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New Exact Solutions of (3+1)-Dimensional Modified Quantum Zakharov-Kuznetsov Equation

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Abstract. In this work, Generalized Kudryashov method (GKM) has been used to obtain exact solutions of (3+1)-dimensional modified quantum Zakharov-Kuznetsov (MQZK) equation. Dark optical soliton solutions of this equation have been found by using this method. Also, the graphical demonstrations clearly display strongness of this method.

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1. Introduction

Nonlinear partial differential equations (NLPDEs) are mostly utilized to clarify great numbers of physical facts in the fields such as quantum field theory, biology, hydrodynamics, meteorology, optical fibers.

Recently, many scientists have developed several methods to obtain exact solutions of NLPDEs such as generalized (G'/G) method [1], Hirota bilinear method [8], generalized simplest equation method [2], extended trial equation method [5], and so on. In this study, GKM [6, 7] will be implemented to reach exact solutions of (3+1)-dimensional MQZK equation.

We submit (3+1)-dimensional modified QZK equation [3]

$$w_t + pw_x w^3 + qw_{zzz} + rw_{xxz} + sw_{yyz} = 0, \tag{1.1}$$

where p, q, r and s are real-valued constants. Here, the behaviour of the weakly nonlinear ion acoustic waves in the structure of an uniform magnetic field is controlled by the QZK equation [4].

In present work, we intend to find exact solutions of (3+1)-dimensional MQZK equation. Thereinafter, we give basic facts of GKM. Lastly, as an application, we obtain exact solutions of (3+1)-dimensional MQZK equation via GKM.

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2. Basic Facts of GKM

We handle the following NLPDE

$$K(w, w_t, w_x, w_y, w_z, w_{xt}, w_{tt}, w_{xx}, w_{yy}, w_{zz}, \cdots) = 0. \tag{2.1}$$

Step 1: Getting the transformation as

$$w(x, y, z, t) = w(\sigma), \sigma = ax + by + cz - dt,$$

Eq. (2.1) is turned into the following nonlinear equation:

$$L(w, w^{'}, w^{''}, w^{'''}, \cdots) = 0. \tag{2.2}$$

Step 2: Taking the following equation for Eq. (2.2) as solution:

$$w(\sigma) = \frac{\sum_{i=0}^{l} a_i \theta^i(\sigma)}{\sum_{j=0}^{n} b_j \theta^j(\sigma)} = \frac{P[\theta(\sigma)]}{Q[\theta(\sigma)]}, \tag{2.3}$$

where $\theta = \frac{1}{1 \pm e^{\sigma}}$ and, $\theta = \theta(\sigma)$ described as;

$$\theta_\sigma = \theta^2 - \theta. \tag{2.4}$$

By taking into consideration Eq. (2.3), we have

$$w^{'}(\sigma) = \frac{P^{'}\theta^{'}Q - PQ^{'}\theta^{'}}{Q^2} = \theta^{'}\left[\frac{P^{'}Q - PQ^{'}}{Q^2}\right] = (\theta^2 - \theta)\left[\frac{P^{'}Q - PQ^{'}}{Q^2}\right],$$

$$w^{''}(\sigma) = \frac{\theta^2 - \theta}{Q^2}\left[(2\theta - 1)(P^{'}Q - PQ^{'} + \frac{\theta^2 - \theta}{Q}\left[Q(P^{''}Q - PQ^{''}) - 2Q^{'}P^{'}Q + 2P(Q^{'})^2\right]\right].$$

Step 3: Taking the following equation for Eq. (2.2) as solution:

$$w(\sigma) = \frac{a_0 + a_1\theta + a_2\theta^2 + \cdots + a_l\theta^l + \cdots}{b_0 + b_1\theta + b_2\theta^2 + \cdots + b_n\theta^n + \cdots}. \tag{2.5}$$

To compute the values n and l in Eq.(2.5), by use of balance principle in Eq. (2.2), we can constitute a relation between n and l . Thus, we can derive some values of n and l .

Step 4: Setting Eq. (2.3) and Eq. (2.4) into Eq. (2.2), a system of $R(\theta)$ can be obtained. We solve this system by using Mathematica to identify the coefficients $a_i, b_j, (0 \le i \le l, 0 \le j \le n), c$.

3. Application of GKM to (3+1)-Dimensional MQZK Equation

Getting the transformation as

$$w(x, y, z, t) = w(\sigma), \sigma = ax + by + cz - dt,$$

Eq. (1.1) demeans

$$-dw^{'} + ap(w^3)^{'} + (c^3q + a^2cr + b^2cs)w^{'''} = 0. \tag{3.1}$$

By integrating Eq. (3.1), we get

$$-dw + ap(w^3) + (c^3q + a^2cr + b^2cs)w^{''} = 0. \tag{3.2}$$

By use of balance principle in Eq. (3.2), we obtain

$$l = n + 1.$$

If we take $n = 1$ so that $l = 2$, we have

$$w(\sigma) = \frac{a_0 + a_1\theta + a_2\theta^2}{b_0 + b_1\theta}, \tag{3.3}$$

$$w^{'} = (\theta^2 - \theta)\left[\frac{(a_1 + 2a_2\theta)(b_0 + b_1\theta) - b_1(a_0 + a_1\theta + a_2\theta^2)}{(b_0 + b_1\theta)^2}\right],$$

$$w^{''}(\sigma) = \frac{\theta^2 - \theta}{(b_0 + b_1\theta)^2}(2\theta - 1)\left[(a_1 + 2a_2\theta)(b_0 + b_1\theta) - b_1(a_0 + a_1\theta + a_2\theta^2)\right]$$
$$+ \frac{(\theta^2 - \theta)^2}{(b_0 + b_1\theta)^3}\left[2a_2(b_0 + b_1\theta)^2 - 2b_1(a_1 + 2a_2\theta)(b_0 + b_1\theta) + 2b_1^2(a_0 + a_1\theta + a_2\theta^2)\right].$$

Thus, a system of $R(\theta)$ can be obtained. We solve this system by using Mathematica to identify the coefficients $a_i, b_j, (0 \le i \le l, 0 \le j \le n), c$.

Case1

$$a_0 = -\frac{a_2 b_0}{2b_1}, a_1 = \frac{1}{2}a_2\left(-1+\frac{2b_0}{b_1}\right), d = -\frac{1}{2}c(c^2q+a^2r+b^2s), p = \frac{2c(c^2q+a^2r+b^2s)b_1^2}{aa_2^2}. \tag{3.4}$$

If we substitute Eq. (3.4) into Eq. (3.3), we find dark optical soliton solution of Eq. (1.1)

$$w_1(x,y,z,t) = -\frac{a_2}{2b_1}\tanh\left[Dx+Ey+Fz+\frac{c(c^2q+a^2r+b^2s)t}{4}\right], \tag{3.5}$$

where $D = \frac{a}{2}, E = \frac{b}{2}, F = \frac{c}{2}$.

Case2

$$a_0 = 0, a_1 = -a_2, b_1 = -2b_0, d = c(c^2q+a^2r+b^2s), p = \frac{-8c(c^2q+a^2r+b^2s)b_0^2}{aa_2^2}. \tag{3.6}$$

According to Eq.(3.6), we obtain dark optical soliton solution of Eq.(1.1)

$$w_2(x,y,z,t) = \frac{-a_2}{4b_0}\left(\coth\left[f(x,y,z,t)\right]-\tanh\left[f(x,y,z,t)\right]\right), \tag{3.7}$$

where $f(x,y,z,t) = Dx+Ey+Fz-\frac{c(c^2q+a^2r+b^2s)t}{2}$.

Case3

$$a_0 = \frac{a_2}{2}, a_1 = -a_2, b_1 = -2b_0, d = -2c(c^2q+a^2r+b^2s), p = \frac{-8c(c^2q+a^2r+b^2s)b_0^2}{aa_2^2}. \tag{3.8}$$

According to Eq.(3.8), we get dark optical soliton solution of Eq.(1.1)

$$w_3(x,y,z,t) = \frac{a_2}{4b_0}\left(\coth\left[g(x,y,z,t)\right]+\tanh\left[g(x,y,z,t)\right]\right),$$

where $g(x,y,z,t) = Dx+Ey+Fz+c(c^2q+a^2r+b^2s)t$.

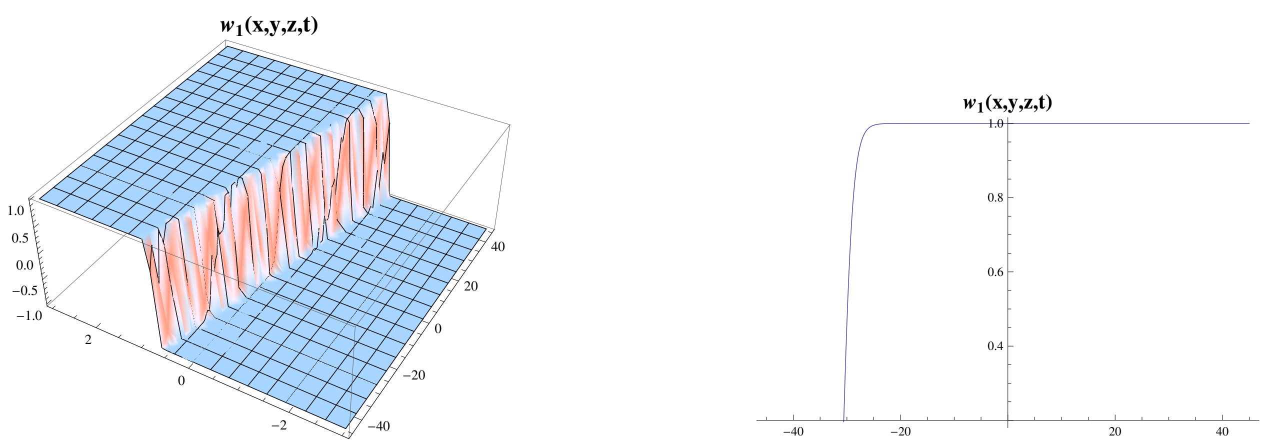

FIGURE 1. Surfaces of Eq. (3.5) for $D = 0.5, E = 1, F = 2, b_1 = -1, a_2 = 2, a = 1, b = 2, c = 4, q = 1, r = 2, s = 3, y = 0.3, z = 0.1, -45 < x < 45, -3 < t < 3$ and $t = 0.5$ for 2D surface.

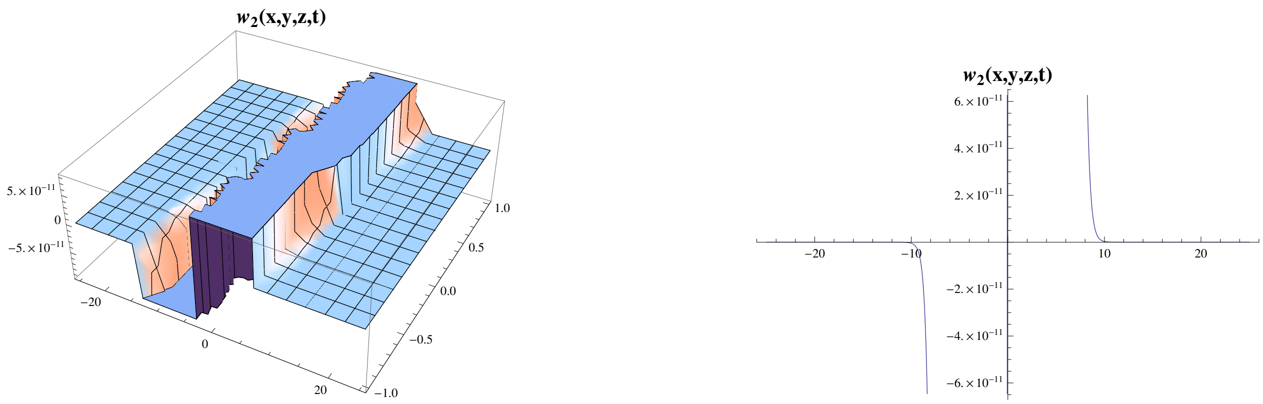

Figure 2. Surfaces of Eq. (3.7) for $D = 1.5, E = 0.5, F = 1, b_0 = 1, a_2 = -4, a = 3, b = 1, c = 2, q = 2, r = -1, s = 5, y = 0.2, z = 0.4, -25 < x < 25, -1 < t < 1$ and $t = 0.1$ for 2D surface.

4. Conclusion

In present work, we research exact solutions of (3+1)-dimensional MQZK equation via GKM. Moreover, we find dark optical soliton solutions of this equation with the aid of this method.

According to these datas, GKM is a powerful mathematical tool for finding the exact solutions of (3+1)-dimensional MQZK equation. The foremost important success of this study lies in the fact that we have worked out in constructing a proper algorithm in an effort to obtain exact solutions of this equation. Thus, we can highlight that not only GKM plays substantial situation in the investigation of NLPDEs but also it is quite forceful to provide exact solutions of NLPDEs. Moreover, this method can be applied to other NLPDEs which arising in description of the physical sciences.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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