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(1/G')-Expansion Method for Exact Solutions of (3+1)-Dimensional Jimbo-Miwa Equation

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ABSTRACT: The purpose of this article is obtaining the exact solutions for (3+1)-dimensional Jimbo-Miwa Equation (3+1DJME). The (1/G')-expansion method which is an effective method in solving nonlinear evolution equations (NLEEs) is used. Then, 3D, contour and 2D graphics are presented by giving special values to the constants in the solutions obtained. These graphics are a special solution of the (3+1DJME) and represent a stationary wave of the equation. Ready computer package program is used to obtain the solutions and graphics presented in this study.

Keywords: (1/G')-expansion method, (3+1)-dimensional Jimbo-Miwa equation, exact solution, traveling wave solution.

(3 + 1) Boyutlu Jimbo-Miwa Denkleminin Tam Çözümleri için (1/G')-Açılım Yöntemi

ÖZET: Bu makalenin amacı (3+1) boyutlu Jimbo-Miwa denklemi için tam çözümler elde etmektir. Lineer olmayan evrim denklemlerinin çözümünde etkili bir yöntem olan (1/G')-açılım yöntemi kullanılmıştır. Daha sonra elde edilen çözümlerdeki sabitlere özel degerler verilerek 3 boyutlu, kontur ve 2 boyutlu grafikler sunulmuştur. Bu grafikler (3 + 1) boyutlu Jimbo-Miwa denkleminin özel bir çözümü olup ve denklemin duragan bir dalgasını temsil etmektedir. Bu çalışmada sunulan çözümler ve grafiklerin elde edilişinde hazır bilgisayar paket programı kullanılmaktadır.

Anahtar Kelimeler: (1/G')-açılım yöntemi, (3+1)-boyutlu Jimbo-Miwa denklemi, tam çözüm, yürüyen dalga çözümü.

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INTRODUCTION

The NLEEs in mathematical physics play a major role in many fields, such as geochemistry, plasma physics, fluid mechanics and solid state physics. The investigation of analytical solutions of NLEEs plays a significant role in the work of nonlinear physical phenomena. Especially in last years, many effective methods have been presented to achieve exact solutions of NLEEs, some of which are new extended direct algebraic method (Rezazadeh et al., 2018), (G'/G) -expansion method (Yokuş and Kaya, 2015; Yokus and Tuz, 2017; Durur, 2020), The tanh–coth method (Wazwaz, 2007), the Clarkson–Kruskal (CK) direct method (Su-Ping and Li-Xin, 2007), Sumudu transform method (Yavuz and Özdemır, 2018), Sub equation method (Durur et al., 2019a), extended sinh-Gordon equation expansion method (Baskonus et al., 2018; Cattani et al., 2018), the modified Kudryashov method (Kumar et al., 2018), $(1/G')$-expansion method (Yokuş and Durur, 2019; Durur and Yokuş, 2019; Durur and Yokuş, 2020; Yokuş et al., 2020a; Yokuş et al., 2020b), Adomian Decomposition methods (Kaya and Yokus, 2002; Kaya and Yokus, 2005; Yavuz and Özdemir, 2018; Yavuz, 2017), collocation method (Aziz and Šarler, 2010), new sub equation method (Kurt et al. 2019), first integral method (Darvishic et al., 2016), improved Bernoulli sub-equation function method (Düşünceli et al., 2020; Dusunceli, 2019), Difference scheme method (Faraj and Modanli 2017; Modanlı, 2019), the modified exp-expansion function method (Yokus et al., 2018), Hirota bilinear method (Manafian, 2018), residual power series method (Durur et al., 2019b), variational iteration algorithms (Ahmad et al., 2020),

In this study, authors attained the exact solutions of the (3+1DJME). Consider the form of the (3+1DJME) (Siddique et al., 2010),

$$u_{xxxy}+3u_yu_{xx}+3u_xu_{xy}+2u_{yt}-3u_{xz}=0. \tag{1}$$

The Jimbo-Miwa equation is first studied by Jimbo and Miwa (Jimbo and Miwa, 1983) and later by several authors. This equation is known as a mathematical model of (3 + 1) dimensional waves in applied sciences and physics. One of the most important features of this equation is that it has soliton solutions. However, it does not carry conventional integration features. Some of these are as follows: Wazwaz have been obtained multiple soliton solutions of two extended (3+1DJME) by applying the simplified Hirota's method (Wazwaz, 2017). Yang and Ma have been obtained Lump-type solutions of the (3+1DJME) by applying Hirota bilinear form (Yang and Ma, 2017). Liu and Jiang have been attained new exact solutions of the (3+1DJME) using the extended homogeneous balance method (Liu and Jiang, 2004). Öziş and Aslan have been obtained analytical and explicit generalized solitary solutions of the (3+1DJME) using the Exp-function method (Öziş and Aslan, 2008). Tang and Liang have been attained variable separation solutions of the (3+1DJME) (Tang and Liang, 2006). Ma has been obtained four classes of lump-type solutions for the (3+1DJME) using Hirota bilinear form (Ma, 2016).

MATERIALS AND METHODS

Method

Consider general form of NLEEs

$$T\left(u,\frac{\partial u}{\partial t},\frac{\partial u}{\partial x},\frac{\partial u}{\partial y},\frac{\partial^2 u}{\partial x^2},\dots\right)=0. \tag{2}$$

Let $u=u(x,y,z,t)=U(\xi)=U,\ \xi=\alpha x+\beta y+\gamma z-ct$ and transmutation Eq. (1) may be converted into following nODE for $U(\xi)$:

$$\mathrm{L}(U,U',U'',UU',U'U'',\dots)=0, \tag{3}$$

where prime refers to derivatives related to ξ. The solution of Eq. (3) is assumed to have the form

$$U(\xi) = a_0 + \sum_{i=1}^{n} a_i \left(\frac{1}{G\prime}\right)^i, \tag{4}$$

where $a_i, (i = 0,1,2, \dots, n)$ are constants, n is a positive integer which is the equilibrium term in Eq. (3) and $G = G(\xi)$ provides the following second order IODE

$$G'' + \lambda G' + \mu = 0, \tag{5}$$

where λ and μ are constants and to be determined later.

$$\frac{1}{G'[\xi]} = \frac{1}{-\frac{\mu}{\lambda} + A\cos h[\xi\lambda] - A\sin h[\xi\lambda]}, \tag{6}$$

where A is integral constant. If the desired derivatives of the Eq. (4) are calculated and replacing in the Eq.(3), a polynomial with the argument $(1/G')$ is attained. An algebraic equation system is created by equalizing the coefficients of this polynomial to zero. These equations are solved using package program and put into place in the default Eq. (3) solution function. Thus, the solutions of Eq. (2) are obtained.

Solutions of The (3+1DJM) Equation

We consider Eq. (1) and using transformation $u = u(x, y, z, t) = U(\xi)$, $\xi = \alpha x + \beta y + \gamma z - ct$, where β, γ, α and c are constants, once obtained (ODE) after integration, we get

$$\alpha^3 \beta U''' + 3\beta\alpha^2 (U')^2 - (2\beta c + 3\alpha\gamma)U' + c_1 = 0, \tag{7}$$

where c_1 is an integration constant to be determined in the future. In Eq. (7), we get term $n = 1$ from the definition of balancing term and the following situation is obtained in Eq. (4),

$$U(\xi) = a_0 + a_1 \left(\frac{1}{G'[\xi]}\right), a_1 \neq 0. \tag{8}$$

Replacing Eq. (8) into Eq. (7) and the coefficients of Eq. (1) are equal to zero, we may establish the following algebraic equation systems

$$\left.\begin{array}{l} Const: c_0 = 0, \\ \frac{1}{G'[\xi]}: -2c\beta\lambda a_1 - 3\alpha\gamma\lambda a_1 + \alpha^3\beta\lambda^3 a_1 = 0, \\ \frac{1}{G'[\xi]^2}: -2c\beta\mu a_1 - 3\alpha\gamma\mu a_1 + 7\alpha^3\beta\lambda^2\mu a_1 + 3\alpha^2\beta\lambda^2 a_1^2 = 0, \\ \frac{1}{G'[\xi]^3}: 12\alpha^3\beta\lambda\mu^2 a_1 + 6\alpha^2\beta\lambda\mu a_1^2 = 0, \\ \frac{1}{G'[\xi]^4}: 6\alpha^3\beta\mu^3 a_1 + 3\alpha^2\beta\mu^2 a_1^2 = 0. \end{array}\right\} \tag{9}$$

Case1.

$$a_1 = -2\alpha\mu, \beta = \frac{3\alpha\gamma}{-2c + \alpha^3\lambda^2}, \tag{10}$$

replacing values Eq. (10) into Eq. (8) and attain the following hyperbolic wave solutions for Eq. (1):

$$u_1(x, y, z, t) = -\frac{2\alpha\mu}{-\frac{\mu}{\lambda} + A\mathrm{Cosh}[\lambda(-ct + x\alpha + z\gamma + \frac{3y\alpha\gamma}{-2c+\alpha^3\lambda^2})] - A\mathrm{Sinh}[\lambda(-ct + x\alpha + z\gamma + \frac{3y\alpha\gamma}{-2c+\alpha^3\lambda^2})]} + a_0. \tag{11}$$

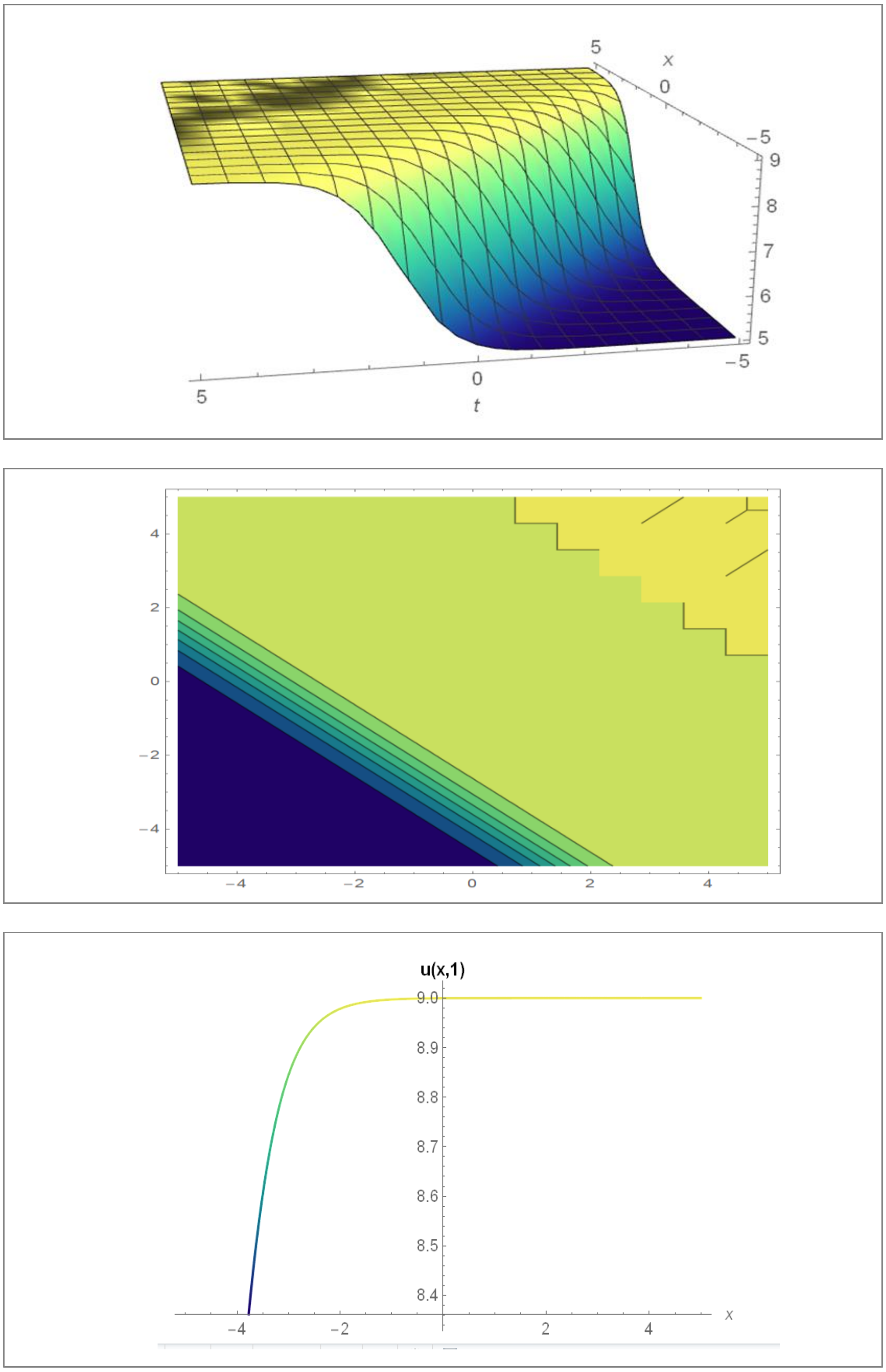

Figure 1. 3D, contour and 2D graphs of $u_1(x,y,z,t)$ respectively for $A=3, \lambda=2, \mu=-1, \alpha=1, \gamma=3, c=-1, y=1, z=1, a_0=5$.

Case2.

$$c_1=0,\ \ \mu=0,\ \ \alpha=0,\ \ \beta=0, \tag{12}$$

replacing values Eq. (12) into Eq. (8) and attain the following exponential wave solution for Eq. (1):

$$u_2(z,t)=a_0+\frac{e^{-ct\lambda+z\gamma\lambda}a_1}{A}. \tag{13}$$

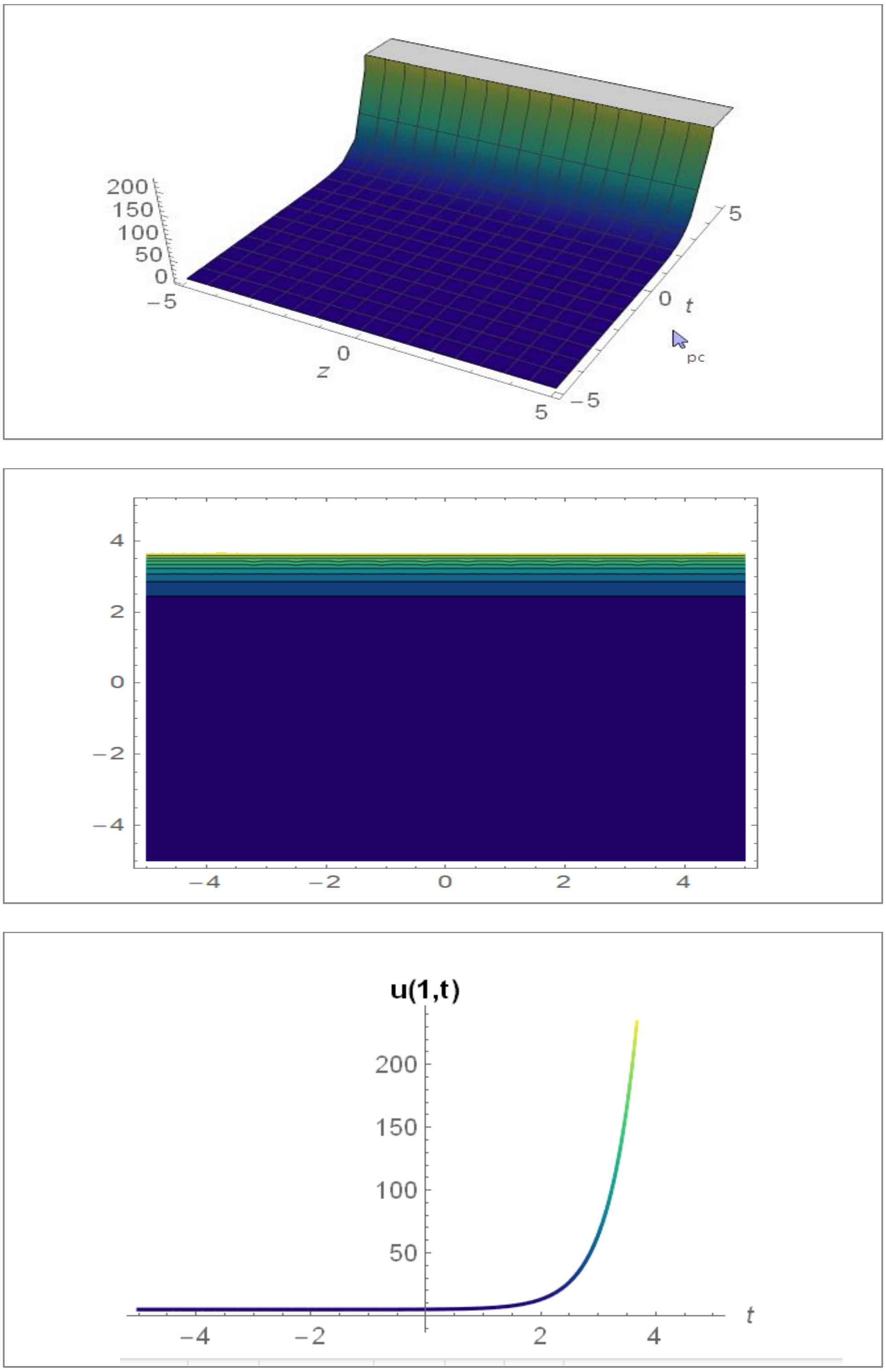

Figure 2. 3D, contour and 2D graphs of $u_2(z,t)$ respectively for $A = 5;\ \lambda = 2;\ c = -1;\ z = 1;\ a_0 = 5;\ a_1 = 0.1;\ \gamma = 1.$

RESULTS AND DISCUSSION

There are several methods to find the analytical solution of NLEEs. One of these methods is the $(1/G')$-expansion method. In this study, we obtained the traveling wave solutions of the the (3+1DJME) by using this method. The solutions obtained in this study are hyperbolic traveling wave solutions. The graphics presented represent the stationary wave. While these graphs are obtained, arbitrary values are given to the constants. The advantage of the method is that a simpler algebraic equation system is obtained compared to other methods. The only disavantage of this method is that it produces a uniform solution function. In the study, it has been shown that this method is easier than other methods in terms of process complexity. So, this method is an effective and easy method to reach the solution. This method can be easily applied to NLEEs.

CONCLUSION

The $(1/G')$-expansion method was used to establish the exacts solution for the (3+1DJME). For the solutions found, 3D, contour and 2D graphics were presented for different values given to the constants. As it is known, each method produces different types of solutions due to its structure. With this method, hyperbolic type traveling wave solutions are produced. For this equation, this method has not been applied and it is aimed to provide different types of solutions to the literature. The advantage of this method is its high reliability and easy application. The disadvantage is that it produces a single type of solution. However, these solutions have an important place in the analysis of the shock wave structure. In addition, having a single point feature is attractive for asymptotic behavior reviewers. In addition, the ready computer package program is used for graphic and computations in this letter.

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