1\left(|u|^{r+2} + |v|^{r+2}\right) \tag{2.3}$$

is satisfied.

Lemma 2.2. *(Sobolev-Poincare inequality) Adams and Fournier (2003). Let q be a number with $2 \leq q < \infty$ $(n = 1, 2)$ or $2 \leq q \leq 2n/(n-2)$ $(n \geq 3)$, then there is a constant $C_* = C_*(\Omega,\ q)$ such that*

$$\|u\|_q \leq C_* \|\nabla u\| \ \text{ for } u \in H_0^1(\Omega).$$

Lemma 2.3. *Messaoudi (2001). Assume that*

$$p \leq 2\frac{n-1}{n-2},\ n \geq 3$$

holds. Then there exists a positive constant $C > 1$ depending on Ω only such that

$$\|u\|_p^s \leq C\left(\|\nabla u\|^2 + \|u\|_p^p\right)$$

for any $u \in H_0^1(\Omega)$, $2 \leq s \leq p$.

The next lemma shows that our energy functional (2.2) is a nonincreasing function along the solution of (1.1).

Lemma 2.4. *Assume that (A1) and (A2) hold. Then* $E(t)$ *is a non-increasing function for* $t \geq 0$ *and*

$$\frac{d}{dt}E(t) = -\gamma_2\left(\|u_t\|^2 + \|v_t\|^2\right) - \left(\|u_t\|_{p+2}^{p+2} + \|v_t\|_{q+2}^{q+2}\right). \tag{2.4}$$

Proof. We multiply the first equation of (1.1) by u_t, the second equation of (1.1) by v_t, and integrating over the domain Ω, we get

$$E(t) - E(0) = -\int_0^t \left[\gamma_2\left(\|u_\tau\|^2 + \|v_\tau\|^2\right) - \left(\|u_\tau\|_{p+2}^{p+2} + \|v_\tau\|_{q+2}^{q+2}\right)\right] d\tau \text{ for } t \geq 0. \tag{2.5}$$

□

Next, we state the local existence theorem that can be established by combining arguments in Georgiev and Todorova (1994); Pişkin (2015b); Taniguchi (2010).

Theorem 2.5. ***(****Local existence****).*** *Assume that* $\min\{p,q\} > r$ *such that*

$$\begin{cases} 0 < p,q, \ 0 < r, \ n = 1,2 \\ 0 < p,q \leq \frac{2}{n-2}, \ 0 < r, \ n \geq 3, \end{cases}$$

and let $(u_0,v_0) \in \left((H_0^1(\Omega) \cap H^2(\Omega)) \times (H_0^1(\Omega) \cap H^2(\Omega))\right)$, $(u_1,v_1) \in H_0^1(\Omega) \times H_0^1(\Omega)$ *be given. Then the problem (1.1) has a local solution*

$$u,v \in C\left([0,T); H_0^1(\Omega) \cap H^2(\Omega)\right) \text{ and } u_t, v_t \in C\left([0,T); H_0^1(\Omega)\right),$$

for any fixed time $T > 0$.

3. Exponential growth

In this part, we deal with the exponential growth results of the solution for the problem (1.1).

Theorem 3.1. *Assume that (A1) and (A2) hold. Let* $r > \max\{2\gamma, \ p, q\}$, $E(0) < 0$.*Then any the solution of the problem (1.1) grows exponentially.*

Proof. For this purpose, we set

$$H(t) = -E(t). \tag{3.1}$$

By using (2.2) and (3.1), we have

$$0 < H(0) \leq H(t). \tag{3.2}$$

We then define

$$\Psi(t)=H(t)+\varepsilon\left(\int_{\Omega}u_t u dx+\int_{\Omega}v_t v dx\right) \tag{3.3}$$

where ε small to be chosen later.

By taking a derivative of (3.3) and using Eq. (1.1), we have

$$\begin{aligned}\Psi'(t) &= \gamma_2\left(\|u_t\|^2+\|v_t\|^2\right)+\left(\|u_t\|_{p+2}^{p+2}+\|v_t\|_{q+2}^{q+2}\right)+\varepsilon\left(\int_{\Omega}|u_t|^2dx+\int_{\Omega}|v_t|^2dx\right)\\ &\quad-\varepsilon\alpha\left(\|\nabla u\|^2+\|\nabla v\|^2\right)-\varepsilon\beta(\|\nabla u\|^{2(\gamma+1)}+\|\nabla v\|^{2(\gamma+1)})+\varepsilon(r+2)\int_{\Omega}F(u,v)dx\\ &\quad-\varepsilon\gamma_2\left(\int_{\Omega}u_t u dx+\int_{\Omega}v_t v dx\right)-\varepsilon\left(\int_{\Omega}u_t u|u_t|^p dx+\int_{\Omega}v_t v|v_t|^q dx\right).\end{aligned} \tag{3.4}$$

From the definition of $H(t)$, it follows that

$$\begin{aligned}&-\beta(\|\nabla u\|^{2(\gamma+1)}+\|\nabla v\|^{2(\gamma+1)})\\ =\ &2(\gamma+1)H(t)+(\gamma+1)\left(\|u_t\|^2+\|v_t\|^2\right)\\ &+\alpha(\gamma+1)\left(\|\nabla u\|^2+\|\nabla v\|^2\right)-2(\gamma+1)\int_{\Omega}F(u,v)dx.\end{aligned} \tag{3.5}$$

Inserting (3.5) into (3.4) and using Young inequality to estimate the last two terms in (3.4), we get

$$\begin{aligned}\Psi'(t) &\geq (1-\varepsilon c(\delta))\left(\|u_t\|_{p+2}^{p+2}+\|v_t\|_{q+2}^{q+2}\right)+(\gamma_2+\varepsilon(\gamma+2))\left(\|u_t\|^2+\|v_t\|^2\right)\\ &\quad+\varepsilon\gamma\alpha\left(\|\nabla u\|^2+\|\nabla v\|^2\right)+2\varepsilon(\gamma+1)H(t)-\varepsilon\delta\left(\|u\|_{p+2}^{p+2}+\|v\|_{q+2}^{q+2}\right)\\ &\quad+\varepsilon(r-2\gamma)\int_{\Omega}F(u,v)dx-\frac{\varepsilon\gamma_2\mu}{2}\left(\|u\|^2+\|v\|^2\right)-\frac{\varepsilon\gamma_2}{2\mu}\left(\|u_t\|^2+\|v_t\|^2\right).\end{aligned} \tag{3.6}$$

By using Sobolev-Poincare's inequality, we have

$$\begin{aligned}\Psi'(t) &\geq (1-\varepsilon c(\delta))\left(\|u_t\|_{p+2}^{p+2}+\|v_t\|_{q+2}^{q+2}\right)+\left(\gamma_2+\varepsilon(\gamma+2-\frac{\gamma_2}{2\mu})\right)\left(\|u_t\|^2+\|v_t\|^2\right)\\ &\quad+\varepsilon\left(\gamma\alpha-\frac{\gamma_2\mu C_*}{2}\right)\left(\|\nabla u\|^2+\|\nabla v\|^2\right)+2\varepsilon(\gamma+1)H(t)\\ &\quad+\varepsilon(r-2\gamma)\int_{\Omega}F(u,v)dx-\varepsilon\delta\left(\|u\|_{p+2}^{p+2}+\|v\|_{q+2}^{q+2}\right).\end{aligned} \tag{3.7}$$

Since

$$r>\max\{p,q\},$$

we have

$$\begin{aligned}\|u\|_{p+2}^{p+2} &\leq C\|u\|_{r+2}^{p+2}\leq C\left(\|u\|_{r+2}+\|v\|_{r+2}\right)^{p+2},\\ \|v\|_{q+2}^{q+2} &\leq C\|u\|_{r+2}^{q+2}\leq C\left(\|u\|_{r+2}+\|v\|_{r+2}\right)^{q+2}.\end{aligned}$$

Thus

$$\begin{aligned}\Psi'(t) \geq\ & (1-\varepsilon c(\delta))\left(\|u_t\|_{p+2}^{p+2}+\|v_t\|_{q+2}^{q+2}\right)+\left(\gamma_2+\varepsilon(\gamma+2-\frac{\gamma_2}{2\mu})\right)\left(\|u_t\|^2+\|v_t\|^2\right)\\ &+\varepsilon\left(\gamma\alpha-\frac{\gamma_2\mu C_*}{2}\right)\left(\|\nabla u\|^2+\|\nabla v\|^2\right)+2\varepsilon(\gamma+1)H(t)\\ &+\varepsilon(r-2\gamma)\int_\Omega F(u,v)dx-\varepsilon\delta C\left(\|u\|_{r+2}+\|v\|_{r+2}\right)^{p+2}\\ &-\varepsilon\delta C\left(\|u\|_{r+2}+\|v\|_{r+2}\right)^{q+2}. \end{aligned} \tag{3.8}$$

Further, since

$$(a+b)^\lambda \leq C\left(a^\lambda+b^\lambda\right),\ \ a,b>0,$$

we have $2\leq p+2\leq r+2$, $2\leq q+2\leq r+2$ and using Lemma 2.3, we conclude that

$$\begin{aligned}\|u\|_{r+2}^{p+2} &\leq C(\|\nabla u\|^2+\|u\|_{r+2}^{r+2}),\\ \|v\|_{r+2}^{q+2} &\leq C(\|\nabla v\|^2+\|v\|_{r+2}^{r+2}).\end{aligned}$$

Therefore, using $c_0\left(|u|^{r+2}+|v|^{r+2}\right)\leq F(u,v)$ in (2.3), we have

$$\begin{aligned}\Psi'(t) \geq\ & 2\varepsilon(\gamma+1)H(t)+\left(\gamma_2+\varepsilon(\gamma+2-\frac{\gamma_2}{2\mu})\right)\left(\|u_t\|^2+\|v_t\|^2\right)\\ &+\varepsilon\left(c_0(r-2\gamma)-\varepsilon\delta C\right)\left(\|u\|_{r+2}^{r+2}+\|v\|_{r+2}^{r+2}\right)\\ &+\varepsilon\left(\gamma\alpha-\frac{\gamma_2\mu C_*}{2}-\varepsilon\delta C\right)\left(\|\nabla u\|^2+\|\nabla v\|^2\right)\end{aligned} \tag{3.9}$$

where $r>2\gamma$ is used.

$$c_0(r-2\gamma)-\varepsilon\delta C>\frac{c_0(r-2\gamma)}{2}$$

and

$$\gamma\alpha-\frac{\gamma_2\mu C_*}{2}-\varepsilon\delta C>\frac{\gamma\alpha}{2}-\frac{\gamma_2\mu C_*}{4}.$$

Therefore, we have

$$\begin{aligned}\Psi'(t) \geq\ & 2\varepsilon(\gamma+1)H(t)+\left(\gamma_2+\varepsilon(\gamma+2-\frac{\gamma_2}{2\mu})\right)\left(\|u_t\|^2+\|v_t\|^2\right)\\ &+\varepsilon\frac{c_0(r-2\gamma)}{2}\left(\|u\|_{r+2}^{r+2}+\|v\|_{r+2}^{r+2}\right)\\ &+\varepsilon\left(\frac{\gamma\alpha}{2}-\frac{\gamma_2\mu C_*}{4}\right)\left(\|\nabla u\|^2+\|\nabla v\|^2\right)\\ \geq\ & \tau\left(\|u_t\|^2+\|v_t\|^2+H(t)+\|\nabla u\|^2+\|\nabla v\|^2+\|u\|_{r+2}^{r+2}+\|v\|_{r+2}^{r+2}\right),\end{aligned} \tag{3.10}$$

where $\tau=\min\left\{\left(\gamma_2+\varepsilon(\gamma+2-\frac{\gamma_2}{2\mu})\right),2\varepsilon(\gamma+1),\varepsilon\frac{c_0(r-2\gamma)}{2},\varepsilon\left(\frac{\gamma\alpha}{2}-\frac{\gamma_2\mu C_*}{4}\right)\right\}$. Consequently we obtain

$$\Psi(t)\geq\Psi(0)>0,\ \forall t\geq 0. \tag{3.11}$$

We now estimate $\Psi(t) = H(t) + \varepsilon\left(\int_\Omega u_t u dx + \int_\Omega v_t v dx\right)$. Applying Young inequality, we obtain

$$\int_\Omega u u_t dx \le \frac{\mu}{2}\|u\|^2 + \frac{1}{2\mu}\|u_t\|^2, \qquad \int_\Omega v v_t dx \le \frac{\mu}{2}\|v\|^2 + \frac{1}{2\mu}\|v_t\|^2. \tag{3.12}$$

Then, it yields by using Poincare's inequalities

$$\Psi(t) \le C\left(\|u_t\|^2 + \|v_t\|^2 + H(t) + \|\nabla u\|^2 + \|\nabla v\|^2 + \|u\|_{r+2}^{r+2} + \|v\|_{r+2}^{r+2}\right). \tag{3.13}$$

By combining of (3.10) and (3.13) we arrive at

$$\Psi'(t) \ge r\Psi(t), \tag{3.14}$$

where r is a positive constant.

A simple integration of (3.14) over $(0,t)$ yields $\Psi(t) \ge \Psi(0)\exp(rt)$. □

4. Lower bounds of blow up time

In this part, we investigated the lower bounds of the blow up time T^* for the problem (1.1).

Theorem 4.1. *(see Pişkin (2015a)) Suppose that $r > \max\{2\gamma, p, q\}$, $E(0) < 0$, and there exists a constant τ such that $\tau \le \frac{2\alpha\gamma}{\gamma_2 C_*}$, where C_* is the constant of the Sobolev embedding theorem. Then the solution of this system blows up in finite time T^*.*

Firstly, we give following lemma that can be established by combining arguments of Peyravi (2017); Pişkin (2017).

Lemma 4.2. *There exist two positive c_1 and c_2 such that*

$$\begin{aligned} \int_\Omega |F_u(u,v)|^2 dx &\le c_1(\|\nabla u\|^2 + \|\nabla v\|^2)^{r+1}, \\ \int_\Omega |F_v(u,v)|^2 dx &\le c_2(\|\nabla u\|^2 + \|\nabla v\|^2)^{r+1} \end{aligned} \tag{4.1}$$

are satisfied.

Theorem 4.3. *Suppose that (A1), (2.1) hold and $(u_0,u_1),(v_0,v_1) \in (H_0^1(\Omega)\cap H^2(\Omega)) \times H_0^1(\Omega)$. Assume further that $0 < p,q < r$ and $\alpha = \beta = 1$. Then the finite blow-up time T^* satisfies the following estimate*

$$\int_{\phi(0)}^{\infty} \frac{d\tau}{(E(0)+\tau) + 2^{2r}(c_1+c_2)((E(0))^{r+1} + \tau^{r+1})} \le T^*$$

where $\phi(0) = \int_\Omega F(u(0),v(0))dx$ and the positive constants c_1 and c_2 are specified in (4.1).

Proof. Define

$$\phi(t) = \int_\Omega F(u,v)dx.$$

By differentiating $\phi(t)$ and using Young's inequality, we obtain

$$\begin{aligned}\phi'(t) &= \int_\Omega u_t F_u + v_t F_v dx \\ &\leq \frac{1}{2}\int_\Omega (u_t^2+v_t^2)dx + \frac{1}{2}\int_\Omega (F_u^2+F_v^2)dx. \end{aligned} \tag{4.2}$$

By the Lemma 4.2, we obtain

$$\phi'(t) \leq \frac{1}{2}\int_\Omega (u_t^2+v_t^2)dx + (\frac{c_1+c_2}{2})(\|\nabla u\|^2+\|\nabla v\|^2)^{r+1} \tag{4.3}$$

Therefore, from (2.2) and Lemma 2.4, we have

$$\begin{aligned}\int_\Omega (u_t^2+v_t^2)dx + (\|\nabla u\|^2+\|\nabla v\|^2) &\leq 2E(t)+2\int_\Omega F(u,v)dx \\ &\leq 2E(0)+2\int_\Omega F(u,v)dx \end{aligned} \tag{4.4}$$

Combining (4.3)-(4.4), we get

$$\begin{aligned}\phi'(t) &\leq \phi(t)+E(0)+2^r(c_1+c_2)\left[\phi(t)+E(0)\right]^{r+1} \\ &\leq \phi(t)+E(0)+2^{2r}(c_1+c_2)\left[(\phi(t))^{r+1}+(E(0))^{r+1}\right]. \end{aligned} \tag{4.5}$$

Integrating (4.5) from 0 to t, we have

$$\int_{\phi(0)}^{\phi(t)} \frac{d\tau}{(E(0)+\tau)+2^{2r}(c_1+c_2)((E(0))^{r+1}+\tau^{r+1})} \leq t.$$

Because of

$$\lim_{t\to T^{*-}} \phi(t) = \infty,$$

we can write

$$\int_{\phi(0)}^{\infty} \frac{d\tau}{(E(0)+\tau)+2^{2r}(c_1+c_2)((E(0))^{r+1}+\tau^{r+1})} \leq T^*.$$

Thus, we obtain the desired result. □

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