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Modeling and Optimal Trajectory Tracking Control of Wheeled a Mobile Robot

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Abstract: Mobile robots have an unlimited workspace, unlike conventional fixed to the robot. Therefore, they are frequently studied from past to present. In this study, it is aimed to model wheeled a mobile robot(WMR) and realize optimal trajectory tracking control. Mathematical model of the robot was obtained. The Linear Quadratic Regulator (LQR) method, one of the optimum control methods for controlling the robot has been proposed. The Q and R parameters affecting the performance of the proposed control method were obtained by using the Firefly optimization algorithm. Both process noise and measurement noise have been added to control the robot in conditions close to the actual ambient conditions. As a result, in order to demonstrate the validity of the obtained model and the proposed control method, the robot was performed control in the simulation environment. The obtained results were given graphically and the results were examined.

Keywords: Wheeled Mobile Robot(WMR), Mathematic Model, Optimal Trajectory Tracking, Lineer Quadratic Regulator(LQR), Firefly Algorithm.

1. INTRODUCTION

Mobile robots have a wide range of application thanks to having an unlimited work area. Mobile robots can be used in many fields such as industry, space, military and social needs (vacuum cleaners and lawnmowers, etc.), which make people's lives easier, for entertainment and other purposes. In the control of mobile robots, the focus is generally on two main targets. These are a stable posture stabilization and trajectory tracking controller. The purpose of posture stabilization is to immobilize the robot to a reference point, the purpose of trajectory tracking is to allow the robot to follow a reference trajectory. In the study conducted in 1983, Brockett stated an opinion regarding whether nonholonomic mobile robot systems could be controlled through a smooth state feedback control (Brockett, 1983).

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Hamel et al. proposed a control method resistant to localization errors of mobile robots (Hamel and Dominique, 1996). In another method proposed in the literature, a technical tracking controller with recoil recursion was used (Jiang and Nijmeijer, 1999; Fu et al., 2013). Xin et al. designed a disturbance observer and an adaptive stabilizer in order to cope with the uncertainties of wheeled mobile robots (Xin et al., 2016). Canale et al. handled the problem of the rapid implementation of a nonlinear predictive control model with function approach techniques (Canale et al., 2010). Bessas et al. proposed the integral sliding mode control method in order to solve the problem of accessing the sliding surface used in the sliding mode control method, and to enable an effective trajectory tracking control (Bessas et al., 2016). Wu and Karkoub proposed the method of hierarchical fuzzy sliding mode adaptive control for trajectory tracking of differentially driven mobile robots (Wu and Karkoub, 2019). In the literature, various studies on mobile robots have been conducted and continue to be conducted by researchers (Yang and Kim, 1999; Kara et al., 1999; Wu et al., 2019, Tian and Sarkar, 2014; Li et al., 2015, Atan, 2019). In this study, it was aimed to model a wheeled mobile robot (WMR) and to perform optimum tracking control. The mathematical model of the robot was obtained. The Linear Quadratic Regulator (LQR) method, which is one of the optimum control methods, was proposed to control the robot. Both process noise and measurement noise were added to control the robot under the conditions close to real environment conditions. As a result, to demonstrate the validity of the obtained model and the proposed control method, the control of the robot was performed in the simulation environment. The results obtained were graphically given and examined. A second-order low-pass filter was designed to improve control performance of the robot under the conditions close to real environment conditions. The control methods applied according to the results of the obtained simulation environment were compared and the results were examined. The two-dimensional general representation of the wheeled mobile robot (WMR) is shown in Figure 1.

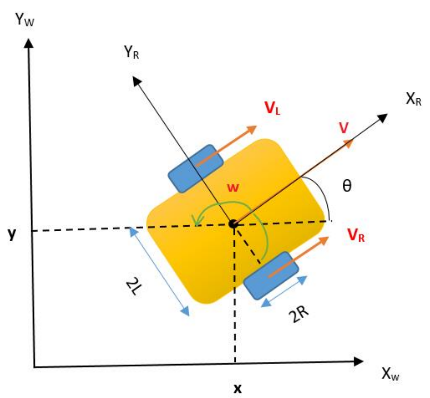


Figure 1. Two-dimensional representation of the wheeled mobile robot (WMR)

2. SYSTEM PREVIEW AND MODELING

In the literature, mobile robots are studied depending on different wheel designs and types. The robot used in this study consists of two independent wheels on the right and left. Generally, the motion of the system is performed by controlling the angular velocities of the dc motors connected to these wheels. Changing the orientations by moving on a curved trajectory or turning around by adjusting the angular velocities of two driving wheels is only one of the abilities of WMRs. To apply high-performance controllers in the control of a robot, the robot must be modeled. On a mobile robot with a differential drive, it is possible to apply the kinematic control approach provided that suitable conditions without sliding are selected. The kinematic model used for a two-wheeled mobile robot in Figure 1 is based on the assumption that the wheels move without sliding. Accordingly, the velocity references given to the actuators on the wheels enable the robot to move at linear and angular velocities corresponding to these references. In Figure 1, the robot coordinate framework was accepted as the center of mass of WMR located on the point C and used as the origin of X_R and Y_R. The robot used in this study has two control variables; these are the angular velocities of the right and left wheels. In Figure 1, the linear velocity of the left wheel is shown with V_L and linear velocity of the right wheel is shown with V_R. Similarly, the angular velocities of the left and right wheels are ω_L and ω_R, respectively. R is the radius of the wheel, 2L is the distance between the wheels and 2L is the distance between the endpoint of the robot and point C which is the geometric center of the robot. The orientation angle of the mobile robot according to the X_W-Y_W coordinate axis is θ. The following equations were obtained for linear and angular velocities

$$V = \frac{V_R + V_L}{2} \tag{1}$$

$$V = \omega * R \tag{2}$$

$$V_R = \omega_R * R, V_L = \omega_L * R \tag{3}$$

$$\omega_R = \frac{V_R}{R - L} \tag{4}$$

$$\omega_L = \frac{V_L}{R + L} \tag{5}$$

State equations of the mobile robot according to the X_W-Y_W coordinate axis were expressed as follows.

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix}=\begin{bmatrix}cos\theta & 0\\ \sin\theta & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}v\\ \omega\end{bmatrix} \quad (6)$$

In this study, the method of Lagrange multipliers was used to obtain the mathematical model of WMR (Bertsekas,1999).

$$M(q_m)\ddot{q}_m + C(q_m,\dot{q}_m)\dot{q}_m + G_m(q_m) + \tau_d = B_m(q_m)\tau - A^T(q_m)\Lambda \quad (7)$$

$q_m(x, y, \theta)$ indicates the position and orientation angle in X and Y directions, respectively. $M(q_m) \in R^{3*3}$ is a positive definite symmetric matrix and shows inertia matrix ; $C(q_m) \in R^{3*3}$ shows Coriolis and centrifugal forces matrix; $G(q_m) \in R^{3*3}$ indicates the forces of gravity. In addition, $B(q_m), A(q_m), \tau\ ve\ \Lambda$ shows the input matrix, kinematic constraint matrix, input vector and Lagrange multiplier. Table I is shown physical parameter of WMR.

3. CONTROLLER DESIGN

The main objective for designing the control system of the mobile robot is stability and low tracking error. In the control of WMR, Linear Quadratic Regulator (LQR) control method was used. The aim of the control methods used is to ensure that the output value of the system tracks the targeted (reference) value. Error is tried to be minimized with the controller applied to the system.

Table 1. Physical Parameter of WMR

Description and symbol	Units and value
Mass of car (m)	1 (kg)
Wheels Distance (L)	0.28 m
Radius of Wheels (R)	0.143 m
Inertia (I)	0.15 g-m^2

3.1. LQR (Linear Quadratic Regulator) Control Method

LQR control method is a modern control method that is used to control a system. This control method is widely used in the literature in optimal control problems (Anderson 2007; Abut,2016). The purpose of the control here is to minimize the integral of the quadratic performance index. In Figure 2, a block diagram of the linear quadratic regulator (LQR) control method is shown.

$$u = -K * x \quad (8)$$

$$J = \frac{1}{2}\int_0^{\infty}(x^T(t)Qx + u^T Ru)\,dt \quad (9)$$

The equation 9 is a function as given. Selecting the parameters of this function in a way to make the function minimum or maximum optimizes the control system. The value of the function indicates to what extent the actual performance of the system corresponds with the desired performance. In other words, performance index is a measure showing the deviation from the ideal performance. This index can be the integral of an error function that needs to be minimized. Optimum performance is nearly achieved through the minimization of the error integral. The main control problem in engineering is to determine the optimal control law that minimizes the performance index given under various safety and economic limitations. In the classical linear optimal control, the control vector u (t) is selected in a way that the performance index becomes the minimum. The performance index selected in the system control is generally quadratic according to both x (t) and u (t). The total expression where the matrices Q and R are located is desired to be minimum. This means the minimization of the equation 11. Here, the matrices Q and R are called weight matrices, and Q is a positive matrix in [2Nx2N] dimension while R is a positive matrix in [mxm] dimension. Q is a positive semi-definite symmetric matrix and R is a positive definite number ($Q \geq 0, R > 0$). The optimal feedback gain matrix K is calculated with the following equation:

$$K = T^{-1}(T^{-1})^{-1}B^T = R^{-1}B^T P \quad (10)$$

The value of the positive definite matrix P is calculated by using the Riccati equation.

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \quad (11)$$

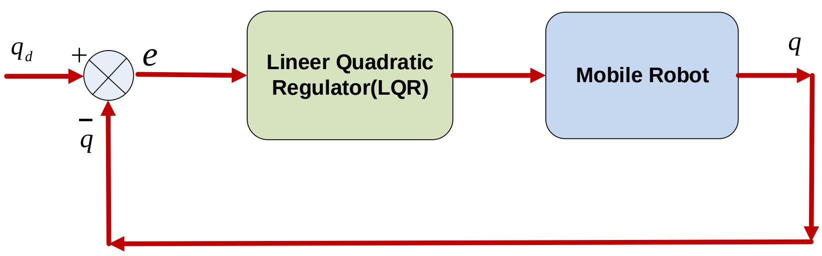


Figure 2. Controller structure of the system

3.2. Firefly Optimization Algorithm

Various methods are applied to design and control the systems at optimal values. In this context, Firefly Optimization Algorithm, which is one of the metaheuristic optimization algorithm types and is actively researched in recent years, is an algorithm type developed by Yang (Yang ,2010; Yang, et al.,2013). This algorithm is used for the optimization of various problems (Olivares, et al.,2014; Patle, et al.,2017; Patle, et al.,2018; Lagunes, et al.,2019). The Firefly Algorithm inspired by the flashing patterns of fireflies in nature is based on the principle of moving to a random direction. According to the level of brightness, fireflies can affect the opposite sex or may scare the predators depending on the speed of flashing. They also use these biological flashing activities to attract their preys. Certain assumptions have been made in this optimization algorithm.

1) All fireflies are accepted as unisexual, so they are attracted to each other independently.

2) Attractiveness is determined by brightness; a less bright firefly moves toward a more bright one.

3) The brightness (objective) function of a firefly is proportional to the fitness function that produces the brightest value.

In this method, there are two important parameters. One of them is the change in the light intensity and the other is the attractiveness of the firefly. In a simple form, according to the inverse-square law, the change of the light intensity obtained at a distance of r from a light source is given in the equation 12.

$$I_0(x) = \frac{\beta}{r^2} \qquad (12)$$

This equation is based on the initial light intensity (I_0), constant absorption coefficient of the light (γ) and distance (r). β_0 indicates the attractiveness when the distance between a firefly and other adjacent firefly is x=0. $\beta(r)$ indicates the attractiveness amount of a firefly at a distance of x and it is expressed in the equation 13.

$$\beta(r) = B_0 e^{-\gamma r^2} \qquad (13)$$

When the distance (xi, xj) between any two fireflies on cartesian coordinates is taken respectively, the distance between fireflies can be calculated by using the equation 14.

$$r_{ij} = \| x_i - x_j \| = \sqrt{\sum_{k=1}^{d} (x_i - x_j)^2} \qquad (14)$$

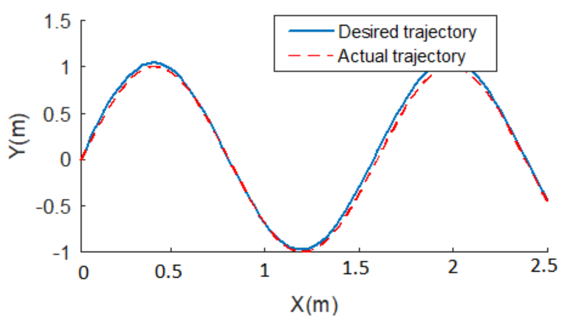
The distance between fireflies, for example, the distance between the ith firefly and jth firefly, can be determined by using the equation 15. Distance is important since it affects light intensity and attractiveness and determines the direction of fireflies.

$$x_i = x_i + B_0 e^{-\gamma r^2}(x_i - x_j) + \alpha \varepsilon_i \qquad (15)$$

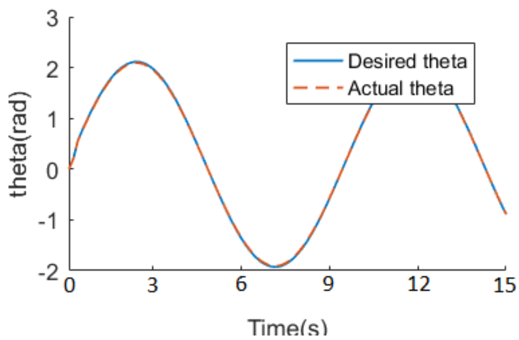
The first term in the right of the equation 15 indicates the current position of the firefly, the second term establishes a relation between the light intensity seen by the adjacent fireflies and attractiveness of the current firefly and the last term represents a random movement to be made when there is not a more attractive firefly around the current firefly. α indicates the coefficient taking a constant value in the range of random parameter [0,1] and ε_i shows a Gauss distribution vector drawn with random numbers in the range of [0,1].

4. SIMULATION RESULTS

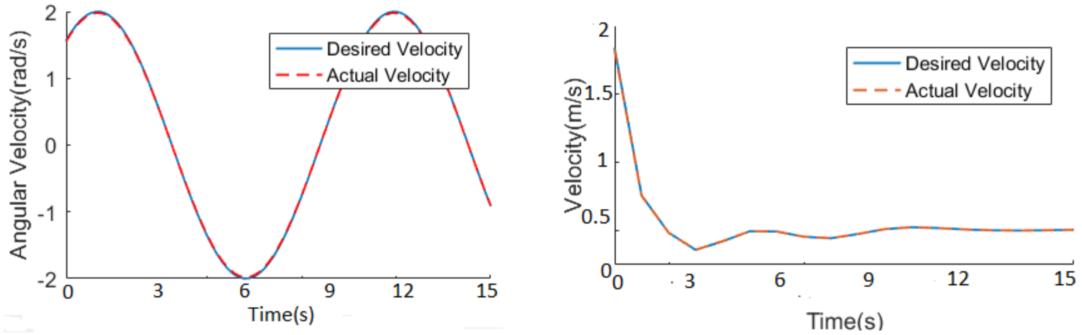
In this section, simulation studies were conducted by using the obtained model equations of WMR. In this section, the performance values of the control method were given graphically. The performance of the Linear Quadratic Regulator (LQR) control method was tested on different trajectories. The control variables of the system X and Y trajectories are the orientation angle values. Figures 3 and 4 show the responses of the WMRs to the method applied for the control of the sinusoidal and randomly created trajectories. The simulation run time was accepted as 15 seconds. The convergence performance of the Firefly Optimization Algorithm is shown in the graph given in Figure 5.



a)

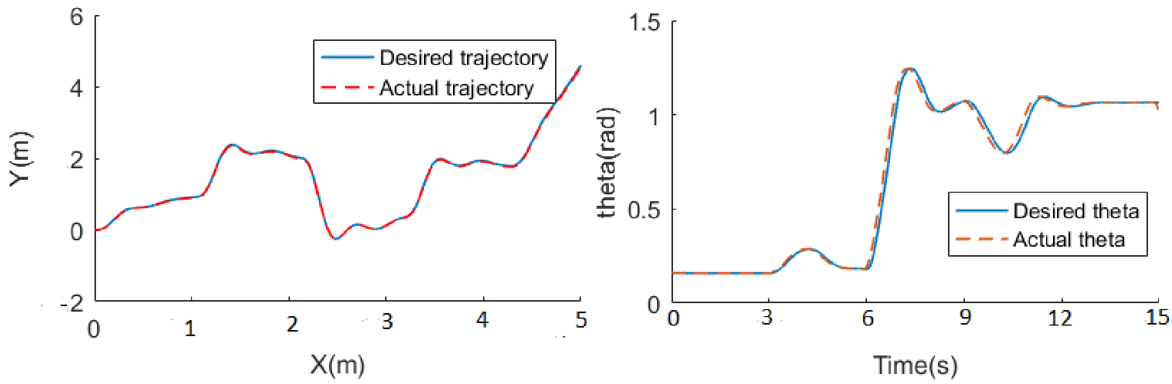


b)

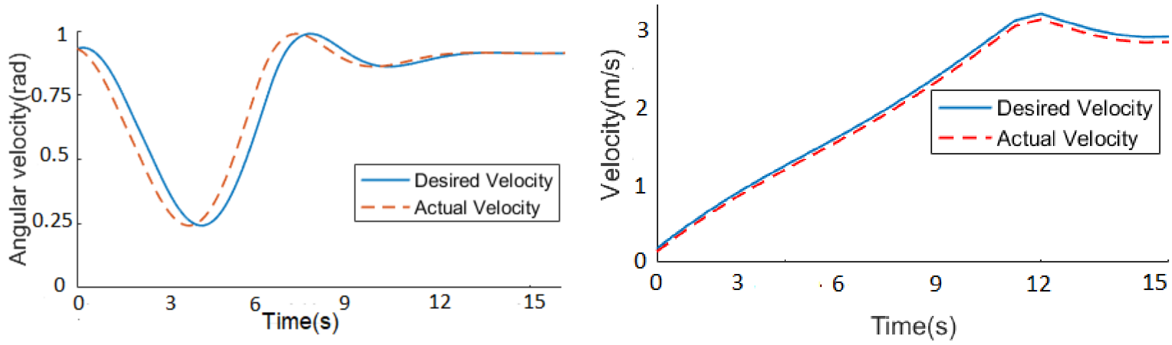


c) d)

Figure 3. a- Trajectory, b- Orientation angle (θ), c- Angular velocity and d- Linear velocity control results



a) b)



c) d)

Figure. 4. a- Trajectory, b- Orientation angle (θ), c- Angular velocity and d- Linear velocity control results,

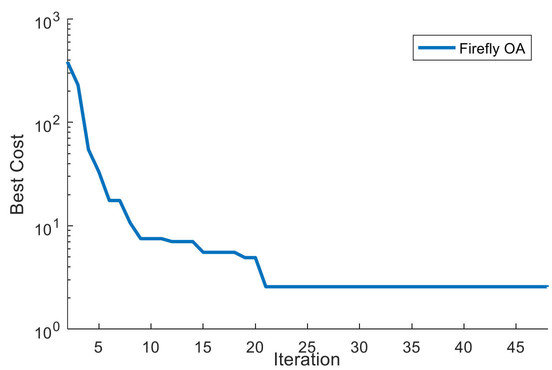


Figure 5. Performance analysis of the proposed algorithm

Figures 3-a, b, c, and d show trajectory, orientation angle, angular velocity, and linear velocity responses, respectively. Figures 4-a, b, c, and d show trajectory, orientation angle, angular velocity, and linear velocity responses, respectively. For the trajectory tracking control of WMR, both kinematic and dynamic models were considered. In the section of kinematic control, the position deviations in the target trajectory were eliminated and accordingly, the desired velocity was generated as output in the dynamic controller. The tracking error of LQR was observed to be low on both trajectories. System iteration number was taken as 50. However, it was observed that the algorithm proposed in the graph given in Figure 5 reached the best solution in the 21st iteration. Another important parameter is that the algorithm used in this study is fast. In the simulations, it was seen that the control performance showing fastness, smoothness and robustness was obtained in the LQR control method.

5. DISCUSSION AND CONCLUSION

In this study, the mathematical model of a wheeled mobile robot (WMR) was obtained and control studies were conducted in the simulation environment. For the control of WMR, the design and simulation of LQR control method were performed. Determining the matrices Q and R when designing an LQR control method is one of the main problems that decrease performance. By using the Firefly Optimization Algorithm, optimum matrices Q and R were obtained and applied successfully. The second-order low-pass filter design was made and applied to enable an effective control under the effect of process and measurement noises added to perform the control of WMR under the conditions close to real environment conditions. The results of the control method showed that the controller gave satisfactory results. In future studies, it is aimed to apply the proposed method on a real robot

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