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STATISTICAL POWER COMPARISONS FOR EQUAL SKEWNESS DIFFERENT KURTOSIS AND EQUAL KURTOSIS DIFFERENT SKEWNESS COEFFICIENTS IN NONPARAMETRIC TESTS

Ötüken SENGER*

Abstract
Mann-Whitney and Kolmogorov-Smirnov two-sample tests are the most appropriate tests when the data, which are obtained from independent two-sample, are asked for testing by the help of nonparametric tests. Both Mann-Whitney and Kolmogorov-Smirnov two-sample tests are nonparametric statistics tests which are used to determine whether independent two-sample belongs to the same or similar populations. In this study, the statistical powers of these two tests are compared by analyzing the change in kurtosis coefficients under the assumption of equal skewness coefficients and analyzing the change in skewness coefficients under the assumption of equal kurtosis coefficients. Variances are assumed as heterogeneous for both situations and variance ratios 2, 3, 4, 1/2, 1/3 and 1/4 are used. Also, equal sample sizes of 4, 5, 8, 10, 12, 15, 16 and 20 are used as small and equal sample sizes. The results of the analyses revealed that Mann-Whitney test is more powerful between small and equal sample sizes of 4 to 10, and Kolmogorov-Smirnov two-sample test is more powerful between small and equal sample sizes of 12 to 20.

Keywords: Mann-Whitney Test, Kolmogorov-Smirnov Two Samples Test, Heterogeneity of Variance, Skewness, Kurtosis, Statistical Power, Monte Carlo Study.
Jel Classification: C12, C14, C15.

* Kafkas University, Faculty of Economic and Administrative Sciences, Department of Numerical Methods, Kars, Turkey, E-mail: otukensenger@gmail.com

1. INTRODUCTION

Nonparametric tests can be used if the test hypothesis is not about population parameter, if the existing data are measured by a weak scale for a parametric test and if the necessary conditions (homogeneity of variance, normal distribution, etc) are not accomplished for parametric tests (Daniel, 1990). Various nonparametric tests are used to test the data of independent two-sample test. Mann-Whitney and Kolmogorov-Smirnov tests are the most used tests within nonparametric tests to analyze independent two-sample. Both of these two tests are used to determine whether independent two-sample belongs to the same population (Siegel & Castellan, 1988) or whether two populations are similar for independent two-sample (Conover, 1999).

In statistical studies, we can look for if two populations have the same distributions or not. In this situation, we can desire to test the differences among means, variances and figures (Magel R. C. & Wibowo S. H., 1997).

Even if, both Mann-Whitney and Kolmogorov-Smirnov two samples tests have the same sufficiencies to establish the group differences, distinctive results can be significantly obtained when these two tests are applied to the same data groups. These distinctivenesses are originated from the differences of sample sizes, heterogeneity of variance or skewness and kurtosis in the population distribution between two samples (Lee, 2007). This study investigates performances of Mann-Whitney and Kolmogorov-Smirnov two-sample tests for different sample sizes, on different distributions and under different skewness and kurtosis coefficients.

2. TESTS USED IN POWER COMPARISON

2.1. Mann-Whitney Test

Wilcoxon considered the condition of equal sample sizes in 1945 and used the rank sum test statistic (Daniel, 1990). In 1947, Mann and Whitney offered slightly more different version of this test for the applicability of both equal or unequal sample sizes and they yielded tables for small sample sizes (Conover, 1999). Siegel and Castellan entitled this test as

Wilcoxon-Mann-Whitney test because both Wilcoxon and Mann and Whitney indepently introduced a nonparametric test with the same principles (Siegel and Castellan, 1988). Mann-Whitney test is one of the nonparametric techniques to be used for determining the differences between similar populations and general two-sample problem under the null hypothesis (Lee, 2007)

Several assumptions have to be occurred in order to be able to apply Mann-Whitney test. These assumptions can be listed as follows:

1) Any sample is randomly selected from its representative population.
2) Fundamentally observed sample score is a continuous variable.
3) Two-sample score is randomly selected and score series are mutually independent.
4) Measurement scale in use is at least ordinal.

Let $X_1, X_2, ..., X_{n1}$ denote randomly selected observation values from the first population and $Y_1, Y_2, ..., Y_{n2}$ denote randomly selected observation values from second population; here the number of X is more than the number of Y and rank 1 is appointed to $n_1 + n_2$ in ascending order when $N = n_1 + n_2$:

$\sum R_1 = Rank$ is sum of first sample group

$\sum R_2 = Rank$ is sum of second sample group(Sheskin, 2000).

Two sided hypotheses to be used in this study are as follows:

H_0: for all x $F(x) = G(x)$; or there is no significant difference between two populations.

H_a: for some x $F(x) \neq G(x)$; or there are significant differences between two populations (Conover, 1999).

W test statistic which is recommended for small samples is used when their sample sizes are equal to or smaller than 10 ($m \leq 10$ and $n \leq 10$). According to W test statistic

$W_x = \sum R_1$ Multivariate rank sums of X selected from the first population

$W_y = \sum R_2$ Multivariate rank sums of Y selected from the second population

$$W_x + W_y = \frac{N(N+1)}{2} \qquad (2.1.1)$$

The small one of W_x and W_y values is selected as W test statistic. Decision rule for this test statistic is as follows:

If the probability of observed W on the table is smaller than specific significance level (α), the null hypothesis (H_0: Mx = My) is rejected and it is concluded that there is a significant difference between two populations.

If the sample size is greater than 10 (m > 10 or n > 10), normal approach formula is used for large samples. This recommended formula for large samples is also used in the conditions when one of the sample sizes is greater than 3 or 4 and other sample is greater than 12. Formula is summarized as follows:

$$z = \frac{W_x \pm 0{,}5 - \frac{m(N+1)}{2}}{\sqrt{\frac{mn(N+1)}{12}}} \qquad (2.1.2)$$

The decision rule in use of the application of W test statistic for large sample conditions is as follows:

If computed absolute z value is greater than the matched z value on the list within α/2 significance level, the null hypothesis (H_0: Mx = My) is rejected and it is concluded that there is a significant difference between the medians of these two populations (Siegel and Castellan, 1988).

2.2. Kolmogorov Smirnov Two-Sample Test

Goodness of fit tests are used to decide what extent do the observed data, which are selected from unknown populations, adapt to the model (Daniel, 1990). One of the most used goodness of fit tests is Kolmogorov-Smirnov test (Gamgam and Altunkaynak, 2008). A. N. Kolmogorov advanced a one-sample goodness of fit test for ordinal data in 1933 (Conover, 1999). Two-sample test was extended by Smirnov in 1939 but this test was called as Kolmogorov test because of its similarity to Kolmogorov one-sample test (Daniel, 1990). Kolmogorov-Smirnov two-sample test is one of the nonparametric techniques which is used to compare two-sample cumulative distribution functions for the purpose of determining whether there is any difference between two sample population distributions (Conover, 1999).

Assumptions required to apply Kolmogorov-Smirnov test are as follows:

1) Any sample is randomly selected from its representative population.
2) Measurement scale in use is at least ordinal.
3) Fundamentally observed sample score is a continuous variable.
4) Two samples are mutually independent (Conover 1999).

Data arrangement proposed by Siegel and Castellan (1988) is as follows:

Let $S_1(x)$ be the cumulative distribution probability function based on random sample scores of X_1, X_2, …, X_{n1}.S_1 (x) is determined for every values of X_1, X_2, …, X_{n1}. Let $S_1(x)=\frac{k}{n_1}$.

Let F(x) be the population which the sample of X is randomly selected.

Let $S_2(x)$ be the cumulative distribution function based on the random sample scores of Y_1, Y_2, …, Y_{n2}. $S_2(x)$ is determined for every values of Y_1, Y_2, …, Y_{n2}. Let $S_2(x)=\frac{k}{n_2}$.

Let G(x) be the population which the sample of Y is randomly selected.

$D_{n1,n2}$ is denoted as a test statistic for Kolmogorov-Smirnov two-samples test and it is the maximum absolute difference between two empirical or cumulative distribution functions (Siegel and Castellan, 1988).

Two sided hypotheses which are used to compare whether there are general differences between two populations are as follows:

H_0: There is no difference between two populations or for all x H_0: F(x) = G(x); from -∞ to + ∞.

Ha: There are several differences between two populations or for at least one value of x F(x) ≠ G(x) (Marascuilo and McSweeney, 1977).

Test statistic of Kolmogorov-Smirnov two sample test for small and large samples is as follows:

$$D = \max\left|S_1(x) - S_2(x)\right| \tag{2.2.1}$$

Decision rule of the hypothesis is summarized below:

If observed D value is greater than or equal to matched D critical value within specific (α) significance level in the list (D ≥ D critical), null hypothesis is rejected. So, it is said that there is a significant difference between two populations (Daniel, 1990).

In this study, tied values for both Mann-Whitney and Kolmogorov-Smirnov tests are eliminated.

3. Statistical Power, Heterogeneity of Variance, Skewness and Kurtosis

The researchers, while structuring their hypothesis, believe that the alternative hypothesis is correct, hoping that the sample data will provide the refusal of the zero hypothesis (Tabachnick and Fidell, 2007). Two types of errors are encountered in hypothesis tests, namely type I error and type II error. The provision of type II error is power and the power is defined by 1-β (Boslaugh and Watters, 2008).The risk of Type II decreases whenever the power of the hypothesis increases, hence the power of the hypothesis should be at the possible ultimate level (Brink, 2010).

Homogeneity of variance is the condition when the populations, which the samples are randomly selected, have similar or equal variances (Vogt, 2005). Zimmerman (2004) used the ratio between the standard deviations of two populations as homogeneity of variance. In this study, standard deviation ratio $\left(\frac{\sigma_1}{\sigma_2}\right)$ was accepted as an indicator in order to determine eliminating homogeneity of variance assumption and power comparisons were investigated by selecting the standard deviation ratios as 2, 3, 4, 1/2, 1/3 and 1/4 respectively.

Skewness is a lack of symmetry in a probability distribution and kurtosis is the measurement of seperation of smoothing probability distribution from a normal distribution shape (Everitt, 2006).Skewness and kurtosis measures were initially developed by Pearson in 1895 (quoted by Balakrishnan and Nevzerov, 2003). According to Pearson skewness and kurtosis are denoted by following formulas respectively:

$$\gamma_1 = \frac{\beta_3}{\sqrt{\beta_2^3}} \qquad (3.1)$$

$$\gamma_2 = \frac{\beta_4}{\sqrt{\beta_2^4}} \qquad (3.2)$$

In these formulas;

β_2 denotes second central moment of population distribution function,

β_3 denotes third central moment of population distribution function and

β_4 denotes fourth central moment of population distribution function.

Algina, Olejnik and Ocanto (1989) suggest that when $\gamma_1 = 0$ and, $\gamma_2 = 3$ a distribution is normal. In addition, Balakrishnan and Nevzorov (2003) assert that when $\gamma_1 > 0$, the distribution is positively skewed, when $\gamma_1 < 0$, the distribution is negatively skewed; when $\gamma_2 = 3$, the distribution is normal, when $\gamma_2 < 3$, the distribution is slightly platykurtic and when $\gamma_2 > 3$ the distribution is platykurtic.

4. SIMULATION STUDY

In the study, SAS 9.00 computer software programme was used to establish Monte Carlo simulation. Fleishman (1978) advanced a power function to assist the researchers for generating extensively different distributions and for simulating empirical distributions. The formula Fleishman used in the power function is as follows:

$$Y = a + [(d \times X + c) \times X + b] \times X \quad (4.1)$$

In this formula, X has a standard normal distribution, and a, b, c and d are constants chosen in such a way that Y has the desired coefficients of skewness and kurtosis. Coefficients a, b, c and d in the formula were identified based on related conditions of the study such as standard deviations, skewness matches and kurtosis. Owing to formula, a is constant and is equal to –c, b, c and d are generated values by Fleishman.

In the study, 12 population distributions of Algina, Olejnik and Ocanto (1989) are used. Of these, the distributions with equal skewness coefficients and different kurtosis coefficitients, and the distributions with equal kurtosis coefficients and different skewness coefficients are determined and analyzed. There is a total of eight distributions that have equal skewness and different kurtosis coefficients. When double combinations of these populations are selected in order to be able to establish dual comparisons, totally 16 different combinations are obtained with equal skewness coefficients and different kurtosis coefficients.

There are totally 10 distributions which have equal kurtosis coefficients and different skewness coefficients. While double combinations are taken into consideration, totally 9 different conditions are determined with equal kurtosis and different skewness coefficients.

In this study, 25 different conditions for 16 equal skewness and different kurtosis and 9 equal kurtosis and different skewness coefficient; 6 different conditions for standard deviation ratios (2, 3, 4, 1/2, 1/3 and 1/4) and 8 different conditions for sample sizes (equal sample sizes of 4, 5, 8, 10, 12, 15, 16, and 20) are in question. Therefore, totally 1200 (25x6x8) syntax is ordered and analyzed in the study.

The simulation steps used in this study are as follows:

- $\mu=0$ ve $\sigma=1$ values and Fleishman's power function have been used and 25 population distributions have been produced from normal distributions by operating SAS/RANNOR program.

- For statistical tests, α level of significance is determined as α =0.05.
- Null (H_0) and alternative hypotheses (H_a) have been determined for comparisons for each test.
- Formulas which will be used in test statistics have been determined for Mann-Whitney ve Kolmogorov-Smirnov two-sample tests.
- Given specific standard deviation rates of two populations (2, 3, 4, 1/2, 1/3 ve 1/4) two untied sample which is from 25 population distribution and whose length is n_1, n_2 and random 8 different samples ((4, 4), (5, 5), (8, 8), (10, 10), (12, 12), (15, 15), (16, 16) and (20, 20)) has been generated.
- Test statistic values of Mann-Whitney ve Kolmogorov-Smirnov two-sample test have been estimated for these samples.
- By being compared these estimated test statistics with critical table values, it has been determined whether H_0 will be accepted or not.
- For each different situation by being repeating 30.000, for Mann-Whitney ve Kolmogorov-Smirnov two-sample tests it has been counted how many times H_0 hypotheses have been refused. SAS/RANNOR procedure has been used again for this process.
- For each test, the number of H_0 hypotheses which has been refused in that test, has been subtracted from 30.000 that is the total number of repetitions and the result has been divided into 30.000 that is the repetition number of result. Therefore, statistical power values have been estimated for each test

12 population distributions from Fleishman's power function (Lee, 2007, 88) are shown in Table 1 and double sample combinations for 25 different conditions are shown in Table 2.

Table1: Fleishman's Power function for μ=0 and σ=1(Source; (Lee, 2007, 88))

Distribution	**Skewness**	**Kurtosis**	**a**	**b**	**c**	**d**
Normal	0.00	0.00	0.00	1.0000000	0.00	0.00
Platykurtic	0.00	-0.50	0.00	1.0767327	0.00	-0.0262683
Normal Platykurtic	0.00	-1.00	0.00	1.2210010	0.00	-0.0801584
Leptokurtic[1]	0.00	1.00	0.00	0.9029766	0.00	0.0313565
Leptokurtic[2]	0.00	2.00	0.00	0.8356646	0.00	0.0520574
Leptokurtic[3]	0.00	3.75	0.00	0.7480208	0.00	0.0778727
Skewed	0.75	0.00	-0.1736300	1.1125146	0.1736300	-0.0503344
Skewed& Platykurtic[1]	0.50	-0.50	-0.1201561	1.1478491	0.1201561	-0.0575035
Skewed &Platykurtic[2]	0.25	-1.00	-0.0774624	1.2634128	0.0774624	-0.1000360
Skewed &Leptokurtic[1]	0.75	3.75	-0.0856306	0.7699520	0.0856306	0.0693486
Skewed &Leptokurtic[2]	1.25	3.75	-0.1606426	0.8188816	0.1606426	0.0491652
Skewed-Leptokurtic	1.75	3.75	-0.3994967	0.9296605	0.3994967	-0.0364670

Table 2: Double Sample Combinations for 25 different conditions

n_1			n_2		
n_1 Distribution	Skewness Coefficcient	Kurtosis Coefficient	n_2 Distribution	Skewness Coefficcient	Kurtosis Coefficient
Normal	0.00	0.00	Platykurtic	0.00	-0.50
Normal	0.00	0.00	Normal Platykurtic	0.00	-1.00
Normal	0.00	0.00	Leptokurtic[1]	0.00	1.00
Normal	0.00	0.00	Leptokurtic[2]	0.00	2.00
Normal	0.00	0.00	Leptokurtic[3]	0.00	3.75
Platykurtic	0.00	-0.50	Normal Platykurtic	0.00	-1.00
Platykurtic	0.00	-0.50	Leptokurtic[1]	0.00	1.00
Platykurtic	0.00	-0.50	Leptokurtic[2]	0.00	2.00
Platykurtic	0.00	-0.50	Leptokurtic[3]	0.00	3.75
Normal Platykurtic	0.00	-1.00	Leptokurtic[1]	0.00	1.00
Normal Platykurtic	0.00	-1.00	Leptokurtic[2]	0.00	2.00
Normal Platykurtic	0.00	-1.00	Leptokurtic[3]	0.00	3.75
Leptokurtic[1]	0.00	1.00	Leptokurtic[2]	0.00	2.00
Leptokurtic[1]	0.00	1.00	Leptokurtic[3]	0.00	3.75
Leptokurtic[2]	0.00	2.00	Leptokurtic[3]	0.00	3.75
Skewed	0.75	0.00	Skewed &Leptokurtic[1]	0.75	3.75

Normal	0.00	0.00	Skewed	0.75	0.00
Platykurtic	0.00	-0.50	Skewed &Platykurtic[1]	0.50	-0.50
Normal Platykurtic	0.00	-1.00	Skewed &Platykurtic[2]	0.25	-1.00
Leptokurtic[3]	0.00	3.75	Skewed &Leptokurtic[1]	0.75	3.75
Leptokurtic[3]	0.00	3.75	Skewed &Leptokurtic[2]	1.25	3.75
Leptokurtic[3]	0.00	3.75	Skewed-Leptokurtic	1.75	3.75
Skewed &Leptokurtic[1]	0.75	3.75	Skewed &Leptokurtic[2]	1.25	3.75
Skewed &Leptokurtic[1]	0.75	3.75	Skewed-Leptokurtic	1.75	3.75
Skewed &Leptokurtic[2]	1.25	3.75	Skewed-Leptokurtic	1.75	3.75

5. SIMULATION RESULTS

5.1. The Conditions of Equal Skewness and Different Kurtosis

Power comparisons were established for 16 different conditions.

When sample pairs, obtained from Normal and Platykurtic distributions, were investigated, it was determined that the power of Mann-Whitney test was higher for the equal sample sizes of 4 to 10. The power of Kolmogorov-Smirnov two-sample test is higher when standard deviations are 4 (0,081>0,073) and 1/4 (0,089>0,073) for (8, 8) sample size, when standard deviations are 1/4 (0,083>0,081) for (10, 10) sample size and finally for equal sample sizes of 12 to 20.

Table 3: Powers of Mann-Whitney and Kolmogorov-Smirnov Two-Sample Tests for Normal & Platykurtic and Normal & Normal Platykurtic Distributions

Distribution	$\frac{\sigma_1}{\sigma_2}$	n_1	n_2	MW	KS-2	Distribution	$\frac{\sigma_1}{\sigma_2}$	n_1	n_2	MW	KS-2
Normal & Platykurtic	2	4	4	0,069	0,038	Normal & Normal Platykurtic	2	4	4	0,068	0,037
	3	4	4	0,080	0,047		3	4	4	0,079	0,045
	4	4	4	0,090	0,057		4	4	4	0,089	0,057
	1/2	4	4	0,066	0,037		1/2	4	4	0,074	0,043
	1/3	4	4	0,081	0,050		1/3	4	4	0,091	0,059
	1/4	4	4	0,094	0,064		1/4	4	4	0,096	0,066
	2	5	5	0,059	0,011		2	5	5	0,061	0,010
	3	5	5	0,065	0,016		3	5	5	0,068	0,016
	4	5	5	0,071	0,021		4	5	5	0,072	0,021
	1/2	5	5	0,064	0,013		1/2	5	5	0,064	0,013
	1/3	5	5	0,070	0,019		1/3	5	5	0,068	0,020
	1/4	5	5	0,075	0,026		1/4	5	5	0,071	0,026
	2	8	8	0,057	0,032		2	8	8	0,058	0,029
	3	8	8	0,067	0,056		3	8	8	0,067	0,053
	4	8	8	0,073	0,081		4	8	8	0,072	0,077
	1/2	8	8	0,059	0,035		1/2	8	8	0,062	0,042

	1/3	8	8	0,067	0,063	1/3	8	8	0,071	0,073
	1/4	8	8	0,073	0,089	1/4	8	8	0,075	0,104
	2	10	10	0,058	0,024	2	10	10	0,061	0,020
	3	10	10	0,070	0,049	3	10	10	0,070	0,044
	4	10	10	0,080	0,076	4	10	10	0,077	0,073
	1/2	10	10	0,063	0,028	1/2	10	10	0,069	0,037
	1/3	10	10	0,075	0,056	1/3	10	10	0,075	0,066
	1/4	10	10	0,081	0,083	1/4	10	10	0,083	0,102
	2	12	12	0,059	0,066	2	12	12	0,059	0,056
	3	12	12	0,070	0,132	3	12	12	0,072	0,127
	4	12	12	0,077	0,209	4	12	12	0,078	0,212
	1/2	12	12	0,063	0,081	1/2	12	12	0,063	0,094
	1/3	12	12	0,071	0,156	1/3	12	12	0,075	0,187
	1/4	12	12	0,078	0,228	1/4	12	12	0,081	0,269
	2	15	15	0,055	0,062	2	15	15	0,058	0,056
	3	15	15	0,068	0,155	3	15	15	0,065	0,146
	4	15	15	0,072	0,249	4	15	15	0,075	0,255
	1/2	15	15	0,060	0,082	1/2	15	15	0,063	0,106
	1/3	15	15	0,068	0,177	1/3	15	15	0,072	0,217
	1/4	15	15	0,079	0,292	1/4	15	15	0,081	0,336
	2	16	16	0,054	0,088	2	16	16	0,054	0,079
	3	16	16	0,064	0,210	3	16	16	0,064	0,205
	4	16	16	0,071	0,337	4	16	16	0,071	0,343
	1/2	16	16	0,057	0,112	1/2	16	16	0,058	0,143
	1/3	16	16	0,065	0,249	1/3	16	16	0,072	0,304
	1/4	16	16	0,074	0,386	1/4	16	16	0,077	0,442
	2	20	20	0,056	0,101	2	20	20	0,058	0,086
	3	20	20	0,067	0,272	3	20	20	0,065	0,262
	4	20	20	0,073	0,455	4	20	20	0,072	0,469
	1/2	20	20	0,058	0,134	1/2	20	20	0,061	0,179
	1/3	20	20	0,071	0,328	1/3	20	20	0,072	0,397
	1/4	20	20	0,079	0,510	1/4	20	20	0,079	0,592

As seen on Table 3, the largest power value was observed for Kolmogorov-Smirnov two-sample test when standard deviation ratio was 1/4. It is also determined that power values of Mann-Whitney test are weak and similar from (12, 12) sample size to (20, 20) sample size. For sample pairs of Normal and Normal Platykurtic distributions, power of Mann-Whitney test is higher for the sample sizes of 4 to 10 excluding the standard deviation ratios of 4 (0,077>0,072), 1/3 (0,073>0,071) and 1/4 (0,104>0,075) for (8, 8) sample size and standard deviation ratio of 1/4 (0,102>0,083) for (10, 10) sample size. Power of Kolmogorov-Smirnov two-sample test is higher for equal sample sizes of 12 to 20 excluding the standard deviation ratio of 2 for (12, 12) (0,059>0,056) and (15, 15) (0,058>0,056) sample sizes.

The highest power value on Table 3 is once again observed in Kolmogorov-Smirnov two-sample test when standard deviation ratio is 1/4. This value is followed by standard deviation ratios of 4, 1/3, 3, 1/2 and 2 for Kolmogorov-Smirnov two-sample test respectively. Power values of Mann-Whitney test is still lower after the sample size of (12, 12).

Power of Mann-Whitney test is higher for sample pairs of Normal and Leptokurtic[1]distributions with equal sample sizes from 4 to 10, excluding the conditions of (8, 8) sample size when standard deviation ratios are 4 (0,085>0,073) and 1/4 (0,072>0,071). As for equal sample sizes from 12 to 20, power of Kolmogorov-Smirnov two-sample test is higher excluding the case of (12, 12) sample size when standard deviation ratio was 1/2.

As illustrated on Table 4, the highest power value was observed for Kolmogorov-Smirnov two-sample test when standard deviation ratio was 4. This value was followed by standard deviation ratios of 1/4, 3, 1/3, 2 and 1/2 for Kolmogorov-Smirnov two-sample test respectively. Power values of Mann-Whitney test are again observed as low after the sample size of (12, 12). While sample pairs of Normal and Leptokurtic[2] distributions are examined, it is determined that power of Mann-Whitney test is higher for equal sample sizes from 4 to 10, excluding the conditions of (8, 8) (0,088>0,071) and (10, 10) (0,085>0,081) sample sizes when the standard deviation ratio is 4. Power of Kolmogorov-Smirnov two-sample test is higher for equal sample sizes from 12 to 20, excluding the conditions of (12, 12) (0,057>0,052) and (15, 15) (0,056>0,048) sample sizes when standard deviation ratio was 1/2.

Power of Kolmogorov-Smirnov two sample test is the highest when standard deviation ratio is 4 on Table 4. Power of Mann-Whitney test is higher for sample pairs of Normal and Leptokurtic[3]distributions with equal sample sizes from 4 to 10, excluding the conditions of (8, 8) sample size when standard deviation ratios are 3 (0,070>0,068) and 4 (0,095>0,070) and (10, 10) (0,088>0,080) sample size when standard deviation ratio is 4. As for equal sample sizes from 12 to 20, power of Kolmogorov-Smirnov two-sample test is higher excluding the case of (12, 12) (0,055>0,044) and (15, 15) (0,050>0,038) sample sizes when standard deviation ratio is 1/2.

Table 4. Powers of Mann-Whitney and Kolmogorov-Smirnov Two-Sample Tests for Normal & Leptokurtic[1] and Normal & Leptokurtic[2] Distributions

Distribution	$\frac{\sigma_1}{\sigma_2}$	n_1	n_2	MW	KS-2	Distribution	$\frac{\sigma_1}{\sigma_2}$	n_1	n_2	MW	KS-2
Normal & Leptokurtic[1]	2	4	4	0,068	0,037	Normal & Leptokurtic[2]	2	4	4	0,072	0,042
	3	4	4	0,084	0,052		3	4	4	0,083	0,052
	4	4	4	0,094	0,061		4	4	4	0,091	0,063
	1/2	4	4	0,063	0,033		1/2	4	4	0,066	0,034
	1/3	4	4	0,078	0,047		1/3	4	4	0,076	0,043

	1/4	4	4	0,088	0,057		1/4	4	4	0,084	0,051
	2	5	5	0,060	0,012		2	5	5	0,064	0,013
	3	5	5	0,066	0,019		3	5	5	0,066	0,017
	4	5	5	0,074	0,025		4	5	5	0,071	0,025
	1/2	5	5	0,063	0,011		1/2	5	5	0,061	0,010
	1/3	5	5	0,066	0,015		1/3	5	5	0,069	0,016
	1/4	5	5	0,069	0,021		1/4	5	5	0,070	0,020
	2	8	8	0,059	0,036		2	8	8	0,058	0,038
	3	8	8	0,065	0,061		3	8	8	0,071	0,066
	4	8	8	0,073	0,085		4	8	8	0,071	0,088
	1/2	8	8	0,058	0,030		1/2	8	8	0,056	0,025
	1/3	8	8	0,062	0,052		1/3	8	8	0,063	0,045
	1/4	8	8	0,071	0,072		1/4	8	8	0,066	0,064
	2	10	10	0,062	0,029		2	10	10	0,064	0,032
	3	10	10	0,078	0,057		3	10	10	0,079	0,062
	4	10	10	0,079	0,078		4	10	10	0,081	0,085
	1/2	10	10	0,061	0,022		1/2	10	10	0,059	0,020
	1/3	10	10	0,067	0,043		1/3	10	10	0,064	0,036
	1/4	10	10	0,076	0,070		1/4	10	10	0,071	0,060
	2	12	12	0,062	0,078		2	12	12	0,061	0,086
	3	12	12	0,073	0,154		3	12	12	0,072	0,160
	4	12	12	0,077	0,223		4	12	12	0,079	0,234
	1/2	12	12	0,061	0,060		1/2	12	12	0,057	0,052
	1/3	12	12	0,071	0,125		1/3	12	12	0,069	0,109
	1/4	12	12	0,076	0,185		1/4	12	12	0,074	0,170
	2	15	15	0,058	0,079		2	15	15	0,062	0,089
	3	15	15	0,070	0,172		3	15	15	0,068	0,181
	4	15	15	0,077	0,272		4	15	15	0,080	0,281
	1/2	15	15	0,059	0,060		1/2	15	15	0,056	0,048
	1/3	15	15	0,067	0,136		1/3	15	15	0,065	0,120
	1/4	15	15	0,069	0,220		1/4	15	15	0,076	0,201
	2	16	16	0,057	0,110		2	16	16	0,062	0,124
	3	16	16	0,068	0,240		3	16	16	0,066	0,250
	4	16	16	0,071	0,369		4	16	16	0,073	0,374
	1/2	16	16	0,054	0,079		1/2	16	16	0,055	0,069
	1/3	16	16	0,063	0,185		1/3	16	16	0,061	0,169
	1/4	16	16	0,068	0,301		1/4	16	16	0,071	0,277
	2	20	20	0,060	0,131		2	20	20	0,058	0,148
	3	20	20	0,069	0,312		3	20	20	0,071	0,330
	4	20	20	0,075	0,481		4	20	20	0,075	0,498
	1/2	20	20	0,056	0,089		1/2	20	20	0,056	0,077
	1/3	20	20	0,065	0,237		1/3	20	20	0,066	0,209
	1/4	20	20	0,071	0,399		1/4	20	20	0,072	0,363

As illustrated on Table 5, the highest power value is observed for Kolmogorov-Smirnov two-sample test when the standard deviation is 4. This value is followed by standard deviation ratios of 3, 1/4, 1/3, 2 and 1/2 respectively for Kolmogorov-Smirnov two-sample test.

Table 5: Powers of Mann-Whitney and Kolmogorov-Smirnov Two-Sample Tests for Normal & Leptokurtic[3] and Platykurtic & Normal Platykurtic Distributions

Distribution	σ_1/σ_2	n_1	n_2	MW	KS-2	Distribution	σ_1/σ_2	n_1	n_2	MW	KS-2
Normal & Leptokurtic[3]	2	4	4	0,074	0,041	Platykurtic & Normal Platykurtic	2	4	4	0,069	0,037
	3	4	4	0,083	0,054		3	4	4	0,085	0,050
	4	4	4	0,092	0,064		4	4	4	0,092	0,060
	1/2	4	4	0,061	0,033		1/2	4	4	0,074	0,042
	1/3	4	4	0,073	0,042		1/3	4	4	0,088	0,056
	1/4	4	4	0,082	0,048		1/4	4	4	0,094	0,064
	2	5	5	0,064	0,014		2	5	5	0,062	0,010
	3	5	5	0,068	0,019		3	5	5	0,071	0,018
	4	5	5	0,069	0,024		4	5	5	0,073	0,023
	1/2	5	5	0,060	0,011		1/2	5	5	0,067	0,014
	1/3	5	5	0,067	0,013		1/3	5	5	0,068	0,019
	1/4	5	5	0,064	0,016		1/4	5	5	0,073	0,025
	2	8	8	0,058	0,042		2	8	8	0,056	0,029
	3	8	8	0,068	0,070		3	8	8	0,069	0,058
	4	8	8	0,070	0,095		4	8	8	0,072	0,083
	1/2	8	8	0,055	0,023		1/2	8	8	0,063	0,042
	1/3	8	8	0,062	0,041		1/3	8	8	0,068	0,070
	1/4	8	8	0,066	0,057		1/4	8	8	0,074	0,099
	2	10	10	0,063	0,034		2	10	10	0,062	0,022
	3	10	10	0,074	0,062		3	10	10	0,075	0,051
	4	10	10	0,080	0,088		4	10	10	0,078	0,081
	1/2	10	10	0,058	0,018		1/2	10	10	0,063	0,031
	1/3	10	10	0,064	0,034		1/3	10	10	0,076	0,065
	1/4	10	10	0,075	0,053		1/4	10	10	0,080	0,097
	2	12	12	0,065	0,098		2	12	12	0,058	0,063
	3	12	12	0,070	0,170		3	12	12	0,068	0,139
	4	12	12	0,082	0,243		4	12	12	0,077	0,225
	1/2	12	12	0,055	0,044		1/2	12	12	0,066	0,092
	1/3	12	12	0,065	0,089		1/3	12	12	0,071	0,178
	1/4	12	12	0,074	0,144		1/4	12	12	0,082	0,262
	2	15	15	0,058	0,097		2	15	15	0,059	0,065
	3	15	15	0,071	0,204		3	15	15	0,070	0,171
	4	15	15	0,082	0,299		4	15	15	0,082	0,287
	1/2	15	15	0,050	0,038		1/2	15	15	0,062	0,096
	1/3	15	15	0,065	0,099		1/3	15	15	0,076	0,219
	1/4	15	15	0,068	0,166		1/4	15	15	0,080	0,333
	2	16	16	0,057	0,139		2	16	16	0,058	0,090
	3	16	16	0,066	0,273		3	16	16	0,063	0,232
	4	16	16	0,074	0,394		4	16	16	0,073	0,386
	1/2	16	16	0,050	0,055		1/2	16	16	0,057	0,130
	1/3	16	16	0,061	0,134		1/3	16	16	0,069	0,295
	1/4	16	16	0,059	0,223		1/4	16	16	0,077	0,438
	2	20	20	0,063	0,174		2	20	20	0,055	0,103
	3	20	20	0,072	0,355		3	20	20	0,067	0,316
	4	20	20	0,077	0,521		4	20	20	0,078	0,519
	1/2	20	20	0,053	0,059		1/2	20	20	0,062	0,166
	1/3	20	20	0,061	0,163		1/3	20	20	0,070	0,394
	1/4	20	20	0,067	0,295		1/4	20	20	0,079	0,583

When sample pairs of Platykurtic and Normal Platykurtic distributions are investigated, power of Mann-Whitney test is higher for equal sample sizes from 4 to 10, excluding (8, 8) sample size when standard deviation ratios are 4 (0,083>0,072), 1/3

(0,070>0,068) and 1/4 (0,099>0,074) and (10, 10) sample size when the standard deviation ratios are 4 (0,081>0,074) and 1/4 (0,099>0,074). Kolmogorov-Smirnov two-sample test is higher for equal sample sizes from 12 to 20.

Power of Kolmogorov-Smirnov two-sample test is the highest when standard deviation ratio is 1/4 on Table 5. This value is followed by standard deviation ratios of 4, 1/3, 3, 1/2 and 2 for Kolmogorov-Smirnov two-sample test respectively. Power of Mann-Whitney test is observed as lower from (12, 12) sample size to (20, 20) sample size again. Power of Mann-Whitney test is higher for equal sample sizes from 4 to 10 of Platykurticand Leptokurtic[1] distributions, excluding (8, 8) (0,094>0,072) and (10, 10) (0,089>0,078) sample sizes when standard deviation ratios are 4 and (8, 8) sample sizes when standard deviation ratios are 1/4 (0,071>0,070). As for equal sample sizes from 12 to 20, power of Kolmogorov-Smirnov two-sample test is higher excluding (12, 12) (0,058>0,054) and (15, 15) (0,057>0,052) sample sizes when standard deviation ratios are 1/2.

As pictured on Table 6, power of Kolmogorov-Smirnov two-sample test is higher when standard deviation ratio is 4. This value is followed by standard deviation ratios of 1/4, 3, 1/3, 2 and 1/2 for Kolmogorov-Smirnov two-sample test, respectively. When sample pairs of Platykurticand Leptokurtic[2], Platykurtic and Leptokurtic[3],Normal Platykurtic and Leptokurtic[1] and Normal Platykurticand Leptokurtic[2] distributions are investigated, power of Mann-Whitney test is higher for equal sample sizes from 4 to 10, excluding the conditions of (8, 8) sample size when standard deviation ratios are 3 (0,072>0,067), (0,073>0,069), (0,074>0,067), (0,080>0,070) and 4 0,097>0,073), (0,104>0,077), (0,107>0,073), (0,110>0,075) and excluding the conditions of (10, 10) sample size when standard deviation ratios are 4 (0,091>0,080), (0,100>0,082), (0,102>0,085), (0,107>0,086). As for equal sample sizes from 12 to 20, power of Kolmogorov-Smirnov two-sample test is higher, excluding the conditions of (12, 12) (0,059>0,049), (0,053>0,040), (0,058>0,048), (0,055>0,041) and (15, 15) 0,052>0,046), (0,053>0,038), (0,058>0,046), (0,054>0,039) sample sizes when standard deviation is 1/2.

Table 6: Powers of Mann-Whitney and Kolmogorov-Smirnov Two-Sample Tests for Platykurtic & Leptokurtic[1] and Platykurtic & Leptokurtic[2] Distributions

Distribution	$\frac{\sigma_1}{\sigma_2}$	n_1	n_2	MW	KS-2	Distribution	$\frac{\sigma_1}{\sigma_2}$	n_1	n_2	MW	KS-2
Platykurtic & Leptokurtic[1]	2	4	4	0,070	0,039	Platykurtic & Leptokurtic[2]	2	4	4	0,074	0,044
	3	4	4	0,089	0,055		3	4	4	0,089	0,057
	4	4	4	0,096	0,066		4	4	4	0,095	0,065
	1/2	4	4	0,065	0,034		1/2	4	4	0,066	0,033
	1/3	4	4	0,076	0,043		1/3	4	4	0,075	0,042
	1/4	4	4	0,085	0,052		1/4	4	4	0,080	0,048
	2	5	5	0,064	0,014		2	5	5	0,065	0,015
	3	5	5	0,071	0,020		3	5	5	0,065	0,020
	4	5	5	0,071	0,025		4	5	5	0,071	0,026
	1/2	5	5	0,059	0,010		1/2	5	5	0,060	0,009
	1/3	5	5	0,062	0,013		1/3	5	5	0,067	0,014
	1/4	5	5	0,068	0,020		1/4	5	5	0,070	0,019
	2	8	8	0,061	0,040		2	8	8	0,061	0,043
	3	8	8	0,067	0,067		3	8	8	0,067	0,072
	4	8	8	0,072	0,094		4	8	8	0,073	0,097
	1/2	8	8	0,059	0,031		1/2	8	8	0,056	0,025
	1/3	8	8	0,064	0,048		1/3	8	8	0,062	0,041
	1/4	8	8	0,070	0,071		1/4	8	8	0,071	0,066
	2	10	10	0,065	0,032		2	10	10	0,063	0,035
	3	10	10	0,076	0,062		3	10	10	0,076	0,063
	4	10	10	0,078	0,089		4	10	10	0,080	0,091
	1/2	10	10	0,062	0,021		1/2	10	10	0,058	0,018
	1/3	10	10	0,071	0,041		1/3	10	10	0,069	0,036
	1/4	10	10	0,072	0,061		1/4	10	10	0,075	0,058
	2	12	12	0,064	0,089		2	12	12	0,063	0,094
	3	12	12	0,073	0,167		3	12	12	0,075	0,177
	4	12	12	0,082	0,243		4	12	12	0,084	0,254
	1/2	12	12	0,058	0,054		1/2	12	12	0,059	0,049
	1/3	12	12	0,064	0,112		1/3	12	12	0,066	0,101
	1/4	12	12	0,073	0,180		1/4	12	12	0,071	0,163
	2	15	15	0,063	0,096		2	15	15	0,060	0,099
	3	15	15	0,069	0,192		3	15	15	0,072	0,206
	4	15	15	0,080	0,299		4	15	15	0,079	0,311
	1/2	15	15	0,057	0,052		1/2	15	15	0,052	0,046
	1/3	15	15	0,063	0,124		1/3	15	15	0,063	0,108
	1/4	15	15	0,075	0,219		1/4	15	15	0,075	0,195
	2	16	16	0,060	0,130		2	16	16	0,058	0,137
	3	16	16	0,068	0,266		3	16	16	0,069	0,285
	4	16	16	0,075	0,397		4	16	16	0,071	0,403
	1/2	16	16	0,055	0,071		1/2	16	16	0,055	0,066
	1/3	16	16	0,062	0,177		1/3	16	16	0,062	0,150
	1/4	16	16	0,070	0,294		1/4	16	16	0,069	0,260
	2	20	20	0,061	0,155		2	20	20	0,058	0,172
	3	20	20	0,067	0,351		3	20	20	0,071	0,369
	4	20	20	0,076	0,523		4	20	20	0,079	0,543
	1/2	20	20	0,052	0,079		1/2	20	20	0,054	0,069
	1/3	20	20	0,067	0,224		1/3	20	20	0,063	0,189
	1/4	20	20	0,071	0,393		1/4	20	20	0,073	0,348

Table 7: Powers of Mann-Whitney and Kolmogorov-Smirnov Two-Sample Tests for Platykurtic & Leptokurtic[3] and Normal Platykurtic & Leptokurtic[1] Distributions

Distribution	$\frac{\sigma_1}{\sigma_2}$	n_1	n_2	MW	KS-2	Distribution	$\frac{\sigma_1}{\sigma_2}$	n_1	n_2	MW	KS-2
Platykurtic & Leptokurtic[3]	2	4	4	0,071	0,042	Normal Platykurtic & Leptokurtic[1]	2	4	4	0,075	0,044
	3	4	4	0,087	0,057		3	4	4	0,087	0,058
	4	4	4	0,094	0,067		4	4	4	0,098	0,070
	1/2	4	4	0,064	0,034		1/2	4	4	0,063	0,032
	1/3	4	4	0,073	0,041		1/3	4	4	0,078	0,044
	1/4	4	4	0,076	0,044		1/4	4	4	0,086	0,052
	2	5	5	0,064	0,015		2	5	5	0,064	0,016
	3	5	5	0,068	0,021		3	5	5	0,072	0,022
	4	5	5	0,073	0,027		4	5	5	0,071	0,026
	1/2	5	5	0,060	0,009		1/2	5	5	0,063	0,010
	1/3	5	5	0,066	0,012		1/3	5	5	0,067	0,014
	1/4	5	5	0,069	0,017		1/4	5	5	0,070	0,019
	2	8	8	0,063	0,047		2	8	8	0,063	0,047
	3	8	8	0,069	0,073		3	8	8	0,067	0,074
	4	8	8	0,077	0,104		4	8	8	0,073	0,107
	1/2	8	8	0,052	0,022		1/2	8	8	0,057	0,025
	1/3	8	8	0,059	0,037		1/3	8	8	0,062	0,043
	1/4	8	8	0,064	0,055		1/4	8	8	0,070	0,067
	2	10	10	0,066	0,039		2	10	10	0,064	0,037
	3	10	10	0,076	0,074		3	10	10	0,080	0,074
	4	10	10	0,082	0,100		4	10	10	0,085	0,102
	1/2	10	10	0,054	0,015		1/2	10	10	0,058	0,017
	1/3	10	10	0,064	0,030		1/3	10	10	0,071	0,037
	1/4	10	10	0,070	0,047		1/4	10	10	0,075	0,063
	2	12	12	0,063	0,106		2	12	12	0,066	0,108
	3	12	12	0,078	0,191		3	12	12	0,074	0,198
	4	12	12	0,083	0,263		4	12	12	0,086	0,280
	1/2	12	12	0,053	0,040		1/2	12	12	0,058	0,048
	1/3	12	12	0,063	0,085		1/3	12	12	0,066	0,107
	1/4	12	12	0,070	0,141		1/4	12	12	0,076	0,179
	2	15	15	0,064	0,120		2	15	15	0,065	0,119
	3	15	15	0,075	0,220		3	15	15	0,073	0,238
	4	15	15	0,079	0,322		4	15	15	0,083	0,347
	1/2	15	15	0,053	0,038		1/2	15	15	0,058	0,046
	1/3	15	15	0,063	0,091		1/3	15	15	0,063	0,116
	1/4	15	15	0,067	0,157		1/4	15	15	0,074	0,211
	2	16	16	0,060	0,159		2	16	16	0,060	0,158
	3	16	16	0,068	0,300		3	16	16	0,069	0,312
	4	16	16	0,073	0,426		4	16	16	0,075	0,453
	1/2	16	16	0,049	0,053		1/2	16	16	0,052	0,061
	1/3	16	16	0,056	0,120		1/3	16	16	0,061	0,167
	1/4	16	16	0,064	0,220		1/4	16	16	0,067	0,292
	2	20	20	0,062	0,201		2	20	20	0,063	0,204
	3	20	20	0,072	0,395		3	20	20	0,074	0,422
	4	20	20	0,080	0,565		4	20	20	0,077	0,600
	1/2	20	20	0,052	0,053		1/2	20	20	0,055	0,068
	1/3	20	20	0,064	0,154		1/3	20	20	0,062	0,214
	1/4	20	20	0,069	0,283		1/4	20	20	0,072	0,400

Table 8: Powers of Mann-Whitney and Kolmogorov-Smirnov Two-Sample Tests for Normal Platykurtic & Leptokurtic[2] and Normal Platykurtic & Leptokurtic[3] Distributions

Distribution	$\frac{\sigma_1}{\sigma_2}$	n_1	n_2	MW	KS-2	Distribution	$\frac{\sigma_1}{\sigma_2}$	n_1	n_2	MW	KS-2
Normal Platykurtic & Leptokurtic[2]	2	4	4	0,076	0,046	Normal Platykurtic & Leptokurtic[3]	2	4	4	0,074	0,045
	3	4	4	0,087	0,057		3	4	4	0,094	0,063
	4	4	4	0,100	0,071		4	4	4	0,098	0,071
	1/2	4	4	0,063	0,031		1/2	4	4	0,061	0,030
	1/3	4	4	0,079	0,043		1/3	4	4	0,071	0,039
	1/4	4	4	0,087	0,052		1/4	4	4	0,082	0,046
	2	5	5	0,068	0,017		2	5	5	0,067	0,015
	3	5	5	0,068	0,022		3	5	5	0,069	0,022
	4	5	5	0,070	0,028		4	5	5	0,067	0,029
	1/2	5	5	0,060	0,010		1/2	5	5	0,060	0,009
	1/3	5	5	0,064	0,012		1/3	5	5	0,066	0,012
	1/4	5	5	0,072	0,019		1/4	5	5	0,069	0,015
	2	8	8	0,062	0,048		2	8	8	0,065	0,056
	3	8	8	0,070	0,080		3	8	8	0,072	0,087
	4	8	8	0,075	0,110		4	8	8	0,076	0,113
	1/2	8	8	0,055	0,023		1/2	8	8	0,052	0,021
	1/3	8	8	0,060	0,039		1/3	8	8	0,059	0,034
	1/4	8	8	0,071	0,063		1/4	8	8	0,065	0,050
	2	10	10	0,068	0,043		2	10	10	0,069	0,045
	3	10	10	0,079	0,076		3	10	10	0,077	0,079
	4	10	10	0,086	0,107		4	10	10	0,087	0,111
	1/2	10	10	0,054	0,016		1/2	10	10	0,058	0,016
	1/3	10	10	0,066	0,031		1/3	10	10	0,066	0,028
	1/4	10	10	0,073	0,056		1/4	10	10	0,069	0,045
	2	12	12	0,066	0,116		2	12	12	0,069	0,131
	3	12	12	0,075	0,206		3	12	12	0,077	0,220
	4	12	12	0,084	0,280		4	12	12	0,089	0,303
	1/2	12	12	0,055	0,041		1/2	12	12	0,053	0,036
	1/3	12	12	0,064	0,091		1/3	12	12	0,067	0,080
	1/4	12	12	0,071	0,160		1/4	12	12	0,071	0,136
	2	15	15	0,067	0,130		2	15	15	0,066	0,147
	3	15	15	0,076	0,245		3	15	15	0,076	0,265
	4	15	15	0,082	0,352		4	15	15	0,081	0,369
	1/2	15	15	0,054	0,039		1/2	15	15	0,053	0,033
	1/3	15	15	0,062	0,099		1/3	15	15	0,062	0,079
	1/4	15	15	0,068	0,190		1/4	15	15	0,067	0,152
	2	16	16	0,062	0,175		2	16	16	0,063	0,192
	3	16	16	0,069	0,334		3	16	16	0,075	0,356
	4	16	16	0,075	0,464		4	16	16	0,074	0,477
	1/2	16	16	0,050	0,052		1/2	16	16	0,052	0,046
	1/3	16	16	0,057	0,133		1/3	16	16	0,054	0,107
	1/4	16	16	0,064	0,254		1/4	16	16	0,064	0,210
	2	20	20	0,064	0,222		2	20	20	0,067	0,249
	3	20	20	0,073	0,433		3	20	20	0,075	0,464
	4	20	20	0,077	0,606		4	20	20	0,081	0,632
	1/2	20	20	0,055	0,057		1/2	20	20	0,056	0,047
	1/3	20	20	0,065	0,177		1/3	20	20	0,057	0,131
	1/4	20	20	0,070	0,343		1/4	20	20	0,066	0,280

Power of Kolmogorov-Smirnov two-sample test is the highest when standard deviation ratio is 4, as illustrated on Table 6, 7 and 8. When these tables are deeply focused, it will be recognized that power sequences are differentiated. This is followed by standard deviations of 3, 1/4, 1/3, 2 and 1/2 respectively for Kolmogorov-Smirnov two sample test on Table 6 and Table 7 and standard deviations of 3, 1/4, 2 and 1/3 respectively for Kolmogorov-Smirnov two-sampler test on Table 7 and Table 8.

Power of Mann-Whitney test is higher for sample pairs of Normal Platykurtic and Leptokurtic[3] distributions with equal sample sizes from 4 to 10, excluding the conditions of (8, 8) and (10, 10) sample sizes when standard deviation ratios were 3 (0,087>0,072), (0,079>0,077) and 4 (0,113>0,076), (0,111>0,087). As for equal sample sizes from 12 to 20, power of Kolmogorov-Smirnov two-sample test is higher excluding the conditions of (12, 12) (0,053>0,036), (15, 15) (0,053>0,033) and (20, 20) (0,056>0,047) sample sizes when standard deviation ratios are 1/2.

It appears on Table 8 that power of Kolmogorov-Smirnov two-sample test is the highest when standard deviation ratio is 4. This value is followed by the standard deviation ratios of 3, 1/4, 2 and 1/3 for Kolmogorov-Smirnov two-sample test respectively. When sample pairs of Leptokurtic[1] and Leptokurtic[2] distributions are investigated, power of Mann-Whitney test is found as higher for the sample sizes from 4 to 10, excluding the condition of (8, 8) sample size, when standard deviation ratio is 4 (0,079>0,069). Furthermore, it is determined that powers of both tests are same for (8, 8) sample size when standard deviation ratio is 1/4 (0,073=0,073). As for equal sample sizes from 12 to 20, power of Kolmogorov-Smirnov test is higher. Power of both tests is found as same again for (12, 12) sample size when standard deviation ratio is 1/2 (0,058=0,058).

As illustrated on Table 9, power of Kolmogorov-Smirnov two-sample test is the highest when standard deviation ratio is 4. This value is followed by standard deviation ratios of 1/4, 3, 1/3, 2 and 1/2 respectively for Kolmogorov-Smirnov two-sample test. Power of Mann-Whitney test is observed lower from (12, 12) to (20, 20) sample size again.

Power of Mann-Whitney test is the highest for sample pairs of Leptokurtic[1] and Leptokurtic[3] and Skewed and Skewed and Leptokurtic[1]distributions with equal sample sizes from 4 to 10, excluding the conditions of (8, 8) (0,085>0,066), (0,126>0,088) and (10, 10) (0,081>0,076), (0,129>0,108) sample sizes when standard deviation ratio is 4. As for equal sample sizes from 12 to 20, power of Kolmogorov-Smirnov two-sample test is higher excluding the conditions of (12, 12) (0,056>0,051), (0,058>0,045) and (15, 15) sample sizes when standard deviation ratio is 1/2.

Table 9: Powers of Mann-Whitney and Kolmogorov-Smirnov Two-Sample Tests for Leptokurtic[1]& Leptokurtic[2] and Leptokurtic[1] & Leptokurtic[3] Distributions

Distribution	$\frac{\sigma_1}{\sigma_2}$	n_1	n_2	MW	KS-2	Distribution	$\frac{\sigma_1}{\sigma_2}$	n_1	n_2	MW	KS-2
	2	4	4	0,069	0,038		2	4	4	0,070	0,039
	3	4	4	0,081	0,049		3	4	4	0,083	0,052
	4	4	4	0,089	0,058		4	4	4	0,091	0,061
	1/2	4	4	0,067	0,037		1/2	4	4	0,069	0,037
	1/3	4	4	0,081	0,048		1/3	4	4	0,072	0,039
	1/4	4	4	0,088	0,056		1/4	4	4	0,082	0,049
	2	5	5	0,060	0,011		2	5	5	0,060	0,010
	3	5	5	0,069	0,017		3	5	5	0,065	0,018
	4	5	5	0,070	0,023		4	5	5	0,066	0,022
	1/2	5	5	0,061	0,011		1/2	5	5	0,065	0,012
	1/3	5	5	0,067	0,015		1/3	5	5	0,064	0,014
	1/4	5	5	0,069	0,022		1/4	5	5	0,065	0,017
	2	8	8	0,059	0,033		2	8	8	0,058	0,037
	3	8	8	0,063	0,056		3	8	8	0,068	0,064
	4	8	8	0,069	0,079		4	8	8	0,066	0,085
	1/2	8	8	0,055	0,029		1/2	8	8	0,052	0,024
	1/3	8	8	0,063	0,049		1/3	8	8	0,060	0,041
	1/4	8	8	0,073	0,073		1/4	8	8	0,065	0,057
	2	10	10	0,061	0,028		2	10	10	0,060	0,029
	3	10	10	0,073	0,051		3	10	10	0,069	0,054
	4	10	10	0,079	0,076		4	10	10	0,076	0,081
	1/2	10	10	0,059	0,023		1/2	10	10	0,057	0,017
Leptokurtic[1] & Leptokurtic[2]	1/3	10	10	0,068	0,042	Leptokurtic[1] & Leptokurtic[3]	1/3	10	10	0,064	0,035
	1/4	10	10	0,077	0,065		1/4	10	10	0,072	0,057
	2	12	12	0,058	0,073		2	12	12	0,064	0,085
	3	12	12	0,067	0,137		3	12	12	0,069	0,150
	4	12	12	0,074	0,204		4	12	12	0,082	0,224
	1/2	12	12	0,058	0,058		1/2	12	12	0,056	0,051
	1/3	12	12	0,067	0,116		1/3	12	12	0,066	0,102
	1/4	12	12	0,075	0,183		1/4	12	12	0,073	0,154
	2	15	15	0,058	0,071		2	15	15	0,060	0,088
	3	15	15	0,066	0,160		3	15	15	0,070	0,173
	4	15	15	0,073	0,247		4	15	15	0,077	0,263
	1/2	15	15	0,059	0,060		1/2	15	15	0,055	0,048
	1/3	15	15	0,069	0,135		1/3	15	15	0,062	0,106
	1/4	15	15	0,071	0,212		1/4	15	15	0,070	0,181
	2	16	16	0,055	0,100		2	16	16	0,057	0,119
	3	16	16	0,064	0,216		3	16	16	0,064	0,235
	4	16	16	0,070	0,329		4	16	16	0,068	0,353
	1/2	16	16	0,054	0,077		1/2	16	16	0,052	0,065
	1/3	16	16	0,061	0,175		1/3	16	16	0,061	0,148
	1/4	16	16	0,067	0,286		1/4	16	16	0,068	0,246
	2	20	20	0,057	0,122		2	20	20	0,060	0,139
	3	20	20	0,068	0,281		3	20	20	0,067	0,307
	4	20	20	0,074	0,432		4	20	20	0,078	0,466
	1/2	20	20	0,056	0,095		1/2	20	20	0,053	0,072
	1/3	20	20	0,064	0,226		1/3	20	20	0,060	0,178

	1/4	20	20	0,069	0,380		1/4	20	20	0,068	0,319

On Table 9, power of Kolmogorov-Smirnov two-sample test is the highest for Kolmogorov-Smirnov two-sample test when standard deviation ratio is 4. This value is followed by standard deviation ratios of 1/4, 3, 1/3, 2 and 1/2 respectively for Kolmogorov-Smirnov two-sample test.

Table 10: Powers of Mann-Whitney and Kolmogorov-Smirnov Two-Sample Tests for Leptokurtic^2 & Leptokurtic^3 and Skewed & Skewed and Leptokurtic^1 Distributions

Distribution	$\frac{\sigma_1}{\sigma_2}$	**n_1**	**n_2**	**MW**	**KS-2**	**Distribution**	$\frac{\sigma_1}{\sigma_2}$	**n_1**	**n_2**	**MW**	**KS-2**
Leptokurtic^2 & Leptokurtic^3	2	4	4	0,067	0,037	Skewed & Skewed and Leptokurtic^1	2	4	4	0,082	0,050
	3	4	4	0,083	0,051		3	4	4	0,098	0,065
	4	4	4	0,087	0,059		4	4	4	0,103	0,076
	1/2	4	4	0,063	0,033		1/2	4	4	0,059	0,029
	1/3	4	4	0,079	0,044		1/3	4	4	0,074	0,044
	1/4	4	4	0,085	0,055		1/4	4	4	0,087	0,053
	2	5	5	0,061	0,011		2	5	5	0,069	0,017
	3	5	5	0,068	0,018		3	5	5	0,076	0,026
	4	5	5	0,066	0,020		4	5	5	0,079	0,030
	1/2	5	5	0,061	0,011		1/2	5	5	0,062	0,010
	1/3	5	5	0,068	0,016		1/3	5	5	0,068	0,014
	1/4	5	5	0,068	0,019		1/4	5	5	0,072	0,018
	2	8	8	0,057	0,032		2	8	8	0,073	0,059
	3	8	8	0,065	0,054		3	8	8	0,084	0,095
	4	8	8	0,069	0,077		4	8	8	0,088	0,126
	1/2	8	8	0,055	0,028		1/2	8	8	0,055	0,024
	1/3	8	8	0,062	0,045		1/3	8	8	0,067	0,048
	1/4	8	8	0,069	0,068		1/4	8	8	0,072	0,072
	2	10	10	0,060	0,027		2	10	10	0,080	0,052
	3	10	10	0,066	0,047		3	10	10	0,097	0,097
	4	10	10	0,078	0,072		4	10	10	0,108	0,129
	1/2	10	10	0,060	0,021		1/2	10	10	0,060	0,017
	1/3	10	10	0,069	0,040		1/3	10	10	0,070	0,040
	1/4	10	10	0,074	0,060		1/4	10	10	0,082	0,070
	2	12	12	0,064	0,077		2	12	12	0,083	0,136
	3	12	12	0,072	0,139		3	12	12	0,103	0,240
	4	12	12	0,076	0,203		4	12	12	0,113	0,328
	1/2	12	12	0,055	0,055		1/2	12	12	0,058	0,045
	1/3	12	12	0,064	0,105		1/3	12	12	0,073	0,111
	1/4	12	12	0,071	0,165		1/4	12	12	0,086	0,194
	2	15	15	0,058	0,077		2	15	15	0,086	0,154
	3	15	15	0,068	0,158		3	15	15	0,106	0,291
	4	15	15	0,076	0,239		4	15	15	0,115	0,401
	1/2	15	15	0,055	0,050		1/2	15	15	0,061	0,044
	1/3	15	15	0,065	0,118		1/3	15	15	0,073	0,129
	1/4	15	15	0,071	0,193		1/4	15	15	0,087	0,238
	2	16	16	0,052	0,099		2	16	16	0,082	0,206
	3	16	16	0,062	0,213		3	16	16	0,100	0,377
	4	16	16	0,069	0,314		4	16	16	0,110	0,511
	1/2	16	16	0,053	0,074		1/2	16	16	0,054	0,060
	1/3	16	16	0,064	0,163		1/3	16	16	0,071	0,183
	1/4	16	16	0,065	0,258		1/4	16	16	0,081	0,315

	2	20	20	0,055	0,120	2	20	20	0,090	0,265
	3	20	20	0,066	0,274	3	20	20	0,112	0,490
	4	20	20	0,072	0,421	4	20	20	0,126	0,652
	1/2	20	20	0,056	0,084	1/2	20	20	0,057	0,070
	1/3	20	20	0,064	0,202	1/3	20	20	0,077	0,238
	1/4	20	20	0,071	0,336	1/4	20	20	0,085	0,420

On Table 10, power of Kolmogorov-Smirnov two-sample test is the highest for Kolmogorov-Smirnov two-sample test when standard deviation ratio is 4. This value is followed by standard deviation ratios of 3, 1/4, 2, 1/3 for Kolmogorov-Smirnov two-sample test and the conditions of Mann-Whitney test when standard deviation ratios are 4 respectively.

When sample pairs of Leptokurtic[2] and Leptokurtic[3] distributions are examined for equal sample sizes from 4 to 10, it is observed that power of Mann-Whitney test is the highest excluding the condition of (8, 8) sample size when standard deviation ratio is 4 (0,077>0,069). As for equal sample sizes from 12 to 20, power of Kolmogorov-Smirnov two-sample test is the highest excluding the condition of (15, 15) sample size when standard deviation is 1/2 (0,055>0,050). Power of both tests are observed as same for (12, 12) sample size when standard deviation ratio is 1/2 (0,055=0,055).

As illustrated on Table 10, power of Kolmogorov-Smirnov two-sample test is the highest when standard deviation ratio is 4. This value is followed by standard deviation ratios of 1/4, 3, 1/3 and 2 respectively for Kolmogorov-Smirnov two-sample test.

5.2. The Conditions of Different Skewness and Equal Kurtosis

Power comparisons have been observed for 9 different conditions.

Power of Mann-Whitney test is higher for sample pairs of Normal and Skewed, Platykurtic and Skewed and Platykurtic[1] and Normal Platykurtic and Skewed and Platykurtic[2] distributions with equal sample sizes from 4 to 10, excluding the conditions of (8, 8) sample sizes when standard deviation ratios are 4 (0,084>0,073), (0,087>0,073), (0,101>0,074) and 1/4 (0,112>0,090), (0,103>0,082), (0,106>0,076) and (10, 10) sample size when standard deviation ratios are 4 (0,084>0,082), (0,088>0,082), (0,100>0,086) and 1/4 (0,114>0,104), (0,107>0,097), (0,104>0,084). As for all equal sample sizes from 12 to 20, Kolmogorov-Smirnov is higher.

As seen on Table 11, power of Kolmogorov-Smirnov two-sample test is the highest when standard deviation ratio is 1/4. This value is followed by standard deviation ratios of 4, 1/3, 3, 1/2 and 2 respectively for Kolmogorov-Smirnov two-sample test.

On Table 12, it is illustrated that power of Kolmogorov-Smirnov two-sample test is the highest when standard deviation ratios are 1/4 and 4. These values are followed by standard deviation ratio conditions of 3 and 1/3 and 2 and 1/2 respectively.

Table 11: Powers of Mann-Whitney and Kolmogorov-Smirnov Two-Sample Tests for Normal & Skewed and Platykurtic & Skewed and Platykurtic[1] Distributions

Distribution	$\frac{\sigma_1}{\sigma_2}$	n_1	n_2	MW	KS-2	Distribution	$\frac{\sigma_1}{\sigma_2}$	n_1	n_2	MW	KS-2
Normal & Skewed	2	4	4	0,069	0,037	Platykurtic & Skewed and Platykurtic[1]	2	4	4	0,067	0,035
	3	4	4	0,083	0,048		3	4	4	0,082	0,050
	4	4	4	0,089	0,059		4	4	4	0,093	0,062
	1/2	4	4	0,083	0,045		1/2	4	4	0,079	0,043
	1/3	4	4	0,093	0,058		1/3	4	4	0,089	0,057
	1/4	4	4	0,107	0,074		1/4	4	4	0,099	0,069
	2	5	5	0,061	0,010		2	5	5	0,062	0,011
	3	5	5	0,070	0,018		3	5	5	0,071	0,017
	4	5	5	0,073	0,023		4	5	5	0,071	0,022
	1/2	5	5	0,069	0,014		1/2	5	5	0,067	0,014
	1/3	5	5	0,076	0,023		1/3	5	5	0,075	0,021
	1/4	5	5	0,080	0,028		1/4	5	5	0,074	0,026
	2	8	8	0,058	0,033		2	8	8	0,059	0,032
	3	8	8	0,069	0,060		3	8	8	0,069	0,062
	4	8	8	0,073	0,084		4	8	8	0,073	0,087
	1/2	8	8	0,072	0,048		1/2	8	8	0,065	0,039
	1/3	8	8	0,084	0,082		1/3	8	8	0,073	0,073
	1/4	8	8	0,090	0,112		1/4	8	8	0,082	0,103
	2	10	10	0,063	0,027		2	10	10	0,064	0,028
	3	10	10	0,071	0,052		3	10	10	0,070	0,055
	4	10	10	0,082	0,084		4	10	10	0,082	0,088
	1/2	10	10	0,078	0,040		1/2	10	10	0,066	0,033
	1/3	10	10	0,096	0,078		1/3	10	10	0,085	0,071
	1/4	10	10	0,104	0,114		1/4	10	10	0,097	0,107
	2	12	12	0,062	0,073		2	12	12	0,063	0,072
	3	12	12	0,070	0,150		3	12	12	0,072	0,160
	4	12	12	0,078	0,231		4	12	12	0,080	0,241
	1/2	12	12	0,081	0,110		1/2	12	12	0,072	0,091
	1/3	12	12	0,095	0,200		1/3	12	12	0,089	0,189
	1/4	12	12	0,108	0,292		1/4	12	12	0,096	0,273
	2	15	15	0,061	0,076		2	15	15	0,063	0,081
	3	15	15	0,065	0,177		3	15	15	0,068	0,187
	4	15	15	0,078	0,290		4	15	15	0,080	0,302
	1/2	15	15	0,084	0,116		1/2	15	15	0,071	0,098

	1/3	15	15	0,103	0,239
	1/4	15	15	0,118	0,361
	2	16	16	0,056	0,105
	3	16	16	0,067	0,248
	4	16	16	0,073	0,382
	1/2	16	16	0,079	0,154
	1/3	16	16	0,098	0,316
	1/4	16	16	0,110	0,457
	2	20	20	0,057	0,133
	3	20	20	0,070	0,337
	4	20	20	0,075	0,525
	1/2	20	20	0,090	0,191
	1/3	20	20	0,113	0,412
	1/4	20	20	0,121	0,607

1/3	15	15	0,085	0,216
1/4	15	15	0,092	0,327
2	16	16	0,055	0,106
3	16	16	0,070	0,261
4	16	16	0,073	0,399
1/2	16	16	0,069	0,133
1/3	16	16	0,080	0,294
1/4	16	16	0,091	0,440
2	20	20	0,058	0,128
3	20	20	0,072	0,344
4	20	20	0,075	0,538
1/2	20	20	0,074	0,162
1/3	20	20	0,086	0,386
1/4	20	20	0,104	0,581

Power of Mann-Whitney test is higher for sample pairs of Leptokurtic[3] and Skewed and Leptokurtic[1] and Leptokurtic[3] and Skewed and Leptokurtic[2] distributions with equal sample sizes from 4 to 10, excluding the condition of (8, 8) sample size when standard deviation ratio is 1/4 (0,081>0,076). As for all equal sample sizes from 12 to 20, Kolmogorov-Smirnov two-sample test is higher.

As illustrated on Table 12, the highest power value referred to Kolmogorov-Smirnov two-sample test is when standard deviation ratio is 1/4. This value is followed by standard deviation ratios of 4, 1/3, 3, 1/2 and 2 respectively for Kolmogorov-Smirnov two-sample test.

On Table 13, it is observed that the highest power value is recognized for Kolmogorov-Smirnov two-sample test when standard deviation ratio is 1/4. This value is followed by standard deviation ratios of 4 and 1/3. The conditions for Kolmogorov-Smirnov two-sample test and Mann-Whitney test have followed former values when standard deviation ratios are 3 and 1/4 respectively. When sample pairs of Leptokurtic[3] and Skewed-Leptokurtic distributions are investigated, power of Mann-Whitney test is higher for equal sample sizes from 4 to 10, excluding the conditions of (8, 8) sample size when standard deviation ratios are 4 (0,083>0,069), 1/3 (0,150>0,140) and 1/4 (0,194>0,156) and the conditions of (10, 10) sample size when standard deviation ratios are 4 (0,079>0,076) and 1/4 (0,213>0,192). As for all equal sample sizes from 12 to 20, Kolmogorov-Smirnov two-sample test is higher.

On Table 13, it is obviously recognized that powers of both tests have generally increased. The highest power value is observed for Kolmogorov-Smirnov two-sample test when standard deviation ratio is 1/4. This value is followed by the conditions of Kolmogorov-

Smirnov two-sample test are 1/3, 4, 1/2, 3 and the condition of Mann-Whitney test when standard deviation is 1/4 respectively. Power of Mann-Whitney test is higher for sample pairs of Skewed and Leptokurtic[1] and Skewed and Leptokurtic[2] distributions for equal sample sizes from 4 to 10, excluding the conditions of (8, 8) sample size when standard deviation ratios are 4 (0,081>0,076) and 1/4 (0,108>0,095). As for all equal sample sizes from 12 to 20, power of Kolmogorov-Smirnov two-sample test is higher.

Table 12: Powers of Mann-Whitney and Kolmogorov-Smirnov Two-Sample Tests for Normal Platykurtic & Skewed and Platykurtic[2] and Leptokurtic[3] & Skewed and Leptokurtic[1] Distributions

Distribution	$\frac{\sigma_1}{\sigma_2}$	n_1	n_2	MW	KS-2	Distribution	$\frac{\sigma_1}{\sigma_2}$	n_1	n_2	MW	KS-2
Normal Platykurtic & Skewed and Platykurtic[2]	2	4	4	0,073	0,041	Leptokurtic[3] & Skewed and Leptokurtic[1]	2	4	4	0,069	0,039
	3	4	4	0,087	0,054		3	4	4	0,078	0,047
	4	4	4	0,097	0,066		4	4	4	0,087	0,053
	1/2	4	4	0,074	0,042		1/2	4	4	0,071	0,038
	1/3	4	4	0,088	0,055		1/3	4	4	0,085	0,052
	1/4	4	4	0,099	0,067		1/4	4	4	0,092	0,062
	2	5	5	0,067	0,012		2	5	5	0,060	0,011
	3	5	5	0,071	0,019		3	5	5	0,066	0,016
	4	5	5	0,073	0,026		4	5	5	0,070	0,020
	1/2	5	5	0,066	0,012		1/2	5	5	0,064	0,011
	1/3	5	5	0,072	0,019		1/3	5	5	0,067	0,016
	1/4	5	5	0,072	0,026		1/4	5	5	0,072	0,024
	2	8	8	0,061	0,038		2	8	8	0,053	0,030
	3	8	8	0,070	0,069		3	8	8	0,063	0,050
	4	8	8	0,074	0,101		4	8	8	0,067	0,069
	1/2	8	8	0,064	0,040		1/2	8	8	0,062	0,037
	1/3	8	8	0,072	0,068		1/3	8	8	0,072	0,060
	1/4	8	8	0,076	0,106		1/4	8	8	0,076	0,081
	2	10	10	0,065	0,029		2	10	10	0,059	0,024
	3	10	10	0,077	0,065		3	10	10	0,071	0,044
	4	10	10	0,086	0,100		4	10	10	0,070	0,063
	1/2	10	10	0,071	0,033		1/2	10	10	0,065	0,029
	1/3	10	10	0,085	0,070		1/3	10	10	0,078	0,051
	1/4	10	10	0,084	0,104		1/4	10	10	0,089	0,080
	2	12	12	0,064	0,083		2	12	12	0,061	0,063
	3	12	12	0,076	0,181		3	12	12	0,068	0,122
	4	12	12	0,080	0,272		4	12	12	0,073	0,172
	1/2	12	12	0,067	0,091		1/2	12	12	0,067	0,077
	1/3	12	12	0,082	0,193		1/3	12	12	0,078	0,140
	1/4	12	12	0,090	0,284		1/4	12	12	0,087	0,200
	2	15	15	0,061	0,092		2	15	15	0,057	0,062
	3	15	15	0,075	0,224		3	15	15	0,065	0,130
	4	15	15	0,081	0,347		4	15	15	0,073	0,205
	1/2	15	15	0,065	0,096		1/2	15	15	0,066	0,073
	1/3	15	15	0,077	0,234		1/3	15	15	0,076	0,150
	1/4	15	15	0,085	0,361		1/4	15	15	0,087	0,239
	2	16	16	0,060	0,123		2	16	16	0,052	0,079
	3	16	16	0,068	0,303		3	16	16	0,062	0,179
	4	16	16	0,079	0,463		4	16	16	0,069	0,281
	1/2	16	16	0,060	0,128		1/2	16	16	0,065	0,102
	1/3	16	16	0,074	0,317		1/3	16	16	0,072	0,204
	1/4	16	16	0,079	0,473		1/4	16	16	0,081	0,317

	2	20	20	0,060	0,153		2	20	20	0,056	0,095
	3	20	20	0,069	0,408		3	20	20	0,064	0,225
	4	20	20	0,079	0,618		4	20	20	0,068	0,364
	1/2	20	20	0,067	0,167		1/2	20	20	0,068	0,119
	1/3	20	20	0,076	0,428		1/3	20	20	0,085	0,268
	1/4	20	20	0,088	0,641		1/4	20	20	0,092	0,414

Table 13: Powers of Mann-Whitney and Kolmogorov-Smirnov Two-Sample Tests for Leptokurtic[3] & Skewed and Leptokurtic[2] and Leptokurtic[3] & Skewed-Leptokurtic Distributions

Distribution	$\frac{\sigma_1}{\sigma_2}$	n_1	n_2	MW	KS-2	Distribution	$\frac{\sigma_1}{\sigma_2}$	n_1	n_2	MW	KS-2
Leptokurtic[3] & Skewed and Leptokurtic[2]	2	4	4	0,070	0,039	Leptokurtic[3] & Skewed-Leptokurtic	2	4	4	0,070	0,040
	3	4	4	0,082	0,047		3	4	4	0,078	0,047
	4	4	4	0,086	0,054		4	4	4	0,084	0,056
	1/2	4	4	0,078	0,045		1/2	4	4	0,108	0,070
	1/3	4	4	0,091	0,058		1/3	4	4	0,129	0,090
	1/4	4	4	0,105	0,073		1/4	4	4	0,143	0,105
	2	5	5	0,060	0,011		2	5	5	0,068	0,014
	3	5	5	0,064	0,014		3	5	5	0,070	0,018
	4	5	5	0,068	0,019		4	5	5	0,070	0,022
	1/2	5	5	0,069	0,016		1/2	5	5	0,095	0,029
	1/3	5	5	0,079	0,023		1/3	5	5	0,109	0,041
	1/4	5	5	0,084	0,029		1/4	5	5	0,117	0,052
	2	8	8	0,055	0,030		2	8	8	0,058	0,038
	3	8	8	0,065	0,049		3	8	8	0,067	0,061
	4	8	8	0,067	0,070		4	8	8	0,069	0,083
	1/2	8	8	0,071	0,047		1/2	8	8	0,109	0,092
	1/3	8	8	0,086	0,075		1/3	8	8	0,140	0,150
	1/4	8	8	0,091	0,104		1/4	8	8	0,156	0,194
	2	10	10	0,058	0,023		2	10	10	0,065	0,033
	3	10	10	0,068	0,041		3	10	10	0,070	0,056
	4	10	10	0,075	0,065		4	10	10	0,076	0,079
	1/2	10	10	0,082	0,041		1/2	10	10	0,134	0,092
	1/3	10	10	0,094	0,071		1/3	10	10	0,170	0,158
	1/4	10	10	0,106	0,100		1/4	10	10	0,192	0,213
	2	12	12	0,058	0,062		2	12	12	0,064	0,090
	3	12	12	0,065	0,117		3	12	12	0,073	0,158
	4	12	12	0,074	0,175		4	12	12	0,077	0,225
	1/2	12	12	0,083	0,101		1/2	12	12	0,149	0,217
	1/3	12	12	0,102	0,182		1/3	12	12	0,198	0,343
	1/4	12	12	0,116	0,260		1/4	12	12	0,214	0,443
	2	15	15	0,055	0,059		2	15	15	0,066	0,096
	3	15	15	0,066	0,130		3	15	15	0,069	0,189
	4	15	15	0,072	0,210		4	15	15	0,073	0,283
	1/2	15	15	0,087	0,109		1/2	15	15	0,166	0,247
	1/3	15	15	0,110	0,215		1/3	15	15	0,217	0,411
	1/4	15	15	0,122	0,313		1/4	15	15	0,244	0,537
	2	16	16	0,053	0,082		2	16	16	0,063	0,138
	3	16	16	0,062	0,180		3	16	16	0,064	0,257
	4	16	16	0,068	0,282		4	16	16	0,070	0,374
	1/2	16	16	0,083	0,145		1/2	16	16	0,169	0,312
	1/3	16	16	0,110	0,284		1/3	16	16	0,215	0,498
	1/4	16	16	0,119	0,404		1/4	16	16	0,239	0,627
	2	20	20	0,059	0,099		2	20	20	0,064	0,173
	3	20	20	0,065	0,226		3	20	20	0,068	0,353
	4	20	20	0,073	0,373		4	20	20	0,073	0,507
	1/2	20	20	0,098	0,177		1/2	20	20	0,201	0,391
	1/3	20	20	0,121	0,359		1/3	20	20	0,262	0,620

	1/4	20	20	0,133	0,512		1/4	20	20	0,294	0,760

Table 14: Powers of Mann-Whitney and Kolmogorov-Smirnov Two-Sample Tests for Skewed and Leptokurtic[1] & Skewed and Leptokurtic[2] and Skewed and Leptokurtic[1] & Skewed-Leptokurtic Distributions

Distribution	$\frac{\sigma_1}{\sigma_2}$	n_1	n_2	MW	KS-2	Distribution	$\frac{\sigma_1}{\sigma_2}$	n_1	n_2	MW	KS-2
Skewed and Leptokurtic[1] & Skewed and Leptokurtic[2]	2	4	4	0,067	0,035	Skewed and Leptokurtic[1] & Skewed-Leptokurtic	2	4	4	0,070	0,040
	3	4	4	0,083	0,050		3	4	4	0,082	0,051
	4	4	4	0,090	0,059		4	4	4	0,089	0,063
	1/2	4	4	0,072	0,042		1/2	4	4	0,105	0,066
	1/3	4	4	0,091	0,058		1/3	4	4	0,126	0,087
	1/4	4	4	0,098	0,066		1/4	4	4	0,141	0,104
	2	5	5	0,062	0,011		2	5	5	0,065	0,013
	3	5	5	0,069	0,016		3	5	5	0,071	0,020
	4	5	5	0,075	0,022		4	5	5	0,071	0,024
	1/2	5	5	0,068	0,015		1/2	5	5	0,096	0,027
	1/3	5	5	0,080	0,021		1/3	5	5	0,108	0,040
	1/4	5	5	0,079	0,027		1/4	5	5	0,112	0,050
	2	8	8	0,061	0,034		2	8	8	0,062	0,038
	3	8	8	0,071	0,056		3	8	8	0,069	0,070
	4	8	8	0,076	0,081		4	8	8	0,078	0,101
	1/2	8	8	0,071	0,042		1/2	8	8	0,112	0,091
	1/3	8	8	0,088	0,076		1/3	8	8	0,142	0,152
	1/4	8	8	0,095	0,108		1/4	8	8	0,157	0,200
	2	10	10	0,063	0,024		2	10	10	0,066	0,030
	3	10	10	0,076	0,051		3	10	10	0,075	0,068
	4	10	10	0,088	0,079		4	10	10	0,083	0,102
	1/2	10	10	0,080	0,038		1/2	10	10	0,132	0,085
	1/3	10	10	0,094	0,070		1/3	10	10	0,171	0,157
	1/4	10	10	0,107	0,104		1/4	10	10	0,192	0,218
	2	12	12	0,062	0,063		2	12	12	0,065	0,093
	3	12	12	0,074	0,134		3	12	12	0,076	0,194
	4	12	12	0,088	0,208		4	12	12	0,088	0,272
	1/2	12	12	0,080	0,095		1/2	12	12	0,146	0,203
	1/3	12	12	0,100	0,180		1/3	12	12	0,191	0,346
	1/4	12	12	0,116	0,266		1/4	12	12	0,215	0,448
	2	15	15	0,060	0,064		2	15	15	0,063	0,109
	3	15	15	0,075	0,152		3	15	15	0,074	0,237
	4	15	15	0,086	0,248		4	15	15	0,086	0,356
	1/2	15	15	0,081	0,096		1/2	15	15	0,164	0,234
	1/3	15	15	0,105	0,214		1/3	15	15	0,220	0,416
	1/4	15	15	0,124	0,324		1/4	15	15	0,241	0,545
	2	16	16	0,056	0,089		2	16	16	0,059	0,149
	3	16	16	0,074	0,210		3	16	16	0,074	0,326
	4	16	16	0,081	0,328		4	16	16	0,085	0,458
	1/2	16	16	0,079	0,135		1/2	16	16	0,168	0,305
	1/3	16	16	0,105	0,282		1/3	16	16	0,221	0,517
	1/4	16	16	0,116	0,410		1/4	16	16	0,242	0,642
	2	20	20	0,060	0,104		2	20	20	0,060	0,204
	3	20	20	0,080	0,270		3	20	20	0,081	0,443
	4	20	20	0,090	0,440		4	20	20	0,091	0,604
	1/2	20	20	0,085	0,160		1/2	20	20	0,201	0,379
	1/3	20	20	0,120	0,367		1/3	20	20	0,269	0,637
	1/4	20	20	0,136	0,530		1/4	20	20	0,296	0,780

As illustrated on Table 14, power of Kolmogorov-Smirnov two-sample test is the highest when standard deviation is 1/4. This value is followed by standard deviation ratios of 4, 1/3, 3, 1/2 and 2 respectively for Kolmogorov-Smirnov test. Power values of Mann-Whitney test are lower from (12, 12) to (20, 20) sample size again. When sample pairs of Skewed and Leptokurtic[1] and Skewed-Leptokurticdistributions are investigated, power of Mann-Whitney test is higher for equal sample sizes from 4 to 10, excluding the conditions of (8, 8) sample size when standard deviation ratios are 3 (0,070>0,069), 4 (0,101>0,078), 1/3 (0,152>0,142) and 1/4 (0,200>0,157) and the conditions of (10, 10) sample size when standard deviation ratios are 4 (0,102>0,083), 1/3 (0,171>0,157) and 1/4 (0,218>0,192). As for all equal sample sizes from 12 to 20, power of Kolmogorov-Smirnov two sample test is higher.

As inferred from Table 14, powers of both Kolmogorov-Smirnov two-sample test and Mann-Whitney test have increased for all standard deviation ratios. The highest power value is observed for Kolmogorov-Smirnov test when standard deviation is 1/4. This value is followed by standard deviation ratios of 1/3, 4, 3 and 1/2 respectively for Kolmogorov-Smirnov two-sample test. Power of Mann-Whitney test is higher for sample pairs of Skewed and Leptokurtic[2] and Skewed-Leptokurtic distributions with sample sizes from 4 to 10, excluding the conditions of (8, 8) sample size when standard deviation ratios are 3 (0,094>0,083), 4 (0,133>0,091), 1/3 (0,151>0,141) and 1/4 (0,201>0,150) and the conditions of (10, 10) sample size when standard deviation are 3 (0,105>0,098), 4 (0,145>0,108) and 1/4 (0,230>0,195). As for all equal sample sizes from 12 to 20, power of Kolmogorov-Smirnov two-sample test is higher.

As recognized on Table 15, powers of both of two tests have reached their maximum values for all standard deviation ratios. The highest power value is observed for Kolmogorov-Smirnov two-sample test when standard deviation ratio is 1/4. This value is followed by the conditions of Kolmogorov-Smirnov test when standard deviation ratios are 4, 1/3, 3 and 1/2 and the conditions of Mann-Whitney test when the standard deviation is 1/4, respectively.

Table 15: Powers of Mann-Whitney and Kolmogorov-Smirnov Two-Sample Tests for Skewed and Leptokurtic[2] & Skewed-Leptokurtic Distributions

Distribution	$\frac{\sigma_1}{\sigma_2}$	**n_1**	**n_2**	**MW**	**KS-2**
Skewed and Leptokurtic[2] & Skewed-Leptokurtic	2	4	4	0,075	0,042
	3	4	4	0,091	0,059
	4	4	4	0,099	0,070
	1/2	4	4	0,101	0,062
	1/3	4	4	0,127	0,086
	1/4	4	4	0,142	0,106
	2	5	5	0,067	0,014
	3	5	5	0,077	0,023
	4	5	5	0,078	0,031
	1/2	5	5	0,095	0,026
	1/3	5	5	0,110	0,040
	1/4	5	5	0,119	0,056
	2	8	8	0,068	0,050
	3	8	8	0,083	0,094
	4	8	8	0,091	0,133
	1/2	8	8	0,119	0,089
	1/3	8	8	0,141	0,151
	1/4	8	8	0,150	0,201
	2	10	10	0,076	0,045
	3	10	10	0,098	0,105
	4	10	10	0,108	0,145
	1/2	10	10	0,133	0,083
	1/3	10	10	0,172	0,166
	1/4	10	10	0,195	0,230
	2	12	12	0,075	0,126
	3	12	12	0,099	0,259
	4	12	12	0,117	0,361
	1/2	12	12	0,146	0,197
	1/3	12	12	0,199	0,363
	1/4	12	12	0,219	0,471
	2	15	15	0,079	0,155
	3	15	15	0,107	0,337
	4	15	15	0,118	0,454
	1/2	15	15	0,167	0,229
	1/3	15	15	0,219	0,435
	1/4	15	15	0,249	0,571
	2	16	16	0,076	0,215
	3	16	16	0,104	0,430
	4	16	16	0,123	0,565
	1/2	16	16	0,170	0,291
	1/3	16	16	0,223	0,533
	1/4	16	16	0,243	0,673
	2	20	20	0,085	0,305
	3	20	20	0,115	0,579
	4	20	20	0,137	0,723
	1/2	20	20	0,202	0,377
	1/3	20	20	0,270	0,662
	1/4	20	20	0,296	0,808

6. RESULTS

According to the analyses of sample pairs obtained from 25 different sample distributions, generally Mann-Whitney test is stronger for the equal and smaller sample sizes from 4 to 10. Kolmogorov-Smirnov two-sample test is stronger for equal and small sample sizes from 4 to 20. It is determined that for all distributions, the highest power values are matched to the standard deviation ratios of 4 or 1/4. It is observed that when standard deviation ratios are increased from 2 to 4, or are decreased from 1/2 to 1/4, powers of both tests tend to increase.

In the case of equal skewness different kurtosis, it is determined that in the sample size (4, 4) for both Mann-Whitney and Kolmogorov-Smirnov two-sample test the highest power value is observed in the sample pairs in which the first sample is obtained from normal and the second sample is obtained from normal Platykurtic distributions. As for the lowest power value in the sample size (4, 4) for both two tests it is observed in the sample pairs in which the first sample is obtained from Platykurtic and the second sample is obtained from Leptokurtic[3] distributions.

It is determined that in the sample size (5, 5) for both Mann-Whitney and Kolmogorov-Smirnov two-sample test the highest power value is observed in the sample pairs in which the first sample is obtained from normal and the second sample is obtained from Platykurtic distributions. In this sample size, the lowest power value is observed in Mann-Whitney test in which the first sample is obtained from normal and the second sample is obtained from Leptokurtic[3] distributions and in Kolmogorov-Smirnov two-sample test in which the first sample is obtained from normal Platykurtic, the second sample is obtained from Leptokurtic[3] distributions.

It is determined that in the sample sizes (8, 8) and (10, 10) the highest power values are observed in the sample pairs in which the first sample is obtained from normal and the second sample is obtained from normal Platykurtic distributions. In these sample sizes for both Mann-Whitney and Kolmogorov-Smirnov tests the lowest power value is in the sample pairs in which the first sample is obtained from normal Platykurtic and the second sample is obtained from Leptokurtic[3] distributions.

It is determined that in the sample sizes (12, 12) and (15, 15) the estimated highest power value is observed in the sample pairs in which the first sample is obtained from Skewed and the second sample is obtained from Skewed and Leptokurtic[1] distributions. In these sample sizes the lowest power value is in the sample pair in which the first sample is obtained from Platykurtic and the second sample is obtained from Leptokurtic[3] distributions. It is determined that in the sample sizes (12, 12) and (15, 15) for Kolmogorov-Smirnov two-sample test the estimated highest power value is observed in the sample pairs in which the first sample is obtained from normal and the second sample is obtained from normal Platykurtic distributions. In these sample sizes the lowest power value is in the sample pair in which the first sample is obtained from normal Platykurtic and the second sample is obtained from Leptokurtic[3] distributions.

It is determined that in the sample sizes (16, 16) and (20, 20) for Mann-Whitney test the estimated highest power value is observed in the sample pairs in which the first sample is obtained from Skewed and the second sample is obtained from Skewed and Leptokurtic[1] distributions. In these sample sizes the lowest power value is in the sample pair in which the first sample is obtained from normal and the second sample is obtained from Leptokurtic[3] distributions.

It is determined that in the sample sizes (16, 16) and (20, 20) for Kolmogorov-Smirnov two-sample test the estimated highest power value is observed in the sample pairs in which the first sample is obtained from normal and the second sample is obtained from normal Platykurtic distributions. In these sample sizes for Kolmogorov-Smirnov two-sample test, the lowest power value is in the sample pair in which the first sample is obtained from normal Platykurtic and the second sample is obtained from Leptokurtic[3] distributions.

As for equal kurtosis different skewness, it is determined that in the sample size (4, 4) for both Mann-Whitney and Kolmogorov-Smirnov two-sample test the highest power value is observed in the sample pairs in which the first sample is obtained from Platykurtic and the second sample is obtained from normal Platykurtic distributions. As for the lowest power value in the sample size (4, 4) for both two tests it is observed in the sample pairs in which the first sample is obtained from normal and the second sample is obtained from Leptokurtic^{2} distributions.

It is determined that in the sample sizes (5, 5), (12, 12) and (16, 16) for both Mann-Whitney and Kolmogorov-Smirnov two-sample test the highest power value is observed in the sample pairs in which the first sample is obtained from Platykurtic and the second sample is obtained from Leptokurtic^{3} distributions. In these sample sizes for both tests, the lowest power value is in the sample pair in which the first sample is obtained from normal and the second sample is obtained from Leptokurtic^{2} distributions.

It is determined that in the sample sizes (8, 8) for both Mann-Whitney and Kolmogorov-Smirnov two-sample test the highest power value is observed in the sample pairs in which the first sample is obtained from Platykurtic and the second sample is obtained from Leptokurtic^{2} distributions. The lowest power value for both tests is in the sample pair in which the first sample is obtained from normal and the second sample is obtained from Leptokurtic^{2} distributions.

It is determined that in the sample sizes (10, 10), (15, 15) and (20, 20) for Mann-Whitney test the estimated highest power value is observed in the sample pairs in which the first sample is obtained from Platykurtic and the second sample is obtained from Leptokurtic^{2} distributions. In these sample sizes, the lowest power value is in the sample pair in which the first sample is obtained from normal and the second sample is obtained from Leptokurtic^{1} distributions. It is determined that in the sample sizes (10, 10), (15, 15) and (20, 20) for Kolmogorov-Smirnov two-sample test the estimated highest power value is observed in the sample pairs in which the first sample is obtained from Platykurtic and the second sample is obtained from Leptokurtic^{2} distributions. In these sample sizes, the lowest power value is in the sample pair in which the first sample is obtained from normal and the second sample is obtained from Leptokurtic^{2} distributions.

According to another result obtained from the study, when skewness coefficient is kept stable and as the kurtosis coefficients decrease, it is observed that the power of both tests is high and as kurtosis coefficients increase the power of tests is low. When kurtosis coefficient is kept stable and as the stewness coefficient increase, it is observed that the power of both two tests is high.

The highest power value (0,808) for power evaluations under equal skewness and different kurtosis and equal skewness and different kurtosis coefficients is reached for the sample pairs of Skewed and Leptokurtic^2 and Skewed-Leptokurtic distributions of Kolmogorov-Smirnov two-sample test when sample-size is (20, 20), and standard deviation ratio is 1/4. The smallest power value (0,009) of the study is observed for the sample pairs of Platykurtic and Leptokurtic^2 distributions of Kolmogorov-Smirnov two-sample test when sample size is (5, 5) and standard deviation ratio was 1/2.

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