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PERFORMANCE OF TURBO CODED SIGNALS OVER FADING CHANNELS

TURBO KODLANMIÞ ÝÞARETLERÝN SÖNÜMLEMELÝ KANALLARDA BAÞARIM ANALÝZÝ

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ABSTRACT

In this paper, the performance of turbo coded signals are investigated for different frame numbers and encoder structures over Rician fading channels. The numerical results demonstrate the error performance degradation due to fading channel frame numbers and the state of the encoder.

Key Words: *Turbo Coding, Rician Fading Channel*

ÖZET

Bu makale de, farklý çerceve uzunluklarý, kodlayýcý özelliklerine gore deðiþtirilerek turbo kodlanmýþ iþaretlerin Rician ortamlarda baþarýmý incelenmiþtir. Benzetim sonuçlarý, söz konusu Rician ortamda farklý parametreler için deðerlendirilmiþtir.

Anahtar Kelimeler: *Turbo kodlama, Rician Sönümleme Kanalý*

I. INTRODUCTION

Turbo codes are a new class of error correction codes that were introduced a long with a practical decoding algorithm in [1]. The importance of turbo codes is that they enable reliable communications with power efficiencies close to the theoretical limit predicted by Claude Shannon [2]. Since their introduction, turbo codes have been proposed for low-power applications such as deep-space and satellite communications, as well as for interference limited applications such as third generation cellular and personal communication services.

The main principle in the turbo coding scheme is to use two codes in parallel. This means that the information sequence is encoded twice, the second time after a scrambling of the information bits. The component codes are chosen as small convolutional codes in the recursive systematic form. With this encoding we are able to decode the two encoded streams with an iterative process using two soft-in soft-out decoders, one corresponding to each of the encoders. For the simulations shown in this paper we have used a MAP decoder. The decoders exchange information as a priori probabilities for the information bits.

II. SYSTEM MODEL

The block diagram of the channel model that we use in this paper is shown in Fig. 1.The channel output u_k is

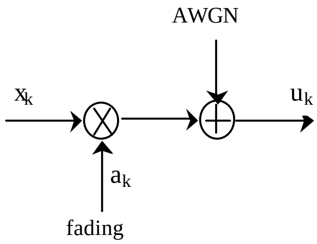

Figure 1. Channel Model

$$u_k = a_k.x_k.+ n_k \tag{1}$$

n_k is Gaussian Noise where the noise variance $\sigma^2 = N_0/2E_s$, a_k is fading amplitude for the signaling interval k.

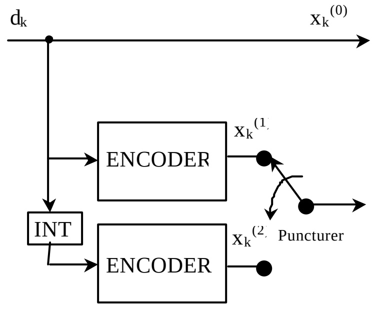

Figure 2. Turbo Encoder for $1 \pm D^n$ / PRFC Model

In the encoder structures we use Recursive Systematic Convolutional (RSC) encoders as Turbo encoders. Consider a half-rate RSC encoder with M memory size. If the d_k is an input at time k the output X_k is equal to

$$X_k = d_k \tag{2}$$

Remainder r(D) can be found using feedback polynomal $g^{(0)}(D)$ and feedforward polynomal is $g^{(1)}(D)$. The feedback variable is

$$r_i = d_k + \sum_{j=1}^{K} r_{k-j} g_j^{(0)} \tag{3}$$

and RSC encoder output Y_k which called parity data [7], is

$$Y_k = \sum_{j=0}^{K} r_{k-j} g_j^{(1)} \tag{4}$$

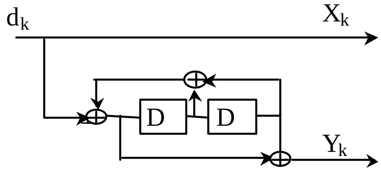

Figure 3. Recursive Systematic Convolutional (RSC) Encoder

RSC encoder with memory M=2 and rate R=1/2 which feedback polynomal $g^{(0)}=7$ and feedforward polynomal $g^{(1)}=5$ thus g[111;101] is illustrated in Fig. 3 and it has a generator matrix

$$G(D) = \begin{bmatrix} 1 & \dfrac{1+D+D^2}{1+D^2} \end{bmatrix} \tag{5}$$

III. TURBO DECODING

The problem of estimating the state sequence of a Markov process observed through noise has two well known trellis-based solutions- Viterbi Algorithm (VA) [8] and the symbol-by-symbol maximum a posteriori (MAP) algorithm. The key difference between algorithms is that the states estimated by the VA must form a connected path through the trellis, while the states estimated by the MAP algorithm need not to be connected. When applied to digital transmission systems, the VA minimizes the frame error rate (FER), while the MAP algorithm minimizes the bit error rate (BER)

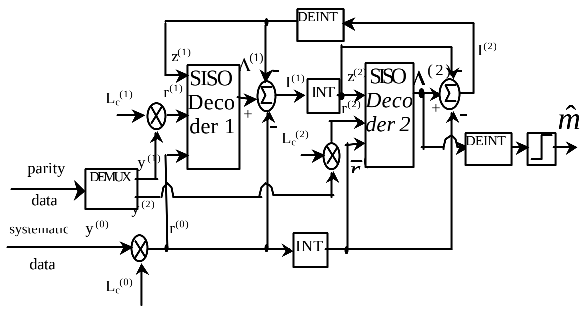

Figure 4. Turbo Decoder

The problem of decoding turbo codes involves the joint estimation of two Markov processes, one for each constituent code. While in theory it is possible to model a turbo code as a single Markov process, such a representation is extremely complex and does not lend itself to computationally tractable decoding algorithms. Turbo decoding proceeds instead by first independently estimating the individual Markov processes. Because the two Markov processes are defined by the same set of data, the estimates can be refined by sharing information between the two decoders in an iterative fashion. More specifically, the output of one decoder can be used as a priori information by the other decoder (Fig. 4). If the outputs of the individual decoders are in the form of hard-bit decisions, then there is little advantage to sharing information. However, if soft-bit decisions are produced by the individual decoders, considerable performance gains can be achieved by executing multiple iterations of decoding [6].

A. Soft-Input, Soft Output (SISO) Decoding

In the MAP decoder, the output is given by

$$\Lambda_k = \ln \frac{P[m_k = 1|\mathbf{y}]}{P[m_k = 0|\mathbf{y}]} \qquad (6)$$

m is message bit and $\mathbf{y}$ is received sequence. There are three inputs for Soft-Input Soft-Output (SISO) decoder. $y_k^{(s)}$ is the systematic observation, $y_k^{(p)}$ is the parity information and z_k is the priori information which is derived from the other decoder's output. The log-likelihood at the output of a SISO decoder using the channel model can be factored into three terms as [6]

$$\Lambda_k = \frac{4a_k^{(s)}\cos(\theta_k^{(s)})E_s}{N_0} y_k^{(s)} + z_k + l_k \qquad (7)$$

where the term l_k is called the extrinsic information, a_k is fading amplitude, E_s is the energy per code symbol and N_0 is the noise power. The priori information at the input of one decoder is found by subtracting two values from its output to prevent "positive feedback problem" as shown in Fig. 4 of M.C. Valenti study [6]. First of all we make the following notation;

$$L_c = \frac{4a_k \cos(\theta_k) E_s}{N_0} \qquad (8)$$

which is called the reliability of the channel, and

$$l_k = \Lambda_k - L_c^{(s)} y_k^{(s)} - z_k \qquad (9)$$

The MAP algorithm attempts to find the most likely individual state s_k given $\mathbf{y}$

$$\hat{s}_k = \arg\left\{ \max_{s_k} \; P[s_k|\mathbf{y}] \right\} \qquad (10)$$

Before finding the a posteriori probabilities (APPs) for the message bits, the MAP algorithm finds the probability of $P[s_k \rightarrow s_{k+1}|\mathbf{y}]$ of each valid state transition given the noisy channel observation $\mathbf{y}$.

$$P[s_k \rightarrow s_{k+1}|\mathbf{y}] = \frac{P[s_k \rightarrow s_{k+1}, \mathbf{y}]}{P[\mathbf{y}]} \qquad (11)$$

B. Log-MAP Algorithm

The maximum a posteriori (MAP) algorithm can calculate the a posteriori probability of each bit with a perfect performance. However there are known problems as the number of calculation depending on the memory states. These problems can be solved by performing the entire algorithm in the log domain [9][10]. To illustrate how performed in the log domain, consider the *Jacobian Logarithm*:

$$\ln(e^x + e^y) = \max(x,y) + \ln(1 + \exp\{-|y-x|\}) \qquad (12)$$
$$= \max(x,y) + f_c(|x-y|)$$

this equation describes the log-MAP algorithm with a correction function f_c.

Let $\bar{\gamma}(s_k \rightarrow s_{k+1})$ denoted the natural logarithm of $\gamma(s_k \rightarrow s_{k+1})$

$$\bar{\gamma}(s_k \rightarrow s_{k+1}) = \ln \gamma(s_k \rightarrow s_{k+1}) \qquad (13)$$

$$= \ln P[m_k] + \ln P[y_k|x_k] \qquad (14)$$

$$\ln P[m_k] = z_k m_k - \ln(1 + e^{z_k}) \qquad (15)$$

and (14) becomes

$$\bar{\gamma}(s_k \rightarrow s_{k+1}) = \ln P[m_k] - \frac{1}{2}\ln(\pi N_0 / E_s)$$
$$- \frac{E_s}{N_0} \sum_{q=0}^{n-1} \left[y_k^{(q)} - (-abs(a_k^{(q)} x_k^{(q)} \cos(\theta_k^{(q)}) + 1) \right]^2$$
$$= l(s_k \rightarrow s_{k+1}) \qquad (16)$$

Now let $\bar{\alpha}(s_k)$ be the natural logarithm of $\alpha(s_k)$,

$$\bar{\alpha}(s_k) = \ln \alpha(s_k) \qquad (17)$$

$$= \ln\left\{ \sum_{s_{k-1} \in A} \exp[\bar{\alpha}(s_{k-1}) + \bar{\gamma}(s_{k-1} \rightarrow s_k)] \right\} \qquad (18)$$

$$= \max_{s_{k-1} \in A} * [\bar{\alpha}(s_{k-1}) + \bar{\gamma}(s_{k-1} \rightarrow s_k)] \qquad (19)$$

where A is the set of states s_{k-1} that are connected to the state s_k.

Now let $\bar{\beta}(s_k)$ be the natural logarithm of $\beta(s_k)$,

$$\bar{\beta}(s_k) = \ln \beta(s_k) \qquad (20)$$

$$= \ln\left\{ \sum_{s_{k+1} \in B} \exp[\bar{\beta}(s_{k+1}) + \bar{\gamma}(s_k \rightarrow s_{k+1})] \right\} \qquad (21)$$

$$= \max_{s_{k+1} \in B} * [\bar{\beta}(s_{k+1}) + \bar{\gamma}(s_k \rightarrow s_{k+1})] \qquad (22)$$

where B is the set of states s_{k+1} that are connected to state s_k, and we can calculate the Log Likelihood Ratio (LLR) by using

$$\Lambda_k = \ln \frac{\sum_{S_1} \exp\left[\bar{a}(s_k) + \bar{g}(s_k \to s_{k+1}) + \bar{b}(s_{k+1})\right]}{\sum_{S_0} \exp\left[\bar{a}(s_k) + \bar{g}(s_k \to s_{k+1}) + \bar{b}(s_{k+1})\right]} \quad (23)$$

where $S_1 = \{s_k \to s_{k+1} : m_k = 1\}$ is the set of all state transitions associated with a message bit of 1, and $S_0 = \{s_k \to s_{k+1} : m_k = 0\}$ is the set of all state transitions associated with a message bit of 0.
At the last iteration we make the hard decision by using the second decoder output $\Lambda^{(2)}$,

$$\hat{m}_k = \begin{cases} 1 & if\ \Lambda^{(2)} \geq 0 \\ 0 & if\ \Lambda^{(2)} < 0 \end{cases} \quad (24)$$

IV. PERFORMANCE OF TURBO CODED SIGNALS

In this section, the performance of turbo coded signals are evaluated for different frame numbers and memory constellations over Rician fading channels.(Figures 5-14).

In our example, 1/2 and 1/3 rate turbo encoders with different memory configurations are investigated. Here we use the generator matrix g=[111:101], g=[1101:1111] and g=[11111:10001], a random interleaver is used and the frame size N=500. It is clear that for a constant iteration number, as K increases, performance improves for the same SNR values. And also the frame number is the important effect for improving the performance as shown in Figure 7 and 8.

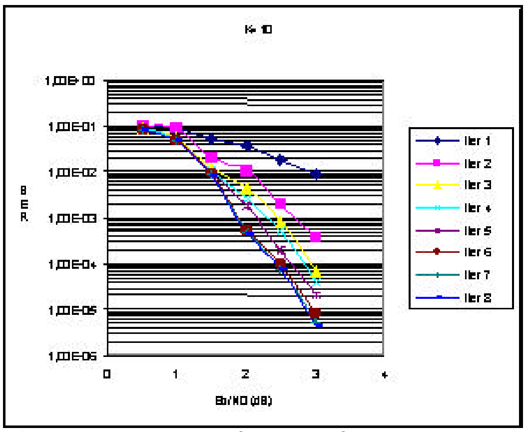

Figure 5. Bit error performance for K=10, g=[111:101] and N=500

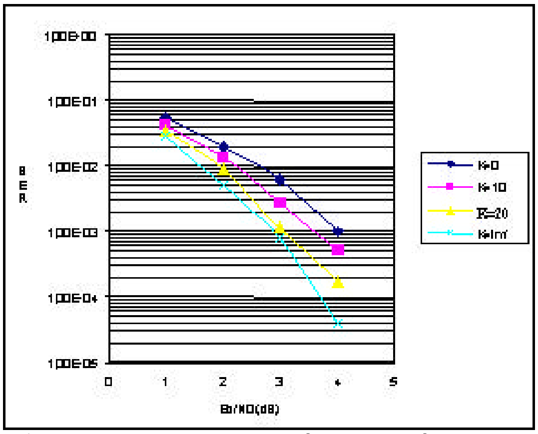

Figure 6. Bit error performance for K=10, g=[111:101] and N=192

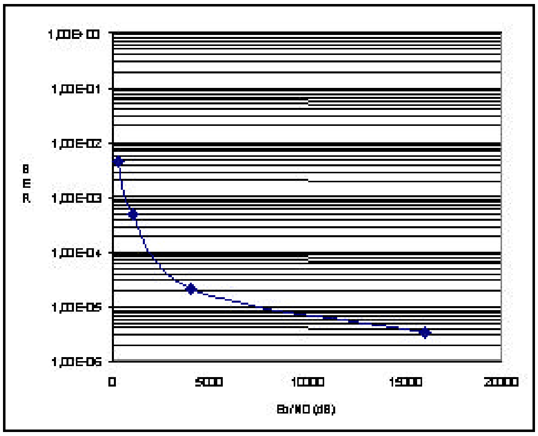

Figure 7. Bit error performance for r=1/3 Eb/N0=4dB g=[111:101] and N=500 over K=0 dB

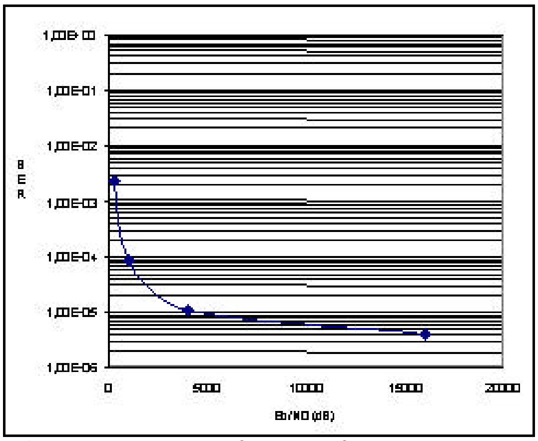

Figure 8.Bit error performance for r=1/3 Eb/N0=1.5dB g=[111:101] and N=500 over AWGN channel

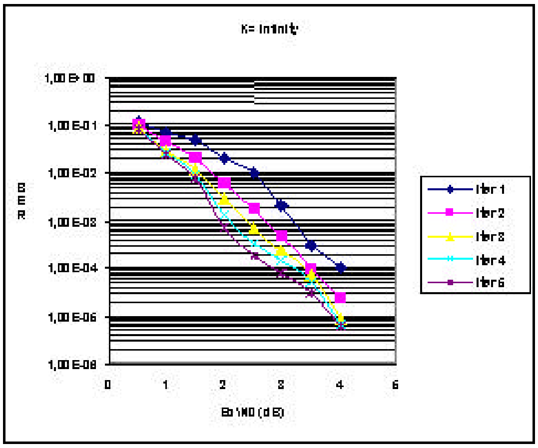

Figure 9. Bit error performance for r=1/2 g=[1101;1111] and N=500 over AWGN channel

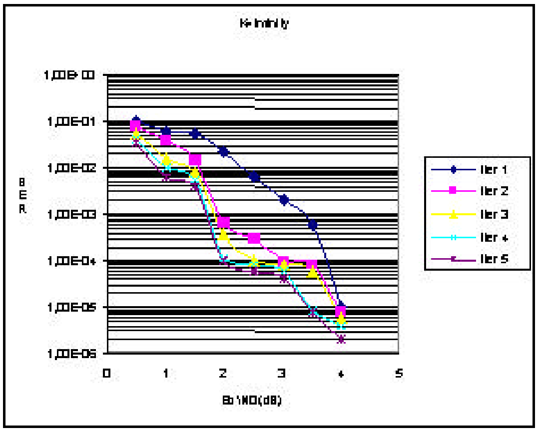

Figure 12. Bit error performance for r=1/2 g=[11111;10001] and N=500 over AWGN channel

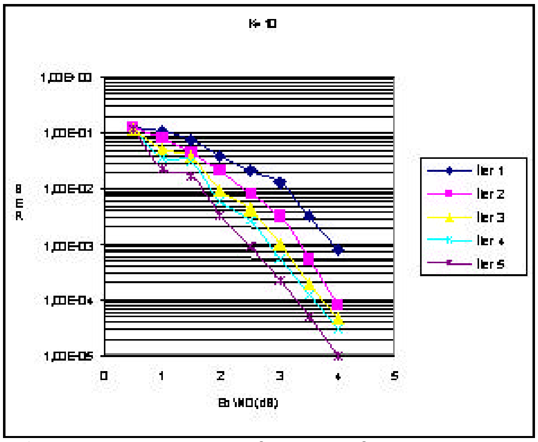

Figure 10. Bit error performance for r=1/2, g=[1101;1111], N=500 and K=10dB

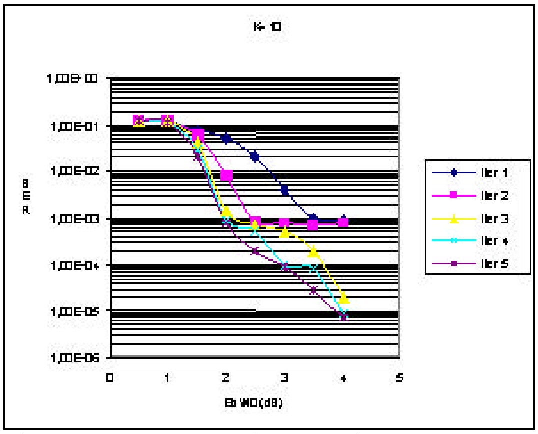

Figure 13. Bit error performance for r=1/2 g=[11111;10001], N=500 and K=10Db

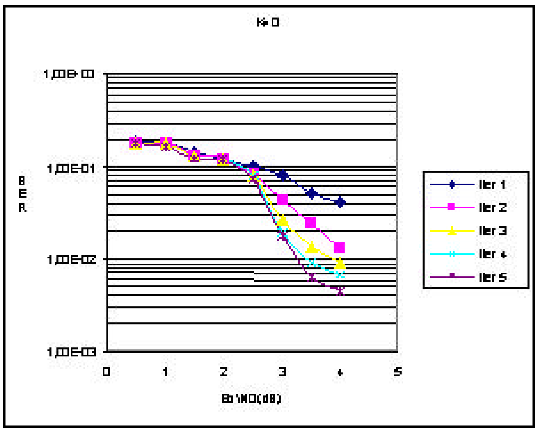

Figure 11. Bit error performance for r=1/2, g=[1101;1111], N=500 and K=0dB

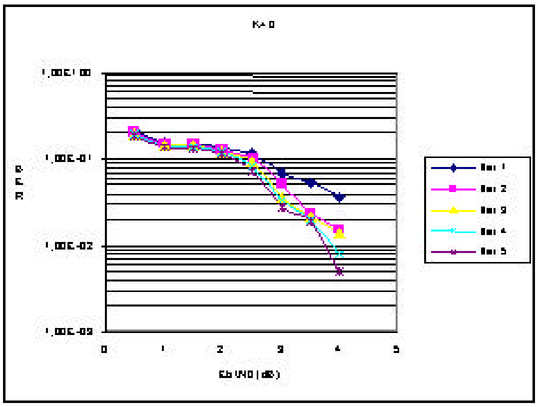

Figure 14. Bit error performance for r=1/2 g=[11111;10001], N=500 and K=0dB

V. CONCLUSION

In this paper we show how encoder structures encoder rate and frame number can be altered the performance while Rician channel is under consideration. As an example, the performance of turbo coded signals are simulated over Rician channel with different fading parameter, iteration number and data block size N.

REFERENCES

[1] C. Berrou, A Glavieux, and P. Thitimasjshima, "Near Shannon-limit error correcting coding and decoding: Turbo codes (1)", in Proc., IEEE Int. Conf. on Commun., (Geneva,Switzerland), pp. 1064-1070, May 1993.

[2] C. E. Shannon, "A mathematical Theory of communication", Bell Sys. Tech. J., vol.27 pp.379-423 and 623-656, 1948.

[3] Jack K. Wolf, Gottfried Ungerboeck,"Trellis Coding for partial -response channels", IEEE Trans. On Commun. Vol. COM-34, No.8, pp.765-773, August 1986.

[4] Mohamed Siala and Ghassan K. Kaleh,"Block and trellis codes for the binary (1-D) partial response channel with simple maximum likelihood decoders", IEEE Trans. On Commun. Vol.44, No.12, pp.1613-1615, December, 1998.

[5] E. Zehavi and G. Kaplan,"Phase noise effects on M-ary PSK trellis codes", IEEE Trans. Commun. COM-39, pp. 373-379, 1991.

[7] M.C. Valenti, "Iterative detection and decoding for wireless communication", prelim report, Virginia Tech, Blacksburg, Virginia, September 1998.

[8] Viterbi, A.J. "Principles of Coherent Communication", McGraww-Hill Book Company, New York, 1966.

[9] W. E. Ryan, "A Turbo Code Tutorial", preprint: submitted to Globecom 1997.

[10] J. Hageneauer, "Iterative decoding of binary block and convolutional codes", IEEE Trans Inform. Theory, vol. 42, pp. 429-445, Mar. 1996

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